CHALLANGES AND CONSEQUENCES OF A POOR POSITION UPDATE OF A SUBMARINE INS IN THE LITTORALS

BY

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Plagiarism Statement

I confirm that this is my own work and does not break the University, school or module conventions on plagiarism as outlined on the School MSc Civil Engineering/Infrastructure Handbook 2014-2015.

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Abstract

There are two main success criteria's within Submarine operations; Safe navigation and the ability to stay undetected.

A Submarine must be able to solve a wide range of operations, from peacetime engagements to war-scenarios. In order to conduct all types of operations the demand for precise navigation is high.

In addition to navigation, does the positioning system also updates the Command & Control- and weapon system.

The employment of both Submarines and surface ships are affected by the physical environment and weather conditions. Operating in a deep open ocean poses different challenges then operating close to shore, or in the littorals.

The high accuracy requirements combined with the need for covered submerged operations is a challenge when it comes to design of the Navigation systems for Submarines. As surface ships have a continued GNSS access, the Submarines does not. There are several techniques and modes of navigating a Submarine submerged (Terrestrial, optical (periscope), use of depth counters, Navigation Sonar etc). However, Dead reckoning by an accurate INS (Inertial Navigation System) is the main navigation method. The INS provides high bandwidth and precision attitude, with sensor quality varying with the size of the submarine and mission length.

The Submarine might be forced to go deep before a proper GNSS fix is obtained. There are mainly two reasons for this: Either to stay undetected (Tactical issue), or in order to avoid a collision (Safety issue). In these cases there will be an inaccurate update of the INS.

The study aims at determining the challenges and consequences of a poor position update of a Submarine INS in the littorals. For this purpose, analyses have been conducted in Inertial Explorer and Excel.

Comparing the position accuracy of different manipulated position updates showed that both different standard errors and position errors affects the performance of the INS/IMU. Some of the trials showed the multipath sometimes have a positive effect on the absolute accuracy.

The INS/IMU complies well with GNSS-updates, even with a few epochs

The analysis showed that it is challenging to the INS/IMU to navigate for long periods without GNSS-input, what is the case for a Submarine.

Acknowledgements

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1.0 Introduction

1.1 Background

Submarine operations in littoral waters can be challenging. High manoeuvrability is critical in shallow and confined waters. Some submarines are able to manoeuvre in a water column of only two to three ship lengths. Operating submerged in these environments for up to 30 days requires accurate navigation. There are several techniques and modes of navigation submerged (Terrestrial, optical, etc). The core of an underwater navigation system is inertial navigation, integrated with a depth pressure sensor and either doppler sonar or a sonar correlation velocity log. The INS provides high bandwidth and precision attitude, with sensor quality varying with the size of the submarine and mission length (Groves 2013). However, during time the accuracy of the INS will decrease, and the accuracy of the INS is depending on the quality of the position update. The nature of submarine operations is to stay undetected. In order to stay undetected the submarine might have to go deep before the INS is updated with an accurate position.

The main success criteria's within submarine operations are safe navigation and the ability to stay undetected. However, sometimes it is necessary for the submarine to expose masts above the sea surface in order to snort (charge batteries), communication or in order to update a position fix. During this time, the Submarine is vulnerable regarding detection.

Navigation is only a small part of Submarine operations, and just a tool in order to get the Submarine in the right position in order to solve its mission. However, it is an important part. If the navigation fails, the mission fails.

There are two main differences between navigation a surface ship and a submarine. A surface ship does only need to solve for two-dimensional navigation (X and Y in a plan) (Heading and speed). The submarine needs to solve for three-dimensional navigation, adding the Z-vector (depth). Further, a surface ship will normally have continuous access to GNSS and other visual navigation aids. When the Submarine is submerged it uses its Inertial Navigation System (INS) as its primary navigation aid, combined with other techniques for navigation.

Operating in the Littorals is challenging in several ways. The submarine is operating in a narrow fjord with normally high traffic density. It is a high risk of detection both from other ships, but also ashore. Operating in narrow fjords surrounded by high steep mountains is also challenging for the GNSS-equipment. Both Multipath and satellite geometry might be challenging. The environment might also extend the Time to First Fix (TTFF), depending on the period the submarine has been submerged. In addition, the submarine antenna is only a few cm above the sea surface, and in rough weather, it is flushed with water.

1.1.1 Threats

Due to the threat (Figure 1) of either being detected (tactical) or in order to avoid collision (safety), the submarine might be forced deep before a proper GNSS-fix is obtained. In these situations the INS will be updated with an incorrect position and/or attitude.

Figure 1: *Threats to the Submarine*

1.1.2 Lack in literature

The accuracy performance of INS is well documented, and several experiments regarding this has been conducted. However, most of these experiments does only cover the performance based on an accurate position input to the INS, and cover systems with a continuously GNSS input. The same applies for the performance of GNSS, and the influence by the environment.

However, there is a lack in literature when combining those two issues and look at the consequences of a poor GNSS update of the INS.

In this project I will look at the influence of a poor GNSS update of an INS in a submarine context.

1.2 Aims and Objectives of the Research

The aim for this project is to reveal the consequences of poor position fixes into an INS. The thesis will be based on analyses of an experiment with a simulated submarine track processed in Inertial Explorer. A second trajectory is also conducted, simulating submarine behavior by car in order to determine:

1. The decrease of accuracy in an INS over time
2. The impact of a poor GNSS update of an INS both regarding;
    a. Position
    b. Attitude
3. Which error has the greatest impact
    c. Position
    d. Attitude
4. If velocity have an impact on the accuracy of the accelerometers

1.2.1 Hypothesis

The following Hypothesis is made for this project:

1. The Accuracy of an INS is depending on the quality of the position update
2. The precision of an INS will decrease over time
    a. A submarine operates submerged for a long period without any position update of the INS

3. The quality of a GNSS fix is influenced by the environment

    b. Multipath
        i. Surroundings (Mountains)
        ii. Sea-surface
    c. Satellite geometry
    d. Time to Frist Fix (TTFF)
5. A poor GNSS fix will give a poor update of an INS
    e. Position
    f. Attitude (Heading)
6. A poor attitude input will influence the INS more than a poor position input
7. The impact of a poor GNSS-update of one IMU will have the same effect on all INS of the same grade (In this case Navigation grade IMU)

1.2.5 Report Layout

Chapter 1

The focus of the project is related to the challenges connected to operating a submerged Submarine in a difficult GNSS-environment for a long period of time. The aims and objectives of the research is also presented.

Chapter 2

Chapter 2 covers the review of previous work literature.

Chapter 3

This chapter describes the experiment designs. This includes methodology, hardware and software.

Chapter 4

The results and analysis are presented in this chapter. The results are divided into three different experiments; 1) The effect of different time-intervals with GNSS-availability. 2) Influence by different standard errors. 3) The consequences of a poor GNSS-update.

Chapter 5

Conclusions and further work

1.3 Challenges with the project

Due to the sensitivity, both military and from the industry it is not possible to conduct the experiment 100% realistic.

Because it was not possible to conduct the test on a submarine, and that the real performance data of a Submarine is unavailable, the following adjustments have been made:

- A trajectory was driven by the NGI van in order to collect IMU data.
- The trajectory was processed, and simulated in Inertial Explorer with Submarine behaviour.
  - Simulating submerged transit without GNSS input.

2 Literature review

A section regarding Kalman smoothing and INS/GNSS integration is added to the Literature Review.

2.1 Navigation

There are several different ways to describe what Navigation actually is. Navigation is defined by Oxford dictionaries as «The Process or activity of accurately ascertaining one's position and planning and following a route». Implied in this definition, the object is mobile. Further, two main concepts needs to be solved. The first concept is the determination of position and velocity of the object in reference to a known point, known as the science of navigation. The second concept is to plan and conduct the movement from one position to another avoiding any obstacles and collisions, known as the art of navigation (Groves 2013). Positioning might be described as the determination of the position of an object. However, positioning is also a part of navigation. There are different ways to determine a position i.e geometry, astronomy, Dead reckoning, GNSS etc. Xiaolin et. al. (2013) has divided navigation into two categories. The first category is Absolute navigation which Calculate current navigation information with no need for previous input. The second category is relative navigation which obtain navigation parameters relaying on previous input. Means of relative navigation is Dead reckoning, INS and visual odometry. There are different positioning techniques. Navigation requires real-time positioning (Groves 2013). A navigation system determines position and velocity automatically. An integrated navigation system uses more than one technology. A navigation sensor is used to give input to the navigation system. This could be doppler/EM-log (speed), GPS (position), Accelerometers (pitch/roll), Gyro (Heading) etc. Based on the inputs from the navigation sensors, the navigation system calculates its output: Navigation solution (Groves 2013).

There are two main differences between navigating a ship and a submarine. First, a ship does only need to solve for two-dimensional navigation (X, Y in a plan (Heading and speed), as the submarine needs to solve for three-dimensional navigation, adding the Z-vector (depth). Second, a ship normally has continues access to GNSS and other visual navigation aids. The submarine, when submerged uses its inertial navigation systems, combined with other techniques for navigation.

2.2 Inertial Navigation System (INS)

2.2.1 General

Inertial Navigation system is in its simple term a self-contained system consisting of two sets of sensors (accelerometers and gyroscopes) that measures three linear accelerometers and three angular rates (Hasan, Samsudin, Ramli and Azamir 2010). Groves (2013) refers to the INS as an example of Dead-reckoning navigation system. These are similar descriptions that underlines the main functions of an INS; a position solution based on integration of velocity and attitude measured by the accelerometers. The system is self-contained, meaning that the navigation can continue without further information from the environment.

The INS call upon inertial detectors (gyros and accelerometers) attached to a platform, which can be either stabilized (gimbaled system), or linked to the platform ("strap-down" systems). In a strap-down system, all output values calculated are based on information delivered by the angular rotation (rate gyros) and linear acceleration (accelerometers) detectors. (SAGEM SA)

An INS consist of an inertial measurement unit (IMU) and a navigation processor (Groves 2013).

Most modern submarines are equipped with a "Strap-down" inertial navigation system.

2.2.2 Inertial Sensor

Accelerometers and gyroscopes (gyros) constitute the inertial sensor in an INS. An accelerometer measures the specific force in a direction, and the gyro measures the angular rate. Both without an external reference. Normally an INS consists of three accelerometers and three gyros. An Inertial measurement unit (IMU) combines the measurements and gyros and produces a three dimensional measurement of specific force and angular rate. In an INS the IMU is the sensor which produces an independent three-dimensional navigation solution (Groves 2013)

2.2.2.1 Accelerometers

2.2.2.1.1 General

An accelerometer in its simplest way consist of proof mass which is free to move inside the accelerometer case along the sensitive axis (figure 1). The proof mass is restrained with springs (suspension). A pick off measures the position of the proof mass with respect to the case. When the case is under influence of an accelerating force, the proof mass will continue with it's initially velocity. The accelerating force will influence the springs attached to the proof mass by stretching one spring and compressing the other. The case will move in respect of the proof mass until the stretching/compressing of the springs matches the force from the acceleration. The position of the proof mass relatively to the case is proportional to the acceleration applied to the case. By measuring the movement of the proof mass relatively to the case, the acceleration is measured (Grove 2013).

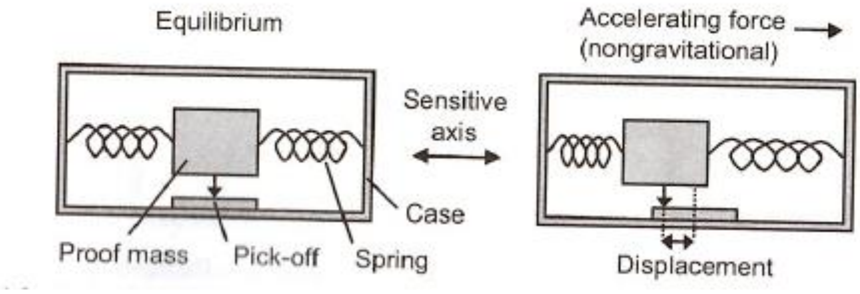

Figure 2: *Simple accelerometer (Grove 2013 page 139)*

2.2.2.1.2 Pendulous Accelerometers

An accelerometer can be linear or pendula. A pendula accelerometer includes the following components (Figure 3):

- A pendulum (B) suspended by a flexible blade (L).
- A torque motor made up of two permanent magnets (A) an the armature winding carried by the pendulum itself)
- A capacitance wheatstone bridge pendulum position detector (C).

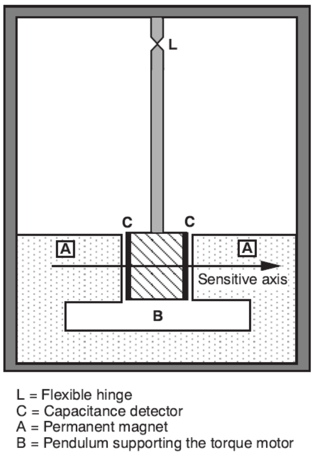

Figure 3: *Constitution of a pendular accelerometer (SAGEM SA)*

When there are no forces, the pendulum is centred and the capacitance bridge is balanced. When subjected to an acceleration, the pendulum moves because of its interia which disbalances the bridge, causing a current proportional to the specific force to be delivered. The specific force is the total of the acceleration linked to any speed variations and the force of gravity. This current, via a slaving circuit, excites the torque motor in order to set the pendulum back to its zero position. It is also digitized so as to be processed by the computer (Sagem SA)

2.2.2.2 Ring laser gyro

A Ring laser gyro consist of a closed loop tube with at least three arms. The tube is filled with a mixture of helium- and neon gas. In each corner of the tube there is a high reflectivity mirror. A cathode and Anode is used to generate an electric field, and thereby apply a high potential difference across the gas.

A ring laser has two laser beams, one in each direction. If there is no rotation, the beams have the same wavelength. A certain rate of rotation induces a small difference between the time it takes light to travers the tube in the two directions. This gives a difference between the frequencies of the two beams. This means that the length of the beam is increased for the beam in the direction of the rotation, and decreased for the beam in the opposite direction (Groves 2013)

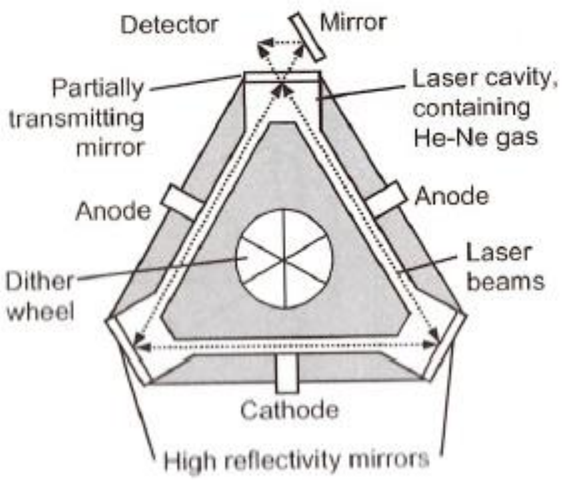

Figure 4: *Ring laser gyro (Groves 2013 page 144)*

The main purpose of a ring laser gyro is to measure a rotational speed about an axis. The SAGNAC effect (figure 5) is a usual method to measure the rotational speed.

When to light beams F+ and F-, originating from the same point A, travel the same closed optical path in opposing directions, the length of their trajectories is identical if the optical system is at rest. If the optical system is in rotation during the time it takes the beams to travel the optical path (point A moves to point A'):

- Beam F+ will travel one turn increased by the distance AA'
- Beam F- will travel one turn decreased by distance AA'

The difference in distance covered will thus be 2AA'. This difference in length is proportional to the dimensions of the optical path and the rotational speed.

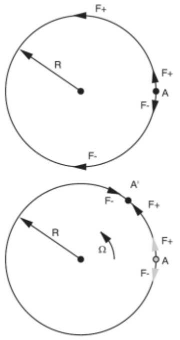

Figure 5: *Principle of SAGNAC effect (SAGEM SE)*

Similarly, the oscillation frequency of both beams is identical for a nil rotational speed. The detection of light beams is obtained by counting the fringes resulting from the interference of the two F+ and F- beams. Counting the fringes indicates the direction and magnitude of the rotation (SAGEM SA)

2.2.2.3 Inertial Measurement Units (IMU)

A typical Inertial Measurement Unit (IMU) consist of: Accelerometers and gyroscopes, the IMU processor, a calibration-parameters store, a temperature sensor and associated power supplies.

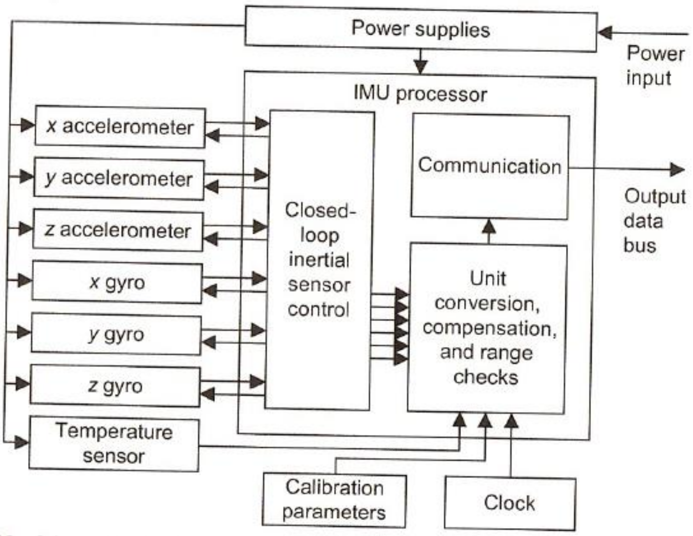

Figure 6: *Schematic of an inertial measurement unit (Groves 2013 p.148)*

Most submarines are equipped with an *Indexed IMU.* In an Indexed IMU the inertial sensor assembly is regularly rotated with respect to the casing, usually in increments of 90°. The rotation is typically performed about two axes or only about the vertical axis (Groves 2013 p. 214). The position and velocity errors developed from the bisases in the accelerometers and gyros are cancelled by the indexing. The growth in the position and velocity errors depends on the attitude of the IMU body frame with respect to the resolving axes of the navigation solution. By reversing the direction of the inertial sensor's sensitive axis, , its biases will lead to pivotal position and velocity errors instead of continuously growing errors. In order to achieve this result, it is more practical to turn the inertial sensor assembly than the submarine. The gyro scale factor and cross-coupling errors will increase attitude errors. To prevent this the rotations about a particular axis should average to zero over time (Groves 2013).

2.2.3 Kalman filter

To determine the values of different parameters in a system, a state estimation algorithm is used. The input to the algorithm is measurements of the properties of the system, such as position and velocity. The Kalman filter forms the basis of most state estimation algorithms used in navigation systems (Groves 2013). The state estimation is important in order to obtain a good navigation solution. A Kalman filter does not just use the most recent measurements, but all measurement inputs over time. To obtain optimal estimates given the information available, the Kalman filter uses knowledge of the deterministic and statistical properties of the system. To enable optimal weighting of the data, the Kalman filter maintains a set of uncertainties in its estimates and a measure of the correlations between the errors in the estimates of the different parameters.

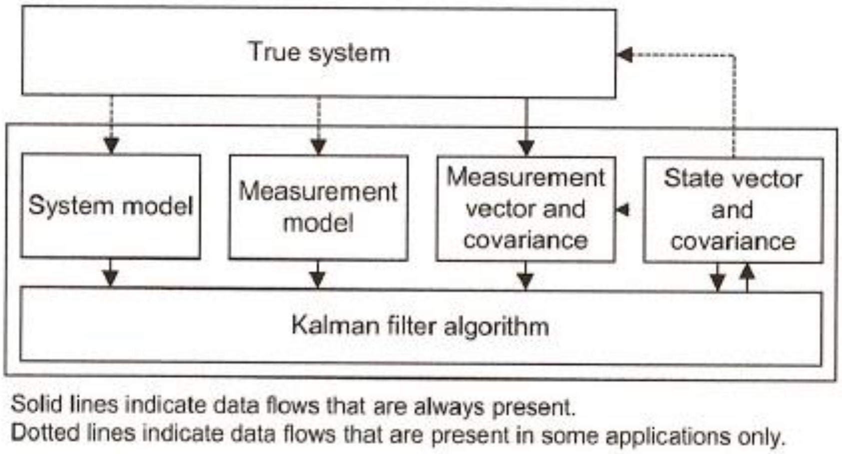

Figure 7: *Elements of the Kalman filter (Groves 2013 page 83)*

The state vector or state, is the set of parameters that describes a system. The states include the components of position or position error. It can also include velocity, attitude and navigation error status. The Kalman filter estimates the states.

The system model describes how the Kalman filter states and error covariance matrix varies over time. Measurement vector is a set of simultaneous measurements of properties of the system which are functions of the state vector. This may be range measurements from a navigation system and differences between an INS under calibration and a reference navigation system. Measurement model describes how the measurement vector varies as a function of the true state vector in the absence of measurement noise. The Kalman filter algorithm uses the measurement vector, measurement model and system model to maintain optimal estimate of the state vector (Groves 2013).

2.2.3.1 Kalman Smoothing

The smoothing has great effect during GNSS-outage areas. Smoothing effectively halves the period of INS drift during GNSS outages, reducing the maximum position error by a factor of up to 4 (Groves p.622).

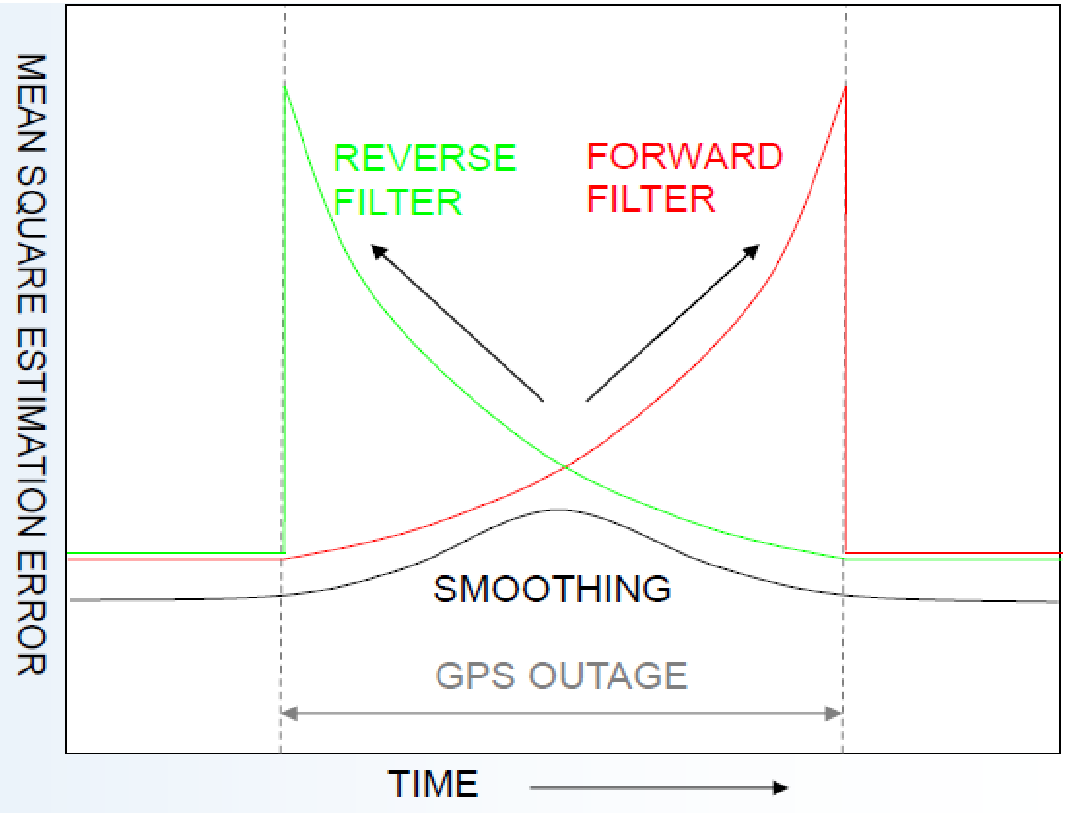

***Figure 8**: GNSS-IMU Smoothing (Dr. Xiaolin Meng, University of Nottingham)*

The Kalman filter smoother is the extension of the Kalman filter that uses measurement information from after the time which state estimates are required as well as before that time. This leads to more accurate state estimates for nonreal-time applications (Groves p.129). In Inertial Explorer the forward-backward filter is used.

The forward-backward filter comprises two Kalman filters, a forward filter and a backward filter. The forward filter is a standard Kalman filter. The Backward filter is a Kalman filter algorithm working backward in time from the end of the data set to the beginning.

Figure 8 shows how the state uncertainty varies with time for the forward, backward and combined filters (Groves p. 130).

2.3 GNSS

2.3.1 Multipath

Multipath is described as “a satellite-emitted signal arrives at the receiver by more than one path” (Hofmann-Wellenhof, Lichtenegger, Wasle, 2008 page 155). The main reasons that multipath occurs is reflecting surfaces near the receiver, secondary it can be reflections at the satellite during the transmission of the signal.

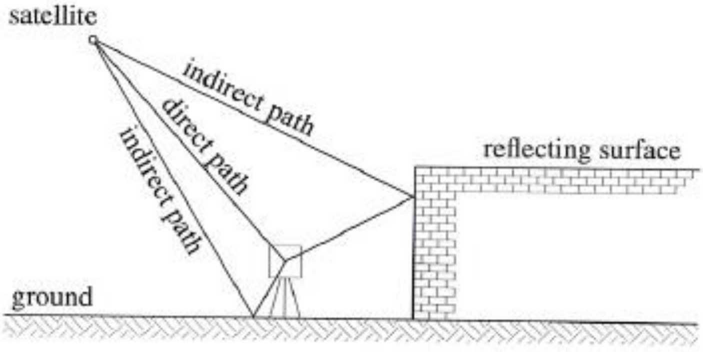

Figure 9: *Multipaht ((Hofmann-Wellenhof, Lichtenegger, Wasle, 2008 page 155).*

Multipath errors of pseduoranges can be divided into three classes (Tranquilla and Carr 1990).

1. Diffuse forward scattering from a widely distributed area (e.g., the signal passes through a cluttered metallic environment)

2. Specular reflection from well-defined objects or reflective surfaces in the vicinity of the antenna.
3. Fluctuations of very low frequency usually associated with reflections from the surface water.

As the submarine often operates in narrow fjords, surrounded by steep mountains class nr 2 might have an influence on the multipath error. Due to the nature of submarines, the receiver antenna has low altitude, and surrounded by water, class nr 3 will most likely have an influence regarding multipath. Since pseudo range is the only actual measurements done by the submarine receiver, I will not cover carrier-phase observations.

The geometry of satellites has a great impact on multipath. Signals received from a satellite with low elevation are more sensitive to multipath than signals from a satellite with high elevation.

Figure 10: *Submarine on periscope depth (photo: Danish Navy)*

2.3.2 Time to first fix (TTFF)

The TTFF parameter defines the time needed for a GNSS receiver between power on and providing the first position information (Hofmann-Wellenhof, Lichtenegger, Wasle, 2008, p: 429). As the submarine operates submerged for a certain time, the TTFF might have a big impact on how quick an accurate position is obtained. Several elements have an effect on the TTFF, such as surroundings (mountains etc), Multipath and satellite visibility. Normally it is possible to get a SPS position within 30 seconds, but disturbing environment might extend this time significantly, or even make it impossible to generate a navigation solution (Li, Zhang, Dempster and Rizos 2011). The GPS receiver can be in three different states: Cold Start, Warm Start or Hot start, all depending on when it was last updated (ephemeris, almanac etc).

Cold start is denoted as a receiver with no information regarding the last computed position and the current time. This means that no ephemeris or almanac data is available. This state occurs when the receiver start up. When the position and the current time is known, the almanac data is valid, but the ephemeris data has expired it is called a Warm start. Typically condition when continuous RTC (real time clock) operates with an accurate last known position available in memory. Hot start is when position, velocity, time and satellite ephemeris is available in the receiver memory. This state may reflect a software reset after a period of continuous navigation (Li, Zhang, Dempster and Rizos 2011). The main difference between these states is the time to get a position. Typically, with hot start it takes a short period (seconds).

Normal condition for a submarine is warm start, as the almanac can be stored in the receiver for about a month. Normally a conventional submarine will get a position fix at least once every 24 hours. When the GNSS antenna is raised above sea level the code phase and Doppler shift for the first four signals must be determined by trial and error. Then the ephemeris data from each satellite has to be downloaded. This process might take more than a minute.

2.3.3 Variance/Covariance

2.3.3.1 Variance

Variance is defined by Investopedia.com as "*A measurement of the spread between numbers in a data set. The variance measures how far each number in the set is from the mean. Variance is calculated by taking the differences between each number in the set and the mean, squaring the differences (to make them positive) and dividing the sum of the squares by the number of values in the set.*"

$$\sigma^2 = \frac{\sum (X - \mu)^2}{N}$$

2.3.3.2 Covariance

Collins dictionary defines Covariance as a measure of the association between two random variables, equal to the expected value of the product of the deviations from the mean of the two variables, and estimated by the sum of products of deviations from the sample mean for associated values of the two variables, divided by the number of sample points. Written as *Cov* (*X, Y*)

2.4 GNSS/INS integration

2.4.1 INS Integration Architectures

There are three main differences regarding the architecture of an INS/GNSS integration:

1. How corrections are applied to the inertial navigation solution
2. What types of GNSS measurements are used
3. How the GNSS user equipment is aided by the INS and integration algorithm

The Inertial Navigator Algorithm can calculate position, velocity and attitude from the IMU outputs. The Kalman filter compares these quantities with the same ones from the GPS, and uses the differences to estimate the errors in the IMU outputs.

2.4.2 Open loop

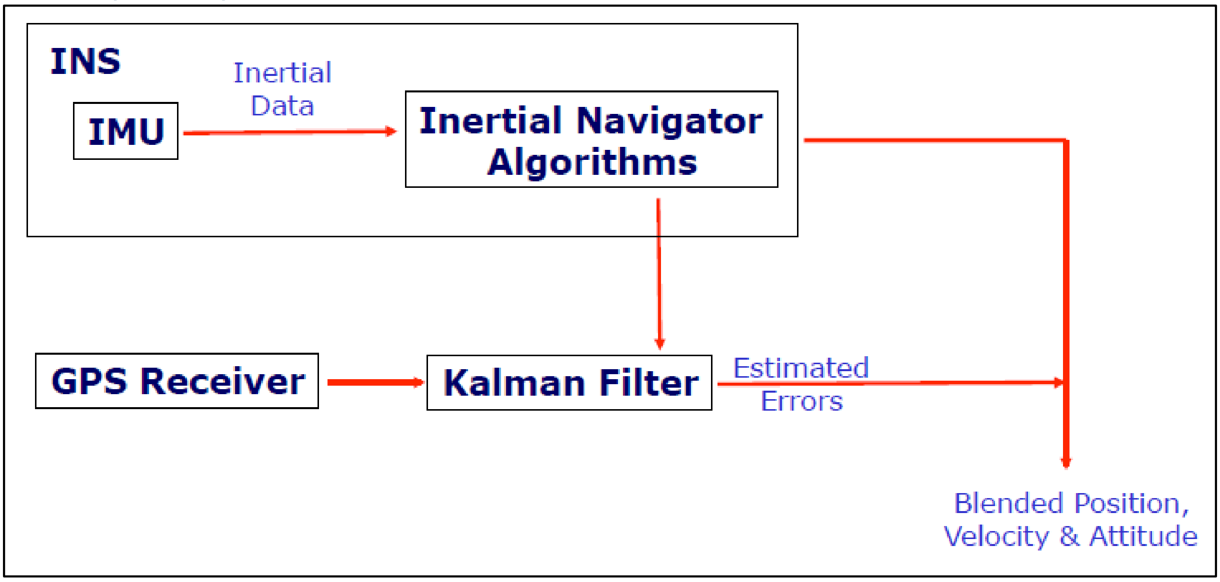

Figure 11: *Open loop (Dr. Chris Hill, University of Nottingham)*

In the open loop scenario the estimated IMU errors become corrections that can be applied to the IMU outputs. The corrections can be used to calculate corrections to the position, velocity and attitude.

Figure 11 show that the corrections are mixed with the output of the Inertial Navigator Algorithms to form the blended solution. In the open loop the IMU errors are allowed to grow and grow, and therefor the corrections always have to account for the full size of the errors.

2.4.3 Closed loop

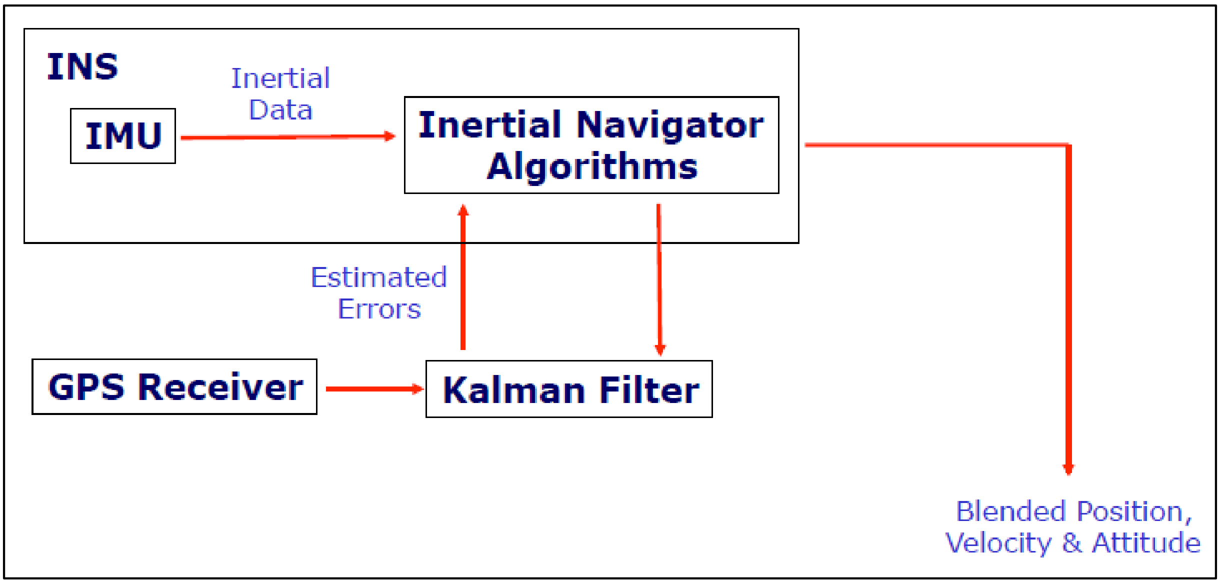

Figure 12: *Closed loop (Dr. Chris Hill, University of Nottingham)*

In a closed loop scenario the IMU errors are corrected at each step, as shown in figure 12. That means that the corrections only have to account for the small growth in the errors since last step.

2.4.4 INS/GNSS integration

In the literature different terms of coupling is used. However, there is no commonly agreed definition of these terms.

The most widely used definition are:

2.4.5 Loosely coupled INS/GNSS system

In a Loosely coupled INS/GNSS solution, the GNSS receiver outputs a position and velocity solution, normally the result of a Kalman filter. The integration filter estimates the INS errors (Integrated solution = INS+corrections). In a Loosely coupled solution there is a need of minimum four satellites in order to obtain a navigation solution from the GNSS receiver.

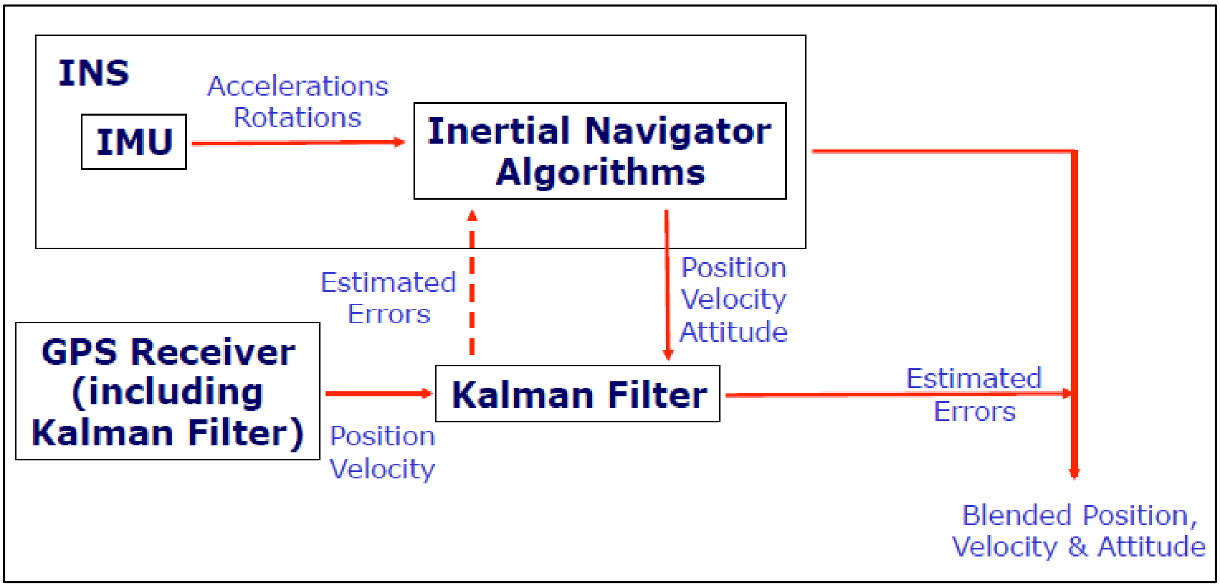

Figure 13: *Loosely coupled INS/GNSS Integration (Dr. Chris Hill, University of Nottingham*

2.4.6 Tightly coupled integration

In a Tightly coupled INS/GNSS solution, the GNSS receiver outputs pseudorange and pseudorange-rate observations. In a Tightly coupled solution the GNSS receiver mainly works as a measurement engine. The main advantage of Tightly coupled integration compared to Loosely coupled integration is the need of only one satellite in order to obtain a solution.

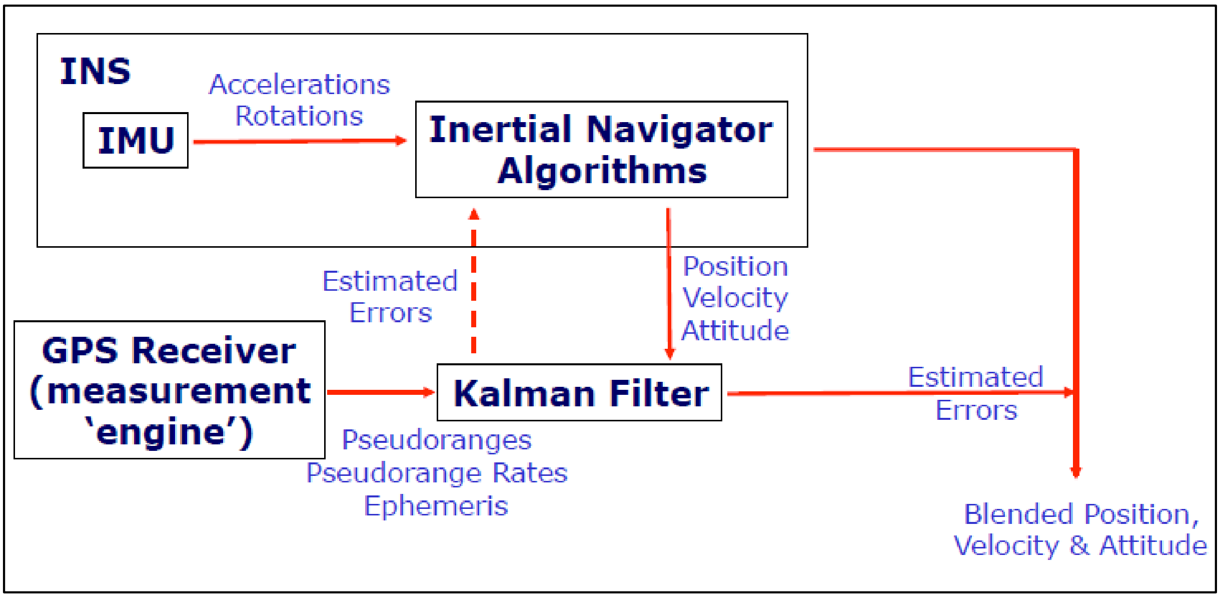

Figure 14: Tightly Coupled INS/GNSS Integration (Dr. Chris Hill, University of Nottingham)

2.4.7 Schuler oscillation

Due to the effect of gravitational that are perpendicular to the direction of motion, Schuler oscillations may occur.

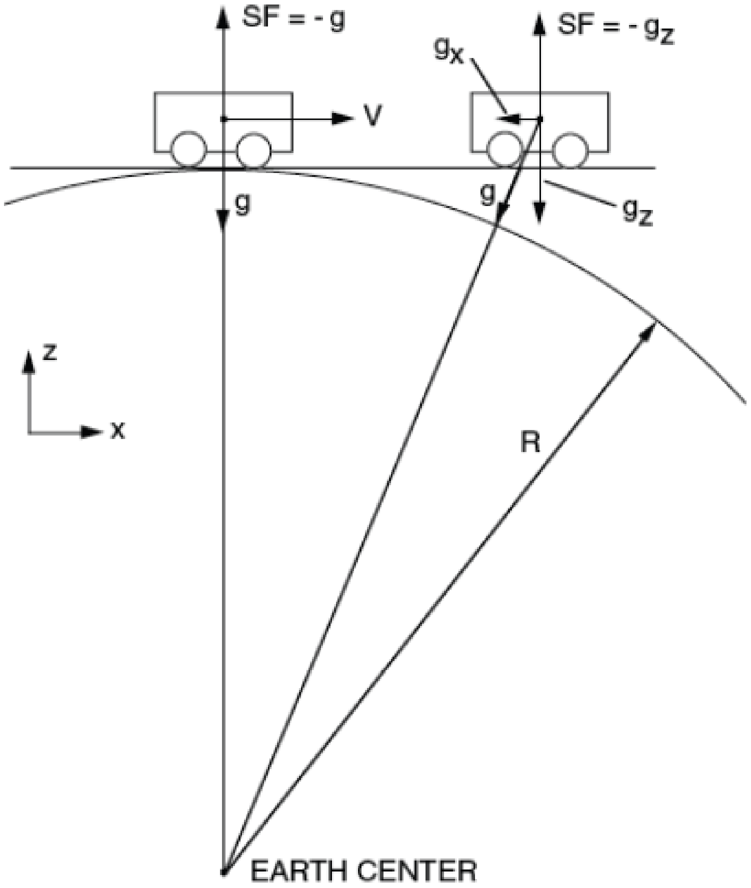

Figure 15: *Schuler oscillations (Paul G. Savage)*

If considering figure 15, illustrating a vehicle rolling without friction on a horizontal plane tangent to the earth

Paul G. Savage defines the Schuler oscillation as "If the vehicle contains an INS having an erroneous initial horizontal velocity V at the point of tangency, the inertial navigation computation will integrate V into a horizontal position change X which will then generate a computed gravitational component gx opposite to the direction of motion proportional to X divided by earth's radius. As the motion continues, the horizontal gravitational acceleration will eventually stop and reverse the motion, generating an undamped oscillation with frequency equal to the square root of earth's gravity magnitude divided by earth's radius ($\sqrt{g/R}$).

This is the so-called Schuler frequency having a period of approximately 84 minutes”

That means that if the INS is further to the North or South than the calculated position, it will be adjusted based on gravitation.

2.5 Submarine operations

2.5.1 General

Submarines are divided types based on either propulsion or task: Nuclear powered and Conventional (Diesel electric propulsion) submarines. Further, they can also be divided into groups based on their tasks.

Nuclear submarines:

- Guided Missile submarines
- Ballistic Missile Submarines
- Attack Submarines

Ballistic Missile submarines (SSBN) are equipped with submarine-launched ballistic missiles with nuclear warheads. Their main mission is strategic nuclear deterrence. The missiles have a range of thousands of kilometers, and thereby they commence a threat wherever they are. Operation area for these submarines are worldwide (navy.mil).

Guided Missile submarines (SSG/SSGN) Submarines equipped with Cruiser Missiles to attack targets ashore.

Conventional Submarines (SSK) are diesel electric powered submarines. SSK’s are smaller than an SSN, which makes them more suitable for operations in shallow and narrow areas.

The main difference between nuclear powered and Diesel Electric submarines, beside the size, is their submerged endurance. A nuclear submarine is able to stay submerged for months, with supply to the crew as the only limitation. Diesel Electric submarines needs to charge their batteries with certain intervals.

2.5.1.1 Typical Submarine missions (Globalsecurity.com 2011):

Peacetime engagement: Deploy submarines in forward areas in order demonstrate a national interest in that area. In particular, it is a show of force.

Surveillance and Intelligence: The submarine is en effective platform to gain information by watching, listening and collect other information undetected.

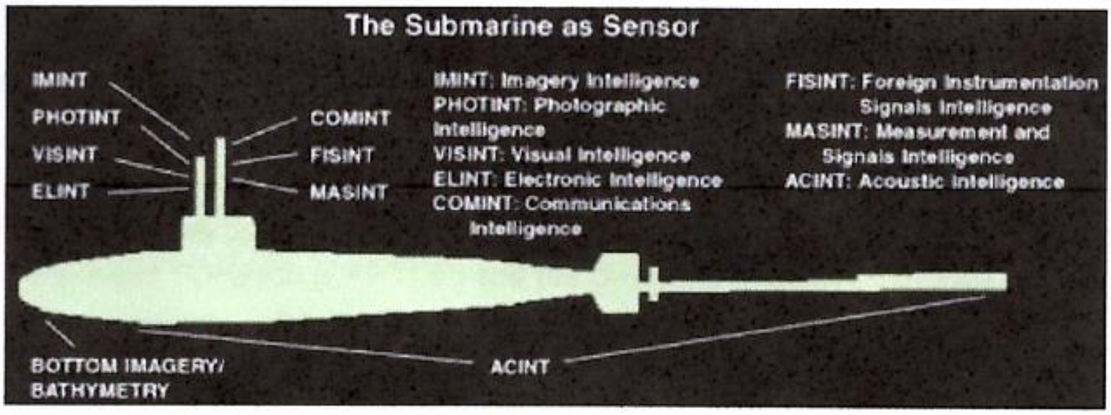

Figure 16: *The submarine as sensor*

Special Operations: Insertion and recovery of Special operation forces (SOF). SOF are able to conduct combat search and rescue operatins, reconnaissance, sabotage, diversionary attacks, monitoring enemy movements and communications behind enemy lines. Only submarines are able to conduct the insertion and recovery covert.

Precision Strike: SSG/SSGN are able to carry Guided missiles, which make them capable to conduct long range precise strikes against targets ashore. During Gulf War I (1991) 12 out of 288 missiles was launched from submarines, both from the Red Sea and the Eastern Mediterranean.

Battle group operations: Attack submarines are integrated into Navy battle group operations. The submaries detached to the Battle Group participate in all pre-deployment operational training and exercises. Typical tasks is Anti submarine Warfare (ASW).

Sea Denial: Stopping enemy surface ships and submarines from using the seas.In order to conduct all these different types of operations the demands for precise navigation is high. In addition to navigation, the positioning system also gives updates to the command & Control- and weapon systems.

2.5.2 Submarine navigation

At all times available technology and the unique requirements of the submarine mission have been discussed. Critical attributes in to a submarine mission are accuracy, availability and covertness. At the moment this is solved through a combination of inertial, sonar, satellite, mapping and computer technologies.

The employment of both submarines and surface ships is affected by the characteristics of the physical environment and weather conditions. Operating in a deep open ocean poses different challenges then operating in close to shore, or littorals.

2.5.2.1 Introduction

The high accuracy requirements combined with the need for covered submerged operations is a great engineering challenge when it comes to design of the navigation systems for submarines (Vaida and Zorn 1998). As surface ships have a continued GNSS access, the submarines does not. Dead reckoning by an accurate INS is the main navigation method.

As technology has developed several attempts to overcome the need for frequent position fixes. The Autonetics (now Boeing) MK2 Ships Inertial Navigation System (SINS) was originally deployed on the first POLARIS submarine in 1960 and was modified several times through 1974 to improve its accuracy (Vaida and Zorn 1998 page 309). The development of LORAN-C, Transit and GNSS improved the accuracy of the position fixes, but still submarines need to expose their antennas above the surface making them vulnerable to detection. Over time the INS will develop errors without updated position fixes. The endurance interval is denoted as the time an INS can operate within required accuracy before a position fix is required. The length of the endurance interval describes the quality of an INS, and establishes the frequency of position fixes required by the navigation system.

The Electrostatically Supported Gyro Monitor (ESGM) was developed and installed in the SINS as a “monitor” in 1974. This technology improved the submarine navigation based on the long-term stability of ESMG. There system did still require position fixes, but the endurance interval increased significantly (Greenspan 1995).

2.5.2.2 Strategic submarine navigation

Strategic submarine navigation has a single mission: *"Provide accurate navigation to initialize the flight of ballistic missiles just prior to their launch from the submarine"* (Vajda and Zorn p. 310). This includes the position, velocity, orientation, gravity and time of the submarine. A simple way to assess the success of a mission is if the missile are hitting the target or not. The initial conditions for success or failure are provided by the navigation system.

2.5.2.3 Submarine navigation in the Littorals

The littorals are denoted as a coastline of land and near-shore waters, or the coastal region (Vego 2010). This area covers a wide specter, from the continental shelf bordering the open ocean to semi-enclosed and enclosed narrow seas. Water depths can be deep or shallow. As an SSN needs at least 50-60 feet of water in order to navigate safely, 30-40 feet is enough for an SSK. Due to their small draft and high maneuverability, the SSK is more suitable for operations in the littorals then an SSN.

A fjord is defined as "*A long, narrow, deep inlet of the sea between high cliffs, as in Norway, typically formed by submergence of a glaciated valley*" by Oxford dictionary's.

The littorals are challenging operation areas in several ways. In addition to operating close to shore, and often inside narrow fjords, the traffic density is normally high (commercial and fishing activity). The challenges is multiply, both regarding navigation and the chances of being detected both from land and ships when operating near high density traffic areas.

Figure 17: *Typical Norwegian Fjord*

2.5.3 INS solution on a Submarine

Due to the information sensitivity in both the Submarine service and the industry it is not possible to go into details regarding the system configurations and capacities of the navigation system on a specific submarine. The following figures and descriptions are based on data from open sources.

SAGEM SA is a known manufacture of INS for submarines. The following is a general description of a typical system configuration on a submarine, and are based on data from SAGEM SA.

SIGMA 40XP (eXtended Performance) is a strapdown ring-laser-gyro inertial navigation system. Performances in according to specifications are (SAGEM.com):

Attitude:

- Heading : <3 arc minutes Sec Lat (RMS)
- Roll and pitch: <1arc minute (RMS)

Autonomous navigation:

- 1.5 NM/24 hours TRMS

Settling time:

- Data availability at dockside: 3 minutes (full accuracy: <15 minutes)
- Data availability at sea: 6 minutes (full accuracy: <30 minutes

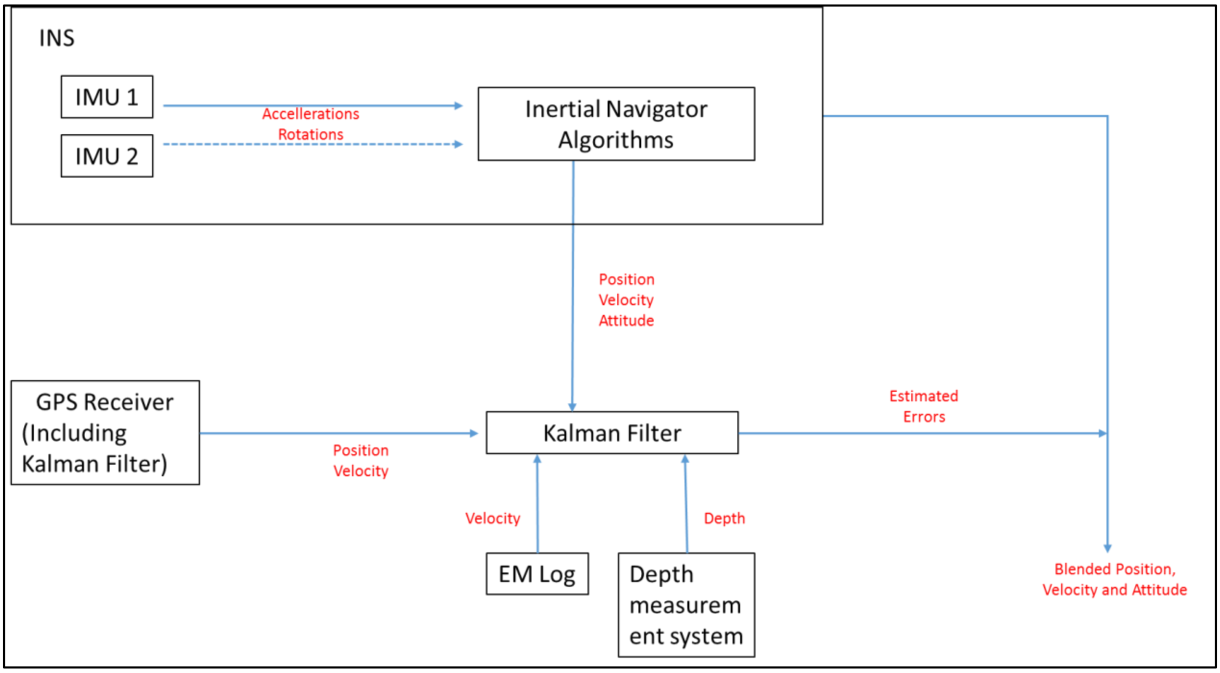

Figure 18: *Example of a loosely coupled INS/GNSS integration on a Submarine (Bard Hess)*

Figure 17 shows an example of INS/GNSS integration on a Submarine. Normally the inputs are from GPS receiver, Electromagnetic log and a depth measurement system. Some systems does also have a mapping system (Bottom mapping).

2.6 Conclusion

Submarine operations in the littorals are challenging in several ways. In order to perform as accurate as possible, the INS requires accurate position fixes. However, the complicity in submarine operations does not always allow this.

The literature review shows that there is a lack in literature regarding the consequences of poor position update of an INS/IMU.

3 Experiment design

3.1 Methodology

The experiment is designed to simulate a Submarine navigation route. It starts with the submarine secured at quay (Static period), then a surfaced transit with GNSS available. When the Submarine dives, it will lose the GNSS-updates, and gets position information from the INS/IMU. During the submerged transit, the submarine returns to Periscope depth in order to update the INS/IMU with a GNSS fix, before it continues the submerged transit. At the end, the Submarine surfaces and conduct a surfaced transit back to harbor.

A second dataset was collected in order to determine if the velocity has any impact on the accuracy of the IMU accelerometers and gyros.

3.2 Data collection

Two different dataset was collected using the NGI (Nottingham Geospatial Institute) van. The van is equipped with both a Tactical grade- and a Navigation grade IMU. NGB2 was used as GNSS reference station for both dataset collections.

3.2.2 Dataset one

Dataset one consist of measurements in GPS timeframe 121 650-126 050 (GPS week 1777) (total 4 400 seconds (73 minutes and 20 Seconds)). There are defined two static periods, one in the beginning and one at the end of the trial (Beginning 121 650-121 950, end 125 750-126 050).

3.2.3 Dataset two

There was a power supply error to the Navigation grade IMU in the van. The POSRS data collected during the trial was useless. Therefore, none analysis are conducted from dataset two. Due to summer holiday at the university, it was not possible to repeat the measurements.

3.3 Routes

3.3.1 Route Dataset one

A route around Nottingham was driven (see figure 19). The trial started and ended with a static period. NGB2 was used as a reference station during the trial. All data was converted to Inertial explorer native format.

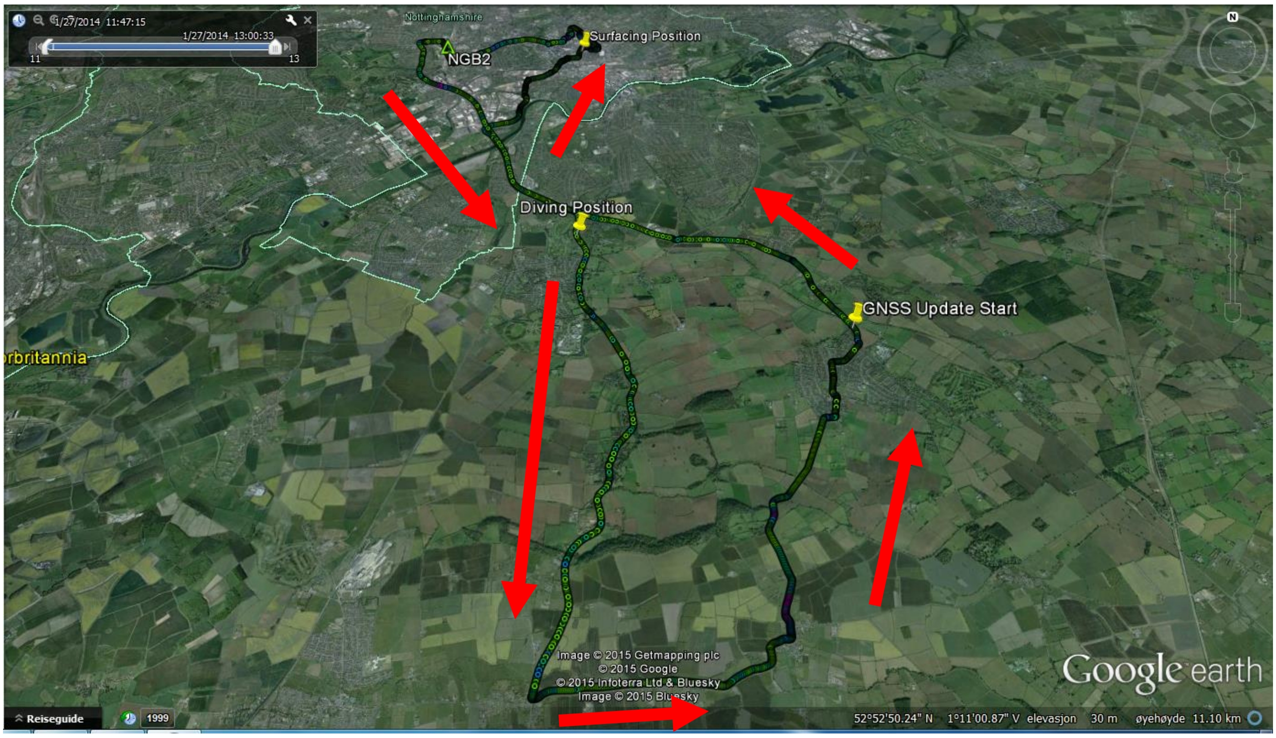

Figure 19: *Route Dataset one (Google Earth)*

The yellow spots in the map is marking the "Diving positon" (IMU-only after this point). The second point is the (GNSS-update start) which is the point of the first Epoch in the GNSS-updates. The last point is the "Surfacing position) which is the point where both GNSS and IMU is available.

The red arrows indicates the direction of the trial.

Table 1: Experiment design

GPS-time	Phase	Mode
121 650-121 950	Static period	GNSS+IMU
121 950-122 700	Surfaced Transit	GNSS+IMU
122 700-124 890	Diving + Submerged Transit	IMU Only
123 830-	GNSS-fix	GNSS+IMU
123 834-124 890	Submerged Transit	IMU only
124 890-125 750	Surfacing + Surfaced Transit	GNSS+IMU
125 750-126 050	Static Period	GNSS+IMU

Table 1 show how the different stages of route 1 have been devided.

3.3.2 Route Dataset two

A route within University Park Campus at University of Nottingham (Figure 20) was driven by NGI van with three different velocities. The velocity for two of the runs was typical speed for a submarine operating inshore. The third run was conducted with as high velocity as possible. All data was converted to Inertial explorer native format.

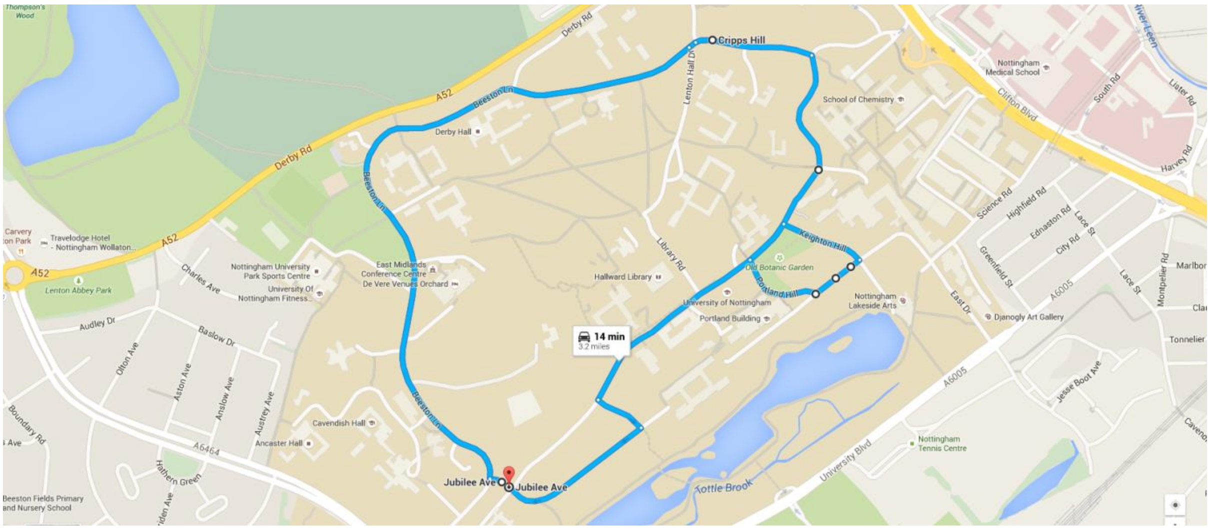

Figure 20: *Route Dataset two (Google maps)*

Velocities Dataset two:

1. *4 Kts = 7.402 kph = 4.599 mph*
2. *8 Kts = 14.816 kph = 9,206 mph*
3. *High speed run (average speed of 30 mph)*

3.2 Hardware

- *Mercedes Vito van*
- *Honeywell* Commercial Inertial Measurement Unit (CIMU)
    - Navigation grade IMU
- LCI – Northrop-Grunnan Litef GMBH
    - Tactical grade IMU

Figure 21: *Mercedes Vito van, Honeywell CIMU and LCI*

Honeywell Commercial Inertial Measurement Unit (CIMU

- GG1320 Digital Laser Gyro
- Honeywell QA2000 Quartz-Flex Accelerometers

Table 2: System specifications Honeywell CIMU (Navigation grade IMU)

	Honeywell
Gyro Bias Continuous	*0.0035 deg/hr*
Angular Random walk	*0.0025 deg/hr*
Accelerometer scale factor	*100 ppm*

Table 1 shows the system specifications for Honewell CIMU – Navigation-grade IMU. It is assumed that all Navigation-grade IMU's have a similar performance, and the results obtained with the Honeywell IMU will reflect the performance of any Navigation-grade IMU.

3.3 Collected Data

- POSRS
- SPAN
- GNSS from reference station (NGB2)

3.4 Software

3.4.1 Inertial Explorer

Inertial Explorer post-processing software suite integrates rate data from six degrees of freedom IMU sensor arrays with GNSS information processed with an integrated GNSS post processor. Inertial Explorer utilizes strapdown accelerometer and angular rate information to produce high rate coordinate and attitude information from a wide variety of IMUs. It is possible to implement either a loose coupling or a tightly coupling of the GNSS and inertial data. (Waypoint, Inertial Explorer User Guide, page 9).

3.4.2 Google earth

Google earth is used to visualize the route and results.

3.4.3 Excel

Excel is used to calculate and visualize the absolute accuracy of the different trials.

3.5 Analyses

The result from the measurements was analyzed in order to determine the impact of a poor position accuracy in the update of INS and the impact of different time-intervals of the GNSS availability.

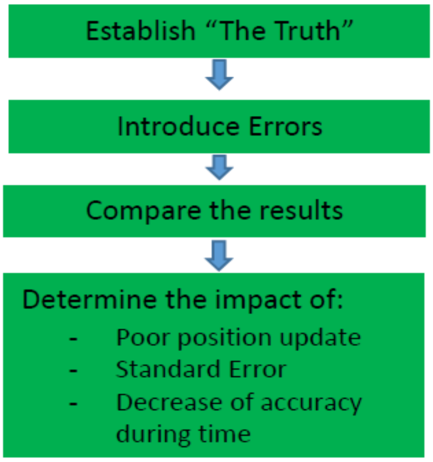

Figure 22: *Model of Analysis (Bård Hess)*

3.7 Workflow

3.7.1 Inertial Explorer Workflow

This is a short description of the workflow; a detailed description is in Annex A.

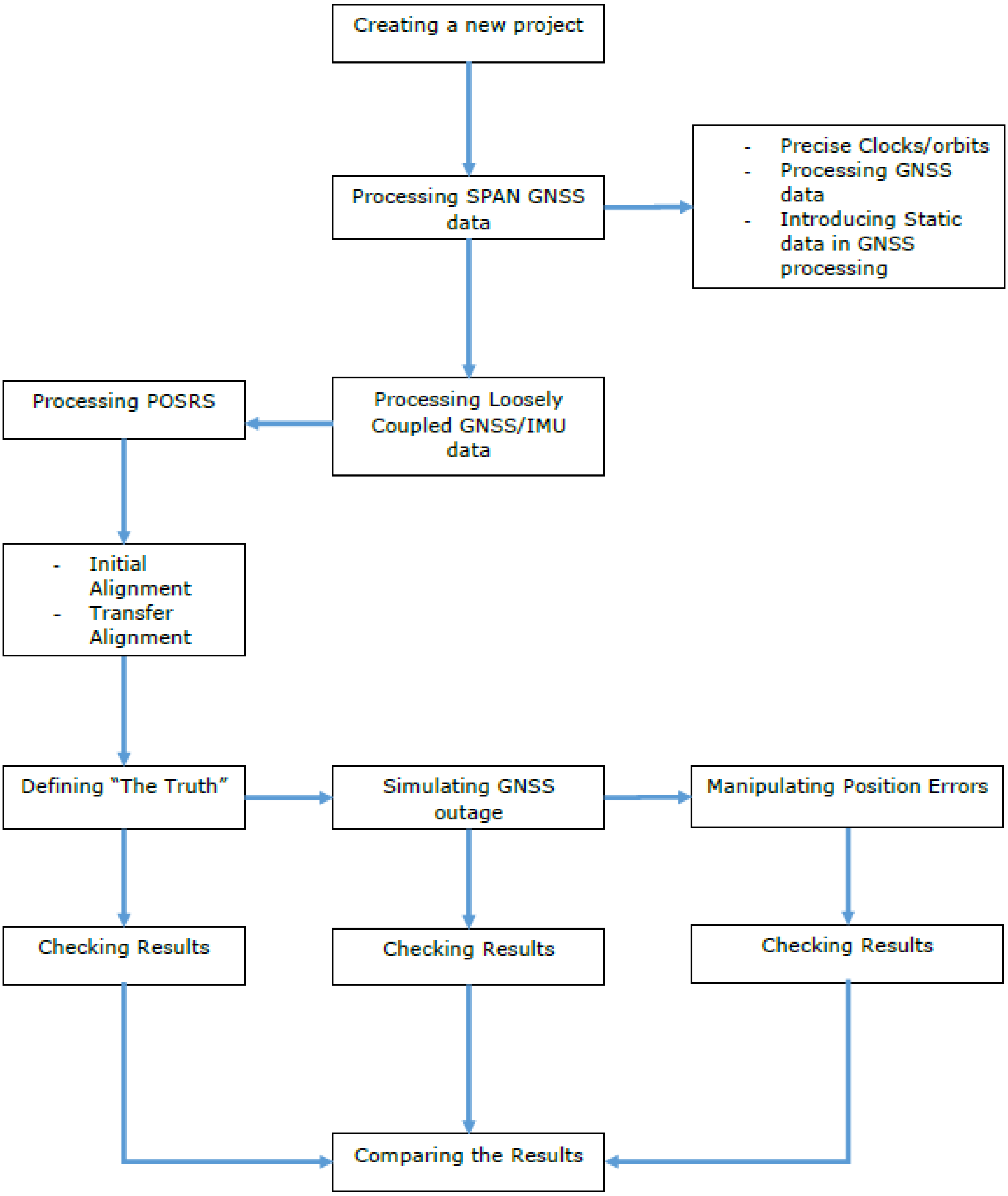

Figure 23: *Inertial Explorer workflow (Bård Hess)*

Figure 23 illustrates the workflow in Inertial Explorer from defining “The Truth”, and introducing the different parts of the experiments. The workflow is also described in Annex A.

3.7.2 Defining “The Truth”

A trial that represents the actual trial of the van has been created and defined as “The Truth”. “The Truth” is used as a reference trial, and all other trials have been compared with “The Truth”.

All available assets were used when defining “The Truth” (GNSS-data with NGB2 as reference station, SPAN data, POSRS data). The Trajectory was processed in Inertial Explorer in a smoothed both way solution with ZUPT (Zero velocity updates) available.

The process of defining “The Truth” is described in Annex A.

3.7.3 Introducing “The Gap”

In order to simulate the submerged transit a gap in the GNSS-data was introduced. The gap was created by removing all the information from “The truth” in the timeframe of “The Gap” (122 700-124 890) Total 2 190 seconds (36 minutes, 30 seconds).
When “The Gap” was introduced, the route was processed in Inertial Explorer with a forward solution only.

3.7.4 GNSS fix

The background of the experiment is to determine if the period with GNSS-availability has any effect on the accuracy of the IMU. Several GNSS-available timeframes were tested, but they did all start at the same time (123 830).

All trials where processed with Forward solution only in Inertial Explorer. The GNSS-updates where created by inserting data for the actual time-frame into “The gap”. The data was collected from “The Truth” dataset in order to give as accurate update as possible. The influence of Multipath is not taken into consideration in this experiment.

Table 3: GNSS update intervals

GNSS-update available	Time-period
0 Seconds	N/A
1 Second, 1Hz	123 830
4 Seconds, 1Hz	123 830 – 123 834
30 Seconds, 1Hz	123 830 – 123 860
60 Seconds, 1Hz	123 830 – 123 890
120 Seconds, 1Hz	123 830 – 123 950
180 Seconds, 1Hz	123 830 – 124 010

Table 3 shows the different time intervals GNSS-was available, and at what time the Updates where conducted.

3.7.5 Introducing Manipulated Errors

Multipath due to the environment will have a great effect on the GNSS-accuracy. When operating in fjords surrounded by high steep mountains it is likely that the area is affected by multipath, and that the accuracy of the GNSS-updates will be degraded.

In order to simulate a multipath environment the positions of 4 epochs, 1Hz was manually manipulated incorrect, and processed in Inertial Explorer. The trials were processed with a forward solution only and ZUPT (Zero velocity updates) was not available, as this is an unlikely support to the IMU on a Submarine.

Table 4: Trials with Manipulated errors

Trials	Manipulated Error	Directions	Standard Error	Number of results
A1, A2, A3, A4	100m	North, South, East, West	0m, 5m, 10m, 50m, 100m	16
AA1	300m	North		1
B1, B2, B3, B4	50m	North, South, East, West	0m, 5m, 10m, 50m, 100m	16
C1, C2, C3, C4	10m	North, South, East, West	0m, 5m, 10m, 50m, 100m	16
D1, D2, D3, D4	5m	North, South, East, West	0m, 5m, 10m, 50m, 100m	16
E	0m	N/A	0m, 5m, 10m, 50m, 100m	5

Table 4 shows an overview of all the different trials with manipulated errors that were processed.

3.7.5.1 Variance-Covariance

In addition to manipulated position errors, all trials were processed with different Variance to determine the influence of different Standard Errors.

Table 5: Standard Errors and Variance

Standard Error (σ)	Variance (σ^2)
0	0
5	25
10	100
50	2500
100	10000

Table 5 shows the connection between Standard error (σ) and Variance (σ^2).

The variance was manually manipulated into the positon with the values from table 5.

4 Results and analysis

4.1 General

In the first part of this chapter, a simulation with different time-interval of GNSS-updates is conducted in order to determine if the length of GNSS-availability will affect the accuracy of the navigation solution.

The second part focus on the effect of different standard errors.

The third part consist of a simulation of different manipulated GNSS-errors added in a 4-epoch, 1Hz updates of the IMU. This is to simulate a poor GNSS-update of the IMU in a Multipath environment with a threat that forces the Submarine deep before a proper GNSS-solution is obtained.

In this section a selection of the results will be presented, a complete collection of the results are in Appendix B and C.

4.2 Different time-intervals of GNSS-updates

4.2.1 Different time-intervals of GNSS-update

Seven different trials with seven different GNSS-intervals was processed in order to determine if the length of the GNSS-availability will have any effect on the accuracy.

Table 6: GNSS-update intervals

Trial	Update time
1	0 Seconds
2	1 Second, 1Hz
3	4 Seconds, 1Hz
4	30 Seconds, 1Hz
5	60 Seconds, 1Hz
6	120 Seconds, 1Hz
7	180 Seconds, 1Hz

4.2.2 Trial 1 – 0 second updates

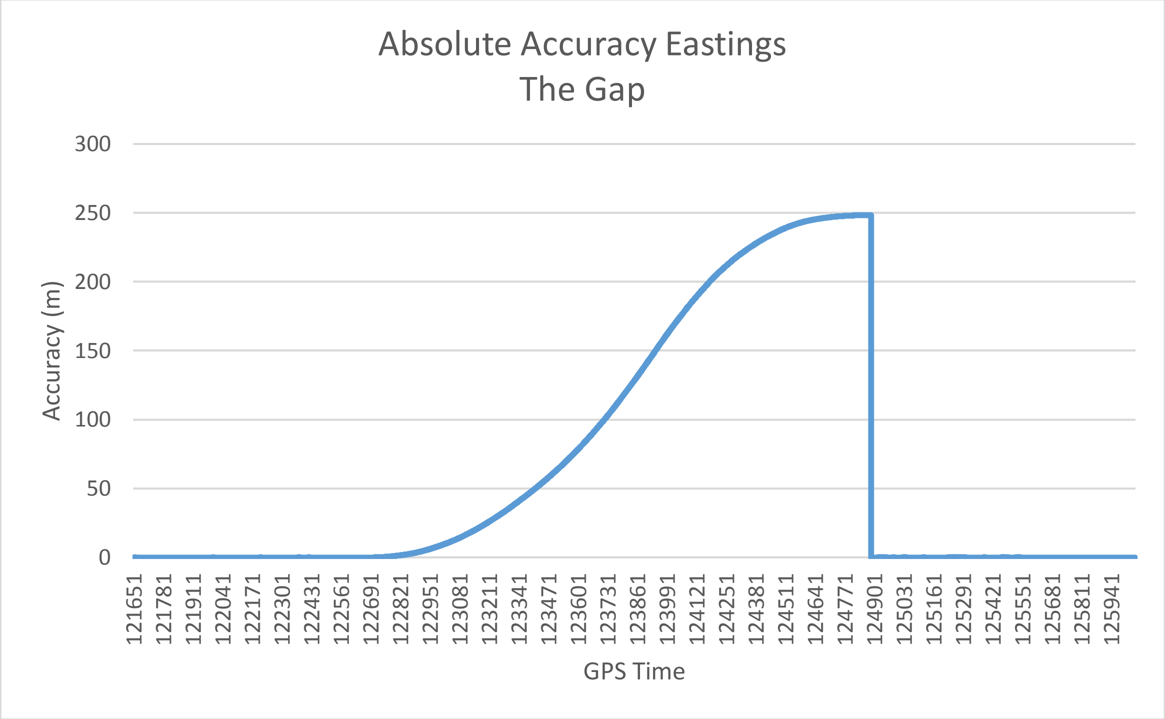

Figure 24: *Absolute Accuracy Easting – The Gap 0 sec update*

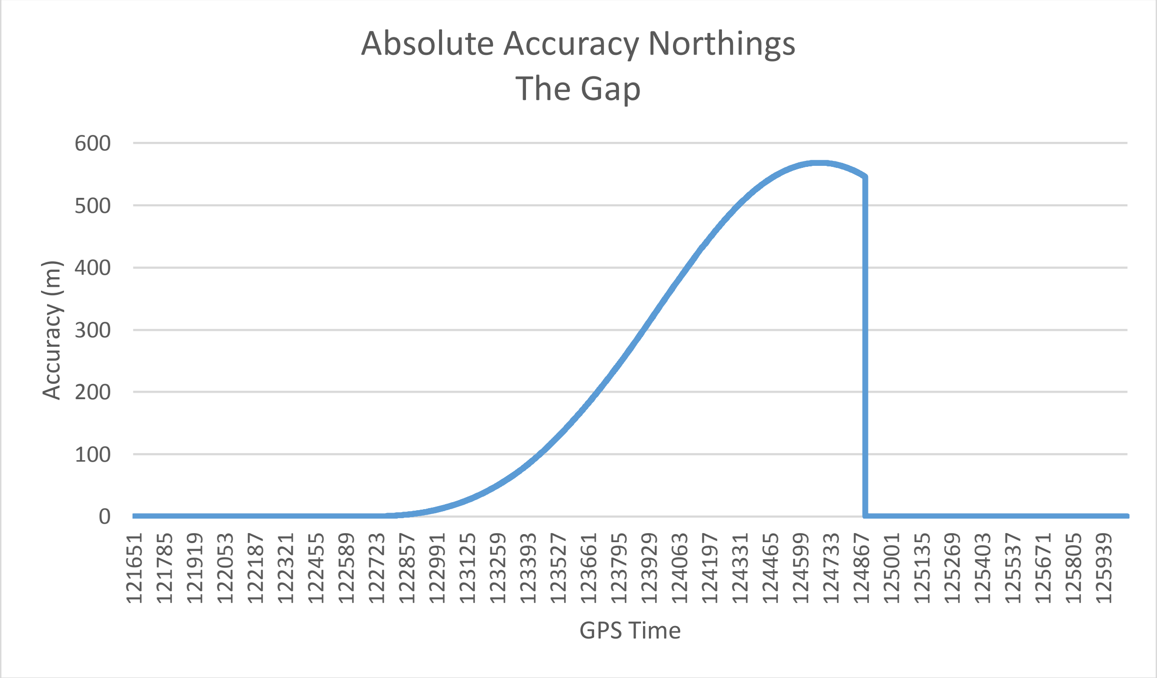

Figure 25: *Absolute Accuracy Northing – The Gap 0 sec update*

Trial 1 was processed in Inertial Explorer with a forward only IMU solution and ZUPT (Zero Velocity Update) unavailable.

Figure 24 and 25 show the absolute accuracy in Easting and Northing during the trial.

Both the Easting and Northing Graph looks like expected for a Forward IMU-only solution in the beginning. However, at time 124 121 the graph starts to converge, and the absolute accuracy improves. The reason for this might be "Schuler oscillations", where the gravity force has an impact on the IMU.

The trial follows the truth perfect until the GNSS-input is removed at time 122 700. The solution is immediately updated when the GNSS is available again at time 124 890.

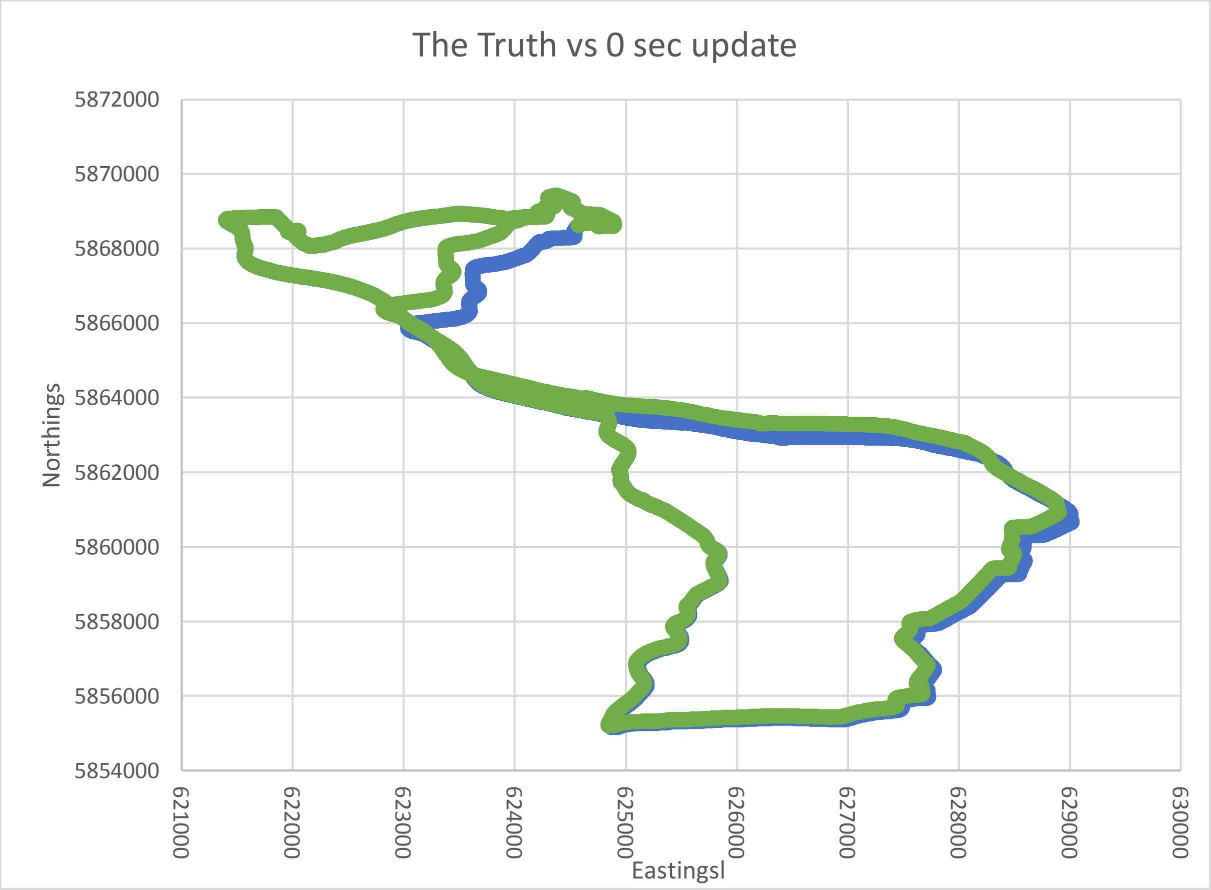

Figure 26: *"The Truth" vs 0 sec updates*

In Figure 26 the green line show the 0 sec update, and the blue line represents "The Truth".

At the end of the trajectory it is the 0 sec update trajectory solution further north than "The Truth", before the absolute accuracy is improved.

The Cross-track is better than the long-track during the trial. That indicates that the gyro is accurate, and that there are some errors in the accelerometers.

When comparing the trials in "Google Earth" it seems like the 0 update trials is behind (further to the north before the crap turn in the southwestern part of the trial).

Figure 27: *"The Corner" (Inertial Explorer/Google Earth)*

By comparing two different epochs at Figure 27, it shows that the error decrease both to the North and to the west after the turn. This might indicate that a dramatic change of course affects the accuracy of the IMU. Most likely there has also been a velocity reduction before the turn and acceleration after the turn. Both the turn-radius, the de-acceleration and the acceleration is not comparable to the dynamics of a Submarine, but it still gives an indication that sudden velocity changes and major turns affects the accuracy of the IMU.

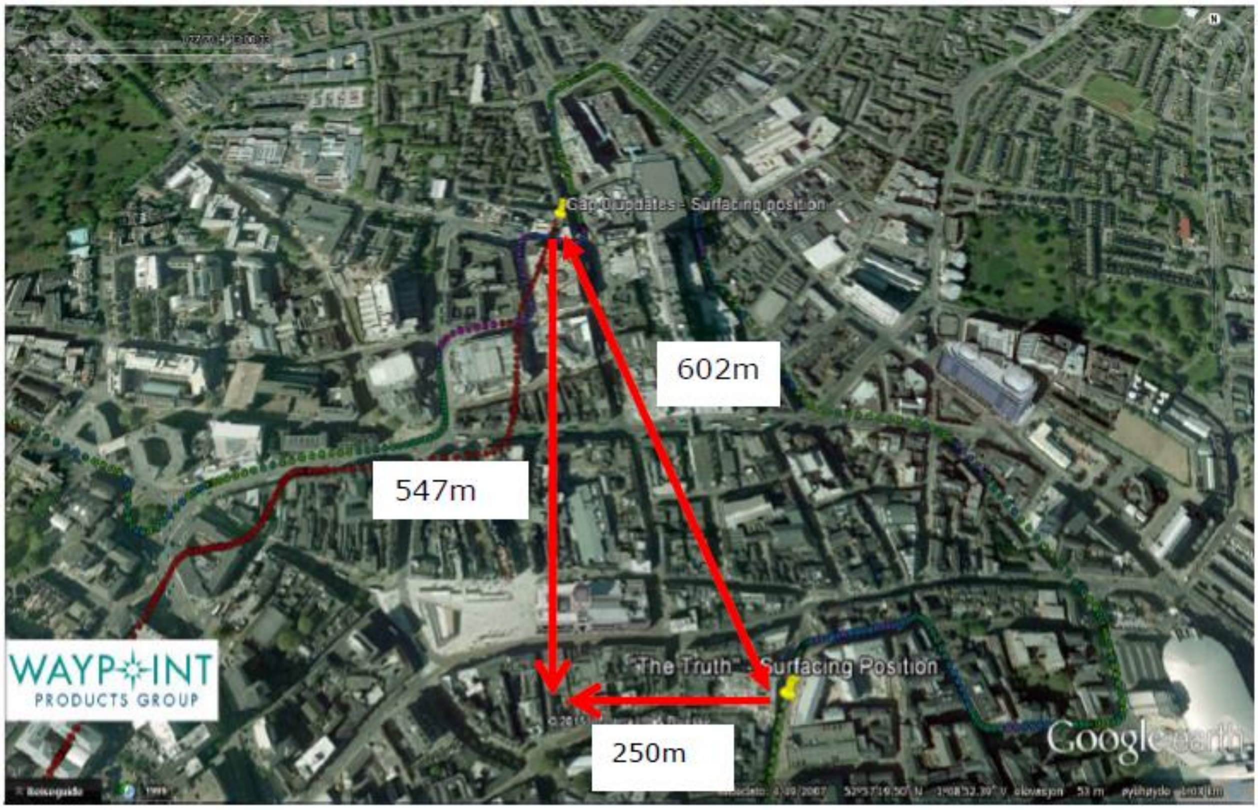

Figure 28: *Surfacing position 0 sec updates*

At the surfacing position (where GNSS is available again), the IMU-solution is 547m to the north of "The Truth" (Figure 28). This is after the curve at figure 24 has converged, and the position has improved. As the trial stops at this point it is unknown how much the accuracy had improved further on in the track.

4.2.3 Comparison of different GNSS-update time-intervals

In order to determine if the time-period with GNSS-availability affected the accuracy of the IMU, different trials with different time-intervals with GNSS-updates was processed.

The trial was processed with an IMU-forward solution only and the submarine dived at epoch 122 700 (start of "The Gap"), at epoch 124 830 the submarine returned to Periscope depth in order to get an GNSS-position update. GNSS-information was available in 1, 2, 30, 60, 120 and 180 seconds with 1 Hz updates. Before the trials it was believed that the accuracy would increase with the length of GNSS-availability.

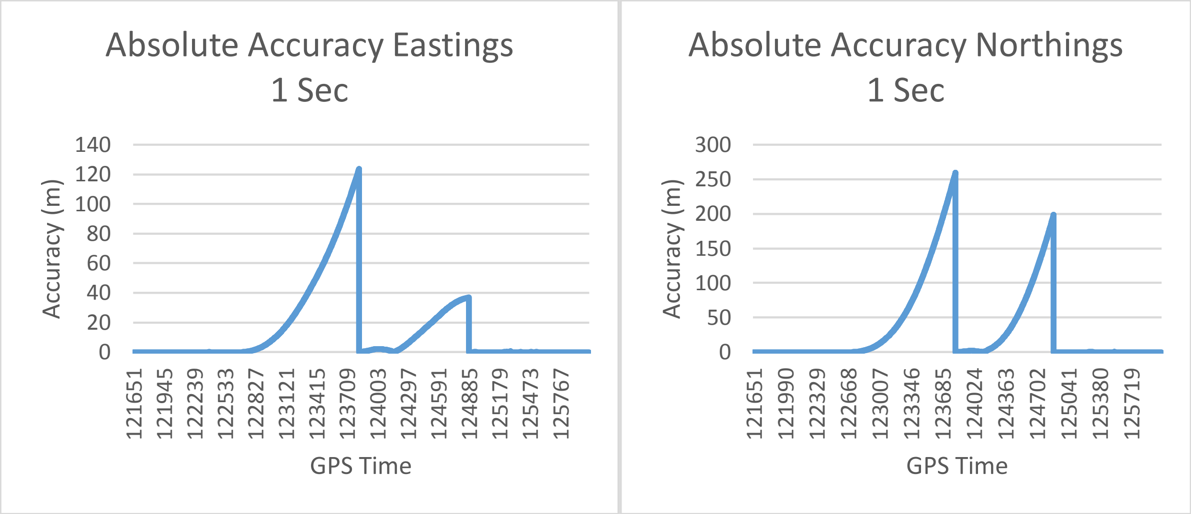

Figure 29: *1 sec GNSS-update*

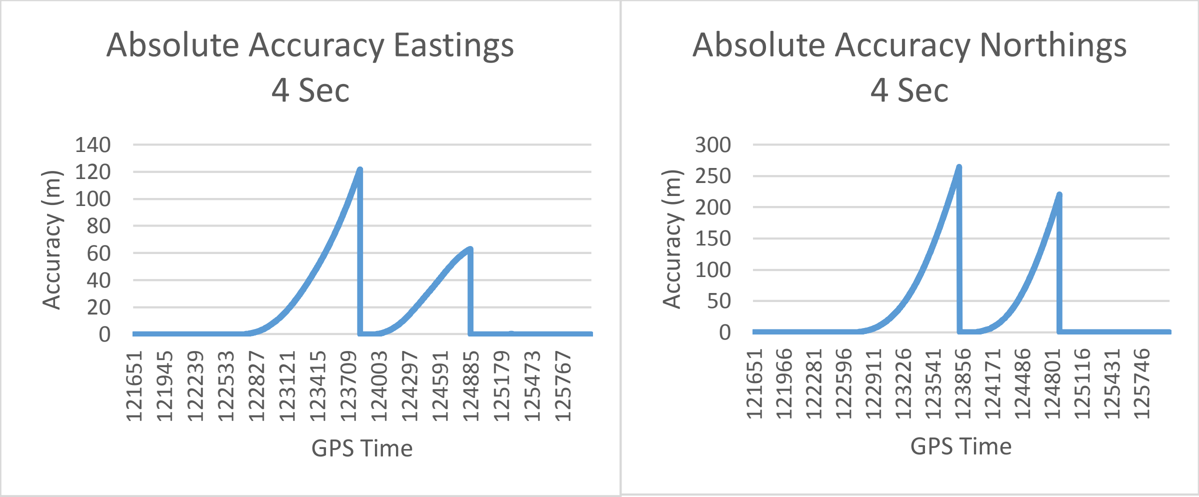

Figure 30: *4 sec GNSS-update*

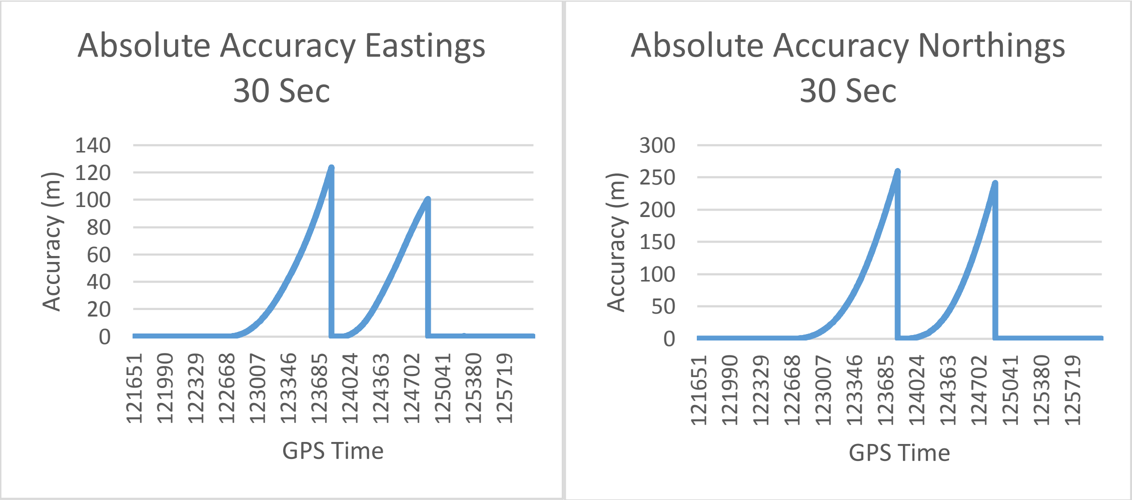

Figure 31: *30 sec GNSS-update*

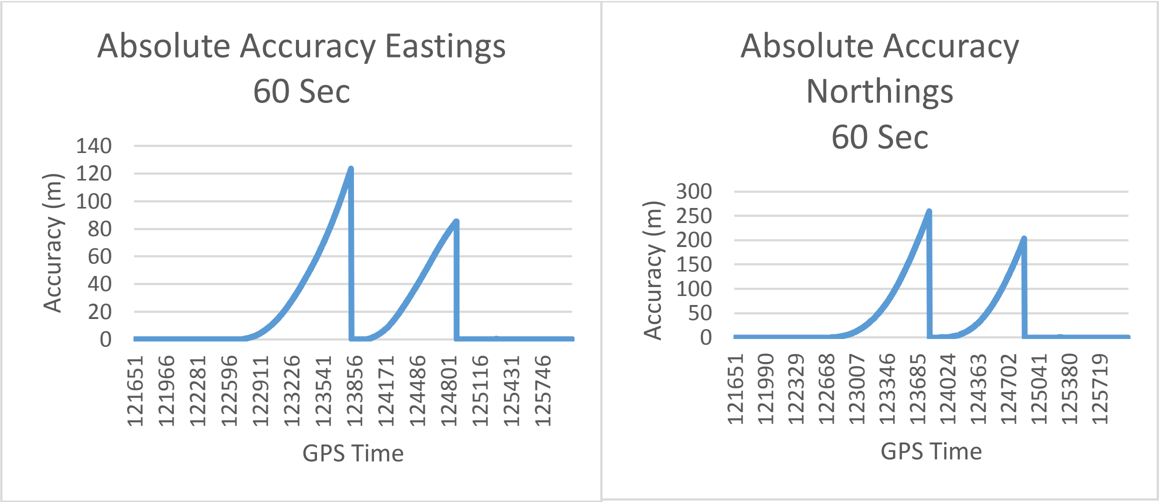

Figure 32: *60 sec GNSS-update*

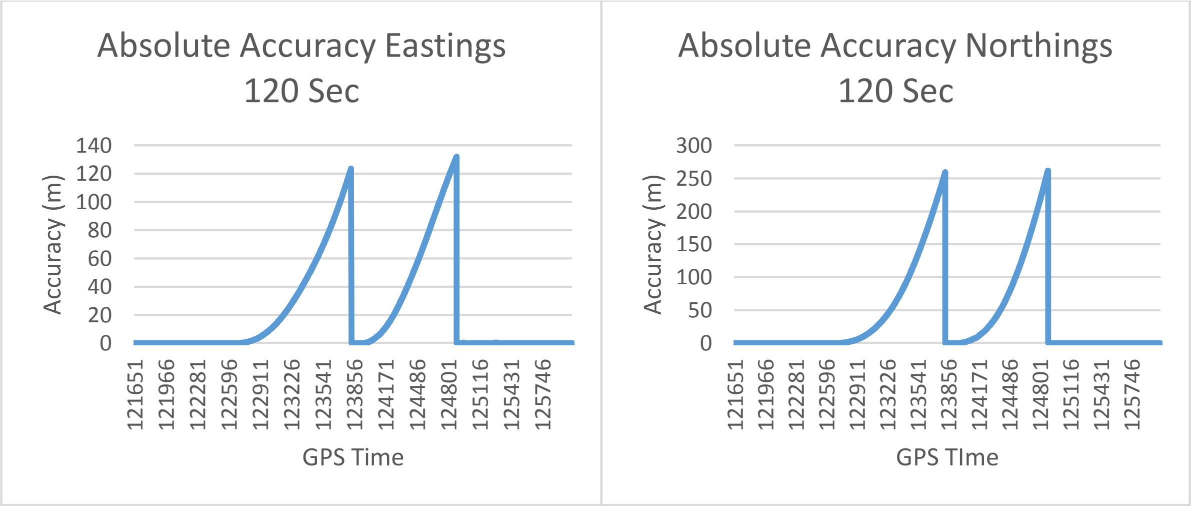

Figure 33: *120 sec GNSS-update*

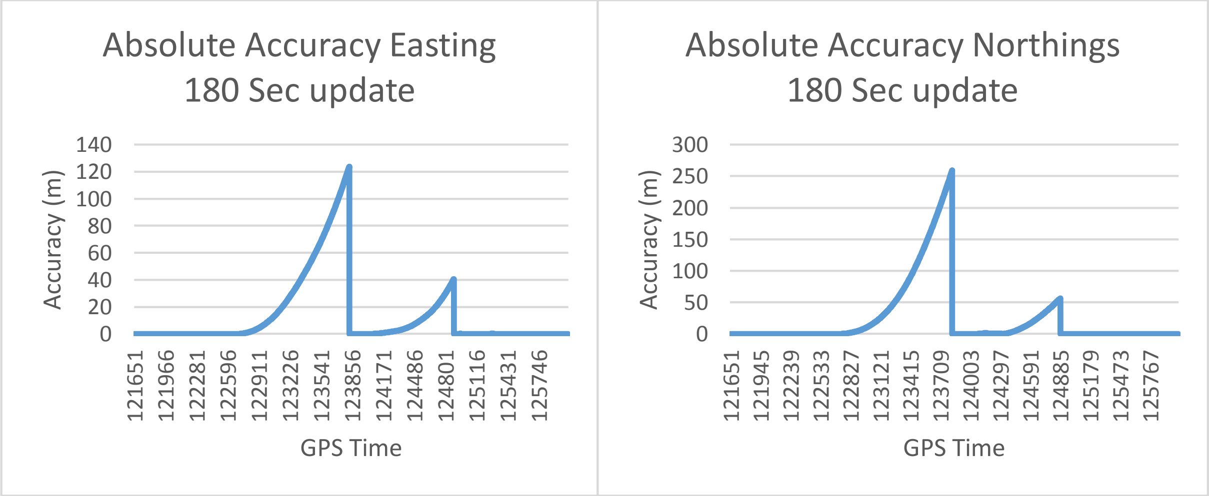

Figure 34: *180 sec GNSS-update*

As expected is the graph from the diving position until the GNSS-update position (GPS time 122 700-124 830) identically for all the trials, as shown in figure 29-34. The shape of the graph is typical for a forward IMU-solution where the accuracy decreases as a factor of time.

However, the results after the GNSS-update shows that the accuracy is better for the 1 second update than the 4, 30, 60 and 120 second updates.

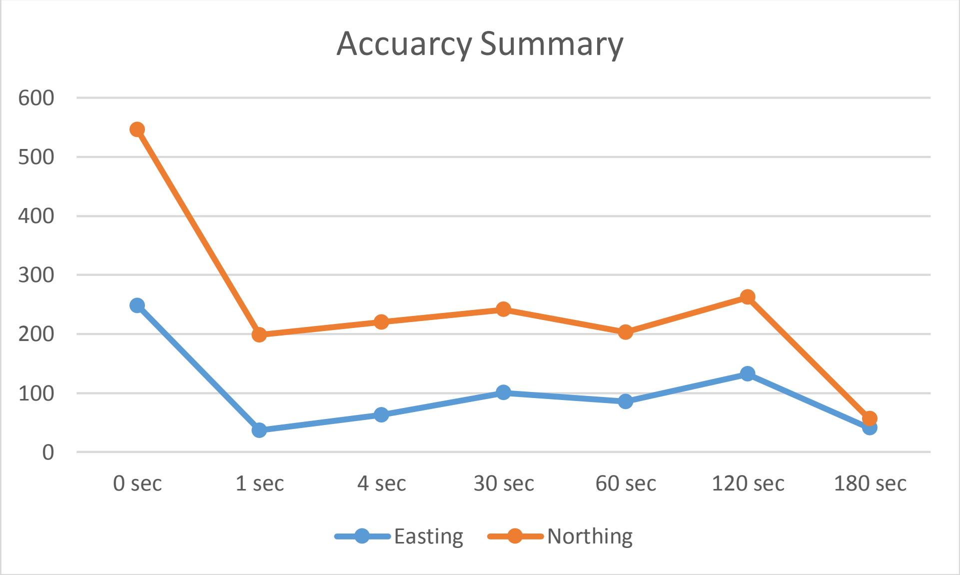

Figure 35: *Summary GNSS-updates*

Figure 35 shows a summary of the results obtain with the different GNSS-update intervals. The effect of an update during the trial is significant when comparing the results with the trial without any updates. The accuracy increases for all the trials with GNSS-update at the surfacing position compared to the trial with no updates..

As expected does the 180 second, 1 Hz update gives the best accuracy. However, it was unexpected that the 120 second, 1 Hz update should give the poorest accuracy.

In a loosely coupled INS/GNSS integration the GNSS-receiver produces a positon and velocity solution. Normally the result of a Kalman filter. The accuracy of the GNSS-update is affected of number of satellites available, satellite geometry and the environment (multipath).

The area around Nottingham is known for a poor GNSS-geometry, something that affects the quality of GNSS-solutions especially in the North/South direction.

This phenomenon might affect the result i.e the GNSS-update in the 1 second 1Hz update is high quality compared with the ones from the later epochs. If the GNSS-updates were of poor quality, the error corrections of the IMU would be also of poor quality.

Even if the phenomenon around Nottingham mostly affects the North/South direction, the quality of the position accuracy in East/West direction will be degraded depending on the heading of the route.

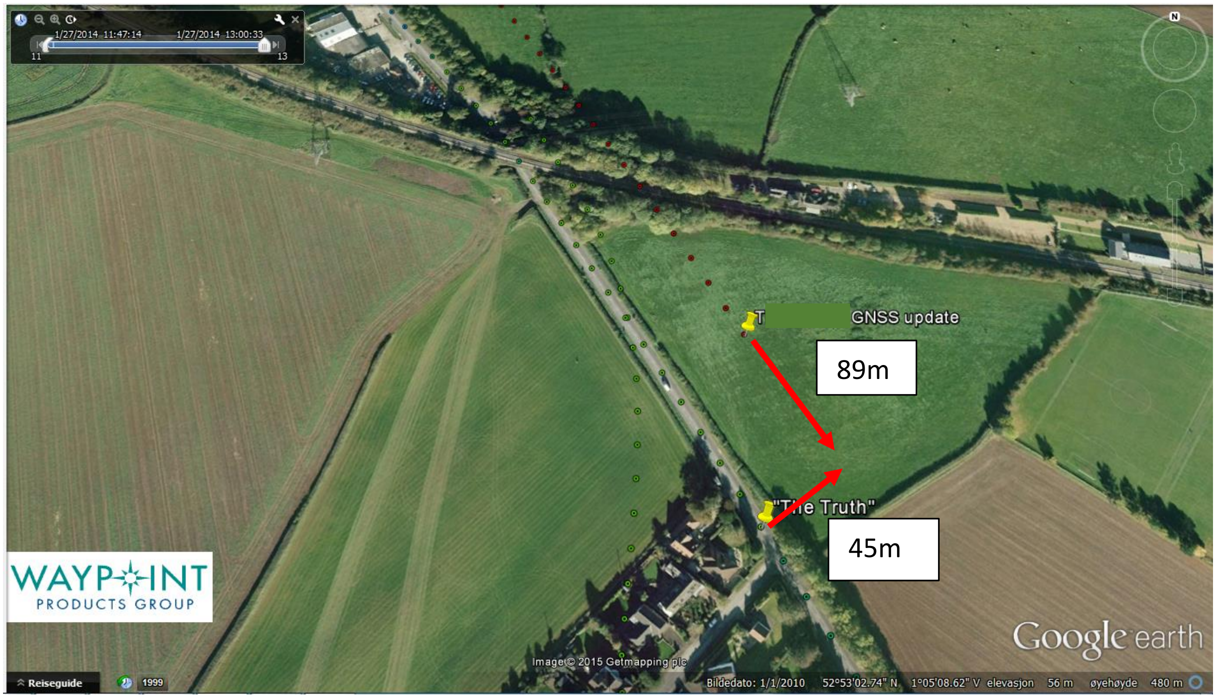

Figure 36: *GNSS update – 100m error to the North*

Figure 36 is an example of how a North/South error in a positon update also affects the accuracy in East/West direction. In the example the trajectory continuous in a Northwest direction after the update. Even if the position error is 100m to the North (No errors in the East/West direction) it generates an error that puts the position 45 meters of the correct trajectory in an Northeast direction.

4.3 Variance/Covariance

4.3.1 General

Each Trial with a manipulated error was processed with different Standard Errors to verify if the variance-covariance have any impact on the accuracy.

The trials were processed in Inertial Explorer with different standard errors for each trial. The trials were conducted in the same way as the previous experiment. The submarine dived at GPS time 122 700 and returned to periscope depth at GPS time 124 830 in order to update the IMU with a GNSS-positon fix.

The update period was 4 seconds, 1Hz for each starting at GPS time 124 830.

The standard errors and variance used are in accordance with table 5.

A trial with Standard Error 0m was also processed, but the absolute accuracy increased quickly to more than 9000m in both Easting and Northing.

In this section only a selected number of trials will be shown and discussed, a complete collection of the trials are in Appendix C.

4.3.2 Effects of different Variance/covariance without manipulated errors

A trial was conducted (Trial E), without any manipulated position errors, but with different standard errors (Table 5) manually added to the GNSS-updates.

When studying the graphs for absolute accuracy in eastings, there are no difference when changing the standard error from 5m up to 100m (Figure 37).

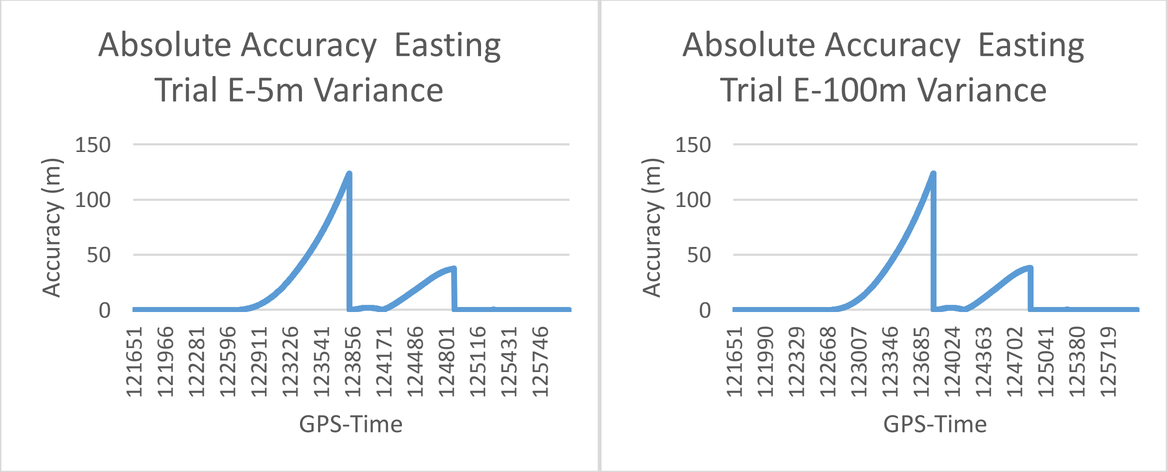

Figure 37: *5m vs 100m Standard Error – Eastings*

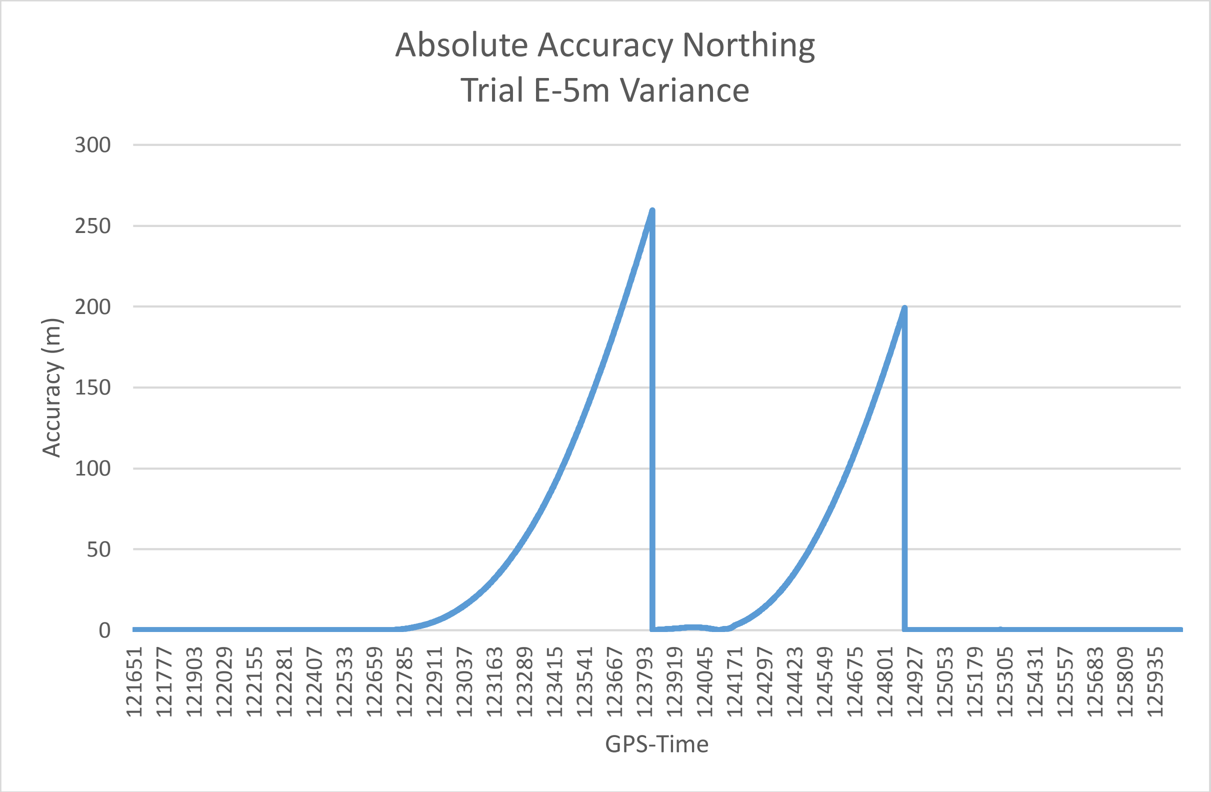

Figure 38: *5m Standard error – Northing*

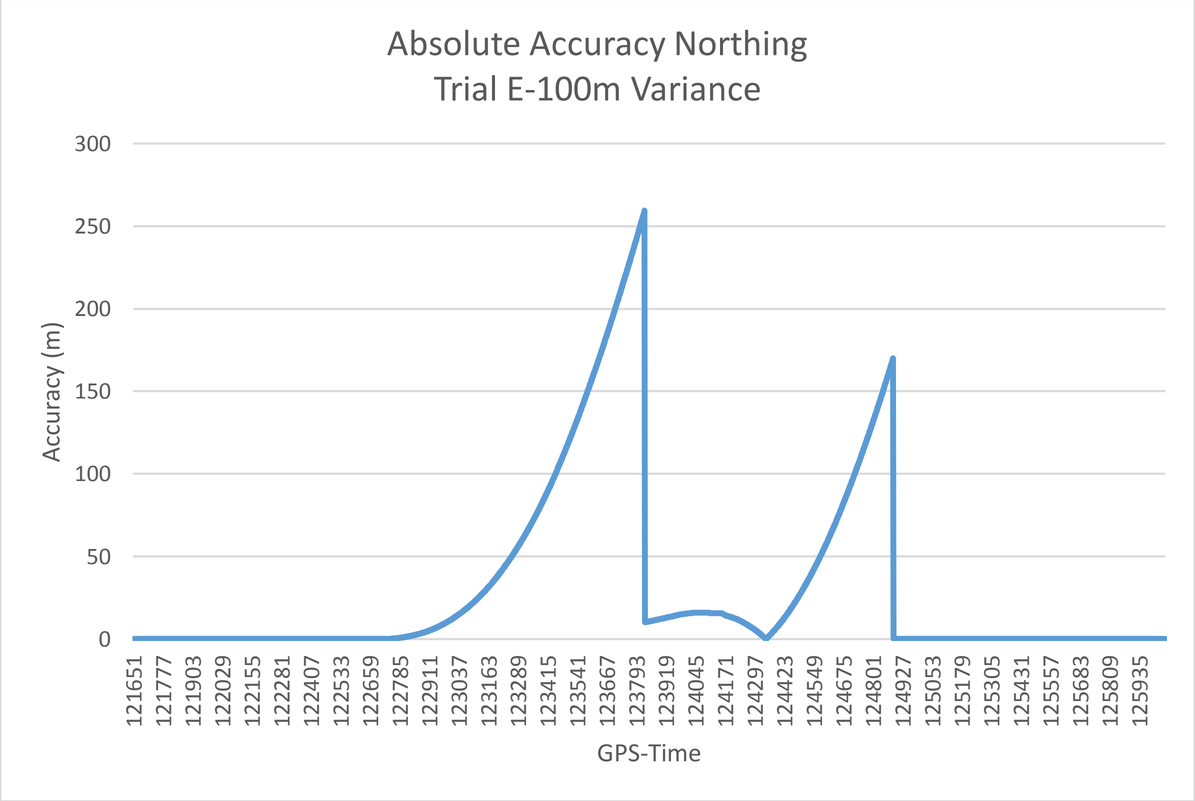

Figure 39: *100m Standard error - Northing*

When comparing the graphs for 5m Standard error (Figure 38) and 100m Standard error (Figure 39), the shapes of the graphs is similar. However, when looking at the 100m Standard error, it has not completely complied with the GNSS-position fix. If so, the Absolute accuracy in Northings should have been 0m at the update time (123 830).

The last position before the GNSS update is 259.511m to the North for both trials. After the GNSS-update, the trial with 5m standard error is 0.023m to the North, and the trial with 100m standard error is 10.161m to the North of the GNSS-update position.

The reason for this result might be due to the size of the search area based on the standard error. The trial with 100m standard error believes it is 259.511m to the North of the updated position, and will not completely agree with the update. It seems like a trial with a large standard error tense to smoothly move towards the updated position but not completely comply with it, based on history or last calculated position. A position with a large standard error will have a larger search are than a position with a low standard error. However, the accuracy of the trial with 100m standard error is better than 5m standard error at surfacing position. The surfacing position for both trials are to the South of "The Truth". Something indicating that the corrections to the IMU is to large.

4.3.3 100m manipulated error north – 5m Standard error vs 100m standard error

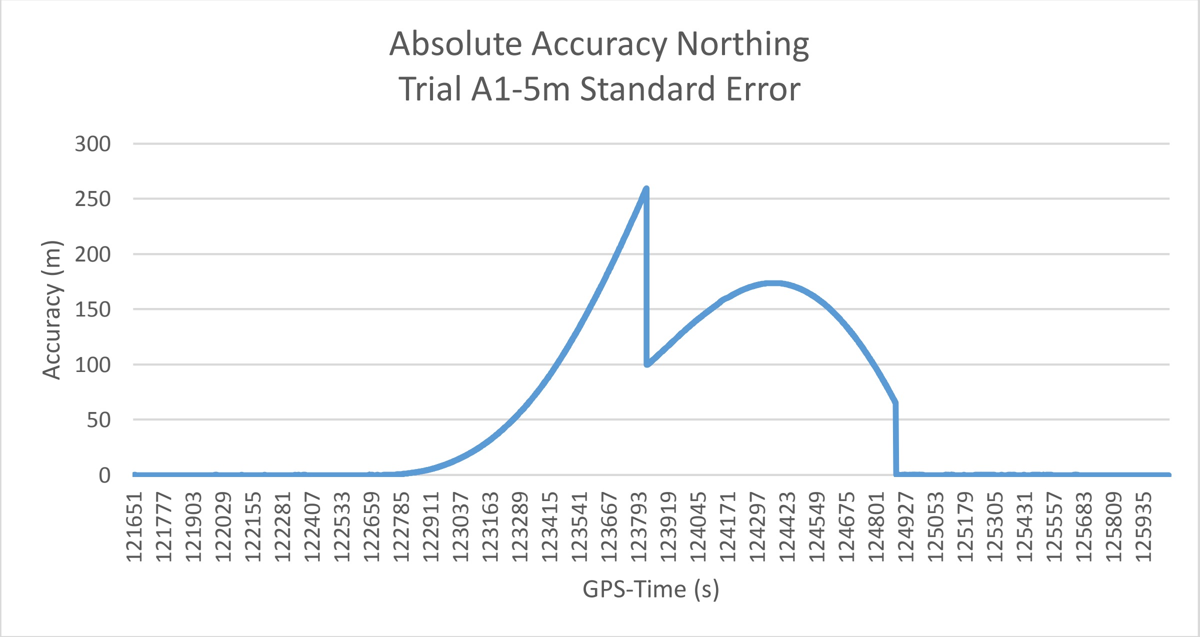

Figure 40: *Trial A1 – 100m error to the North, 5m Standard Error*

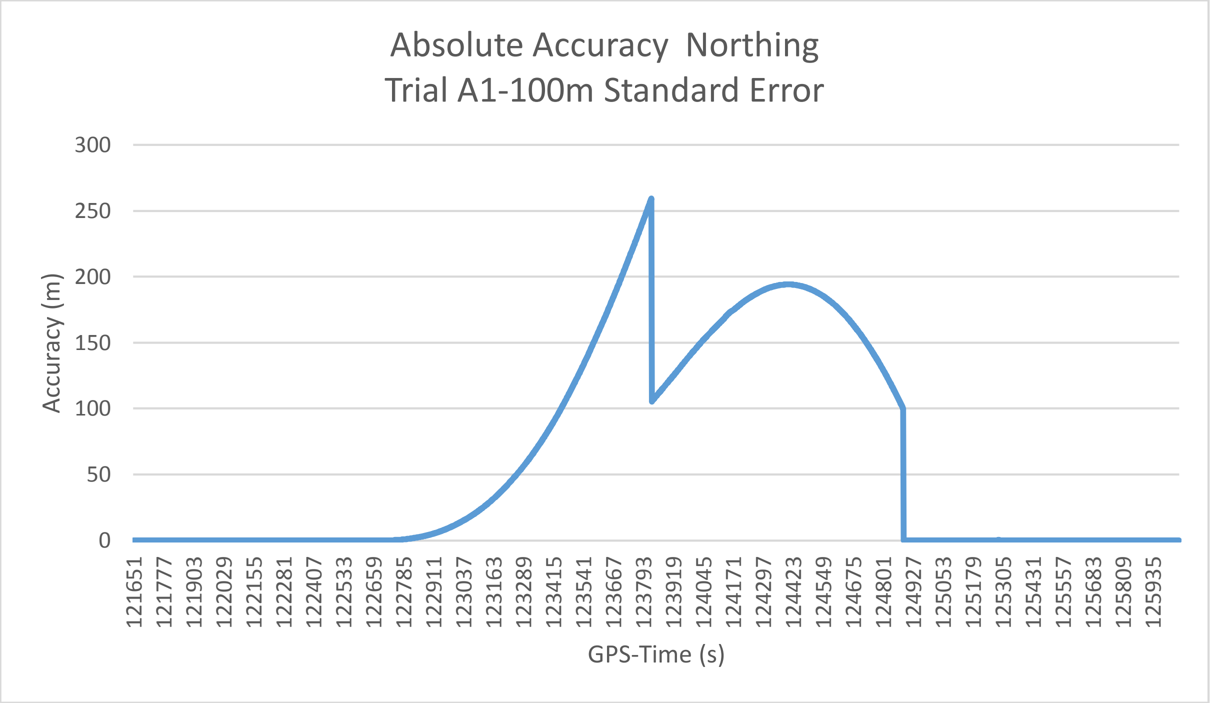

Figure 41: *Trial A1 – 100m error to the North, 5m Standard Error*

Figure 40 and 41 shows the absolute accuracy a trial with a manipulated error of 100m to the North. Figure 40 shows the result with a 5m standard error, and Figure 41 shows the result with a 100m standard error.

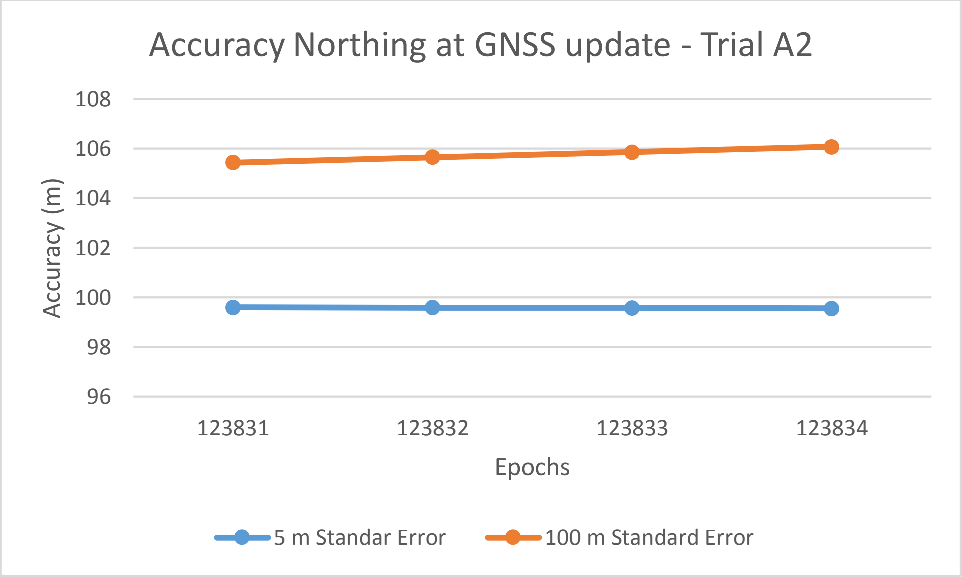

Figure 42: *Accuracy Northing at GNSS update –Trial A1 5m vs 100m Standard Error*

The standard error has an influence of the accuracy when the IMU is updated by a GNSS-fix (Figure 42). Unlike the trial without errors, The 5m standard error has a better absolute accuracy then the 100m standard error, when a 100m position error is added.

When adding the 100m error to the north, the difference between the last calculated INS/IMU position is closer to the manipulated error then "The Truth". The distance between the calculated position and "The Truth" is now reduced to 159.511m.

The result for this trial shows that a small standard error is more accurate than a large standard error, unlike the trial with "The Truth".

This indicates that a small standard error is preferable if the distance between the calculated position and update position is small, and a large standard error is preferable if the distance between the calculated position and update position is large.

The graphs follows the same pattern after the updated position, but the trial with lowest standard error (5m) have the best accuracy throughout the trial.

4.3.4 100m manipulated error south – 5m Standard error vs 100m standard error

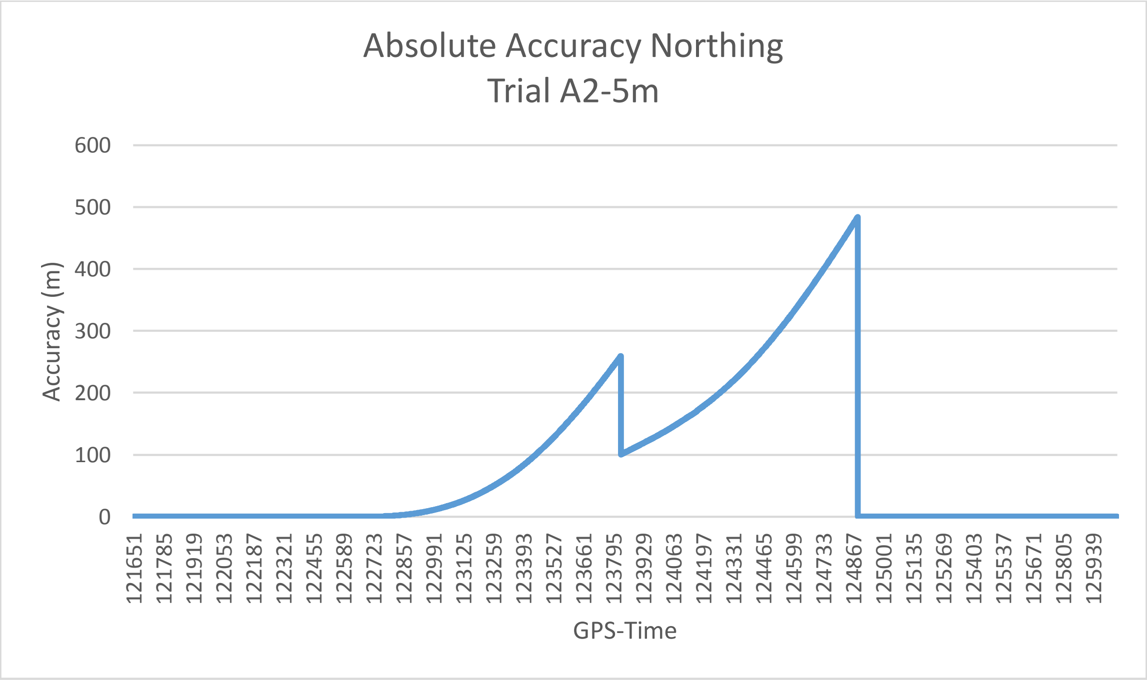

Figure 43: *Trial A2 – 100m error to the South, 5m Standard Error*

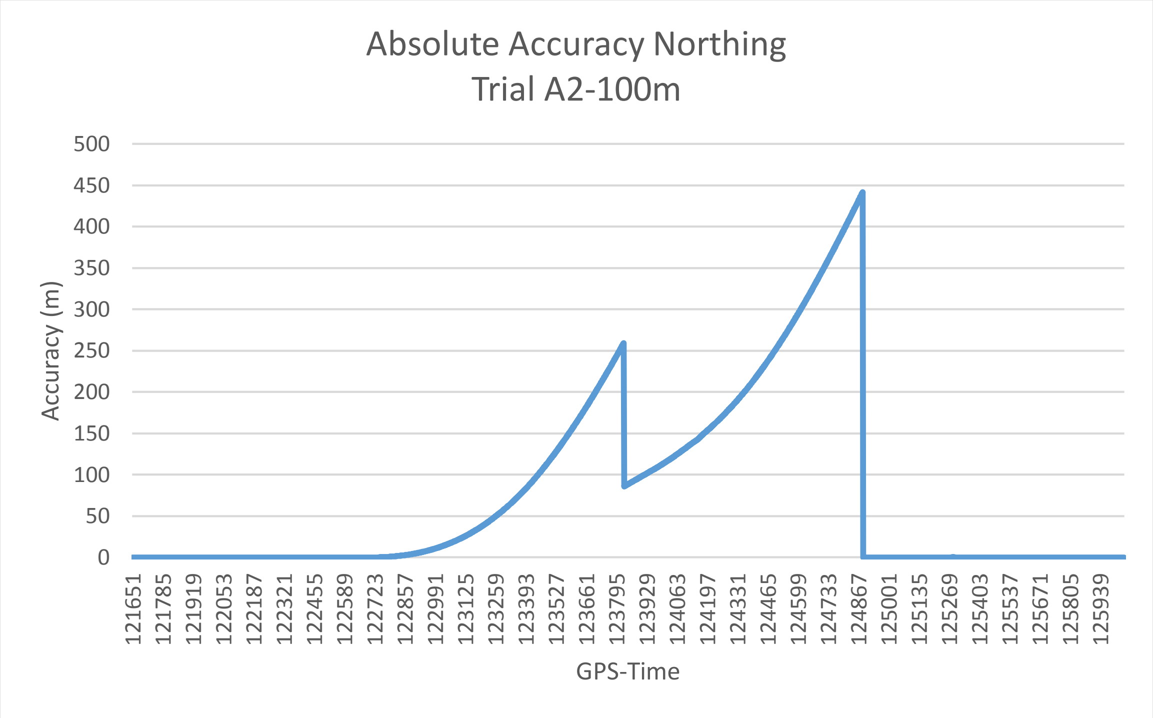

Figure 44: *Trial A2 – 100m error to the South, 5m Standard Error*

Figure 43 and 44 shows the absolute accuracy a trial with a manipulated error of 100m to the South. Figure 43 shows the result with a 5m standard error, and Figure 44 shows the result with a 100m standard error.

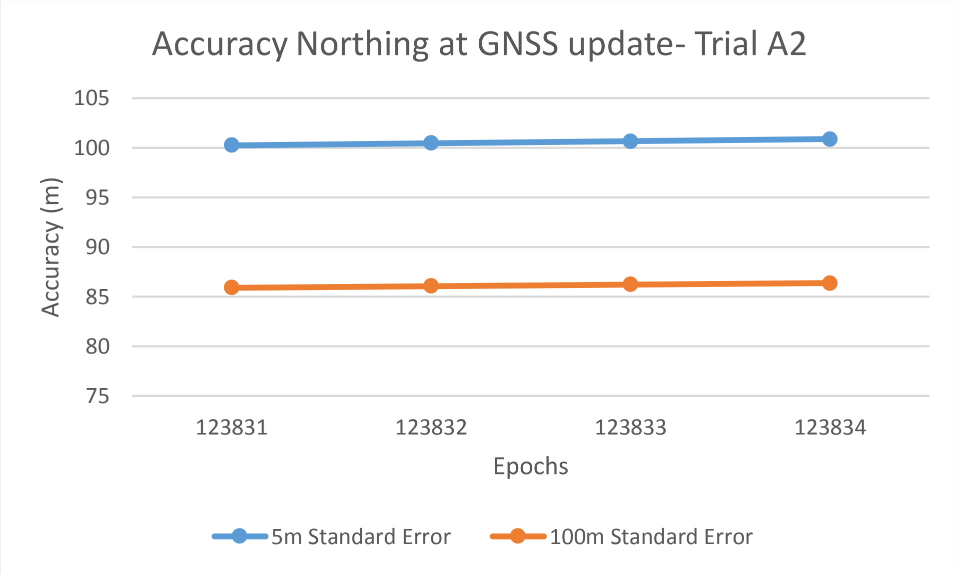

Figure 45: *Accuracy Northing at GNSS update –Trial A2 5m vs 100m Standard Error*

When the manipulated error is 100m to the south of the Truth the distance between the last calculated position and the update position is 359.511m.

The trial with standard error 100m has a better absolute accuracy at the surfacing position than the trial with 5m standard error.

After the GNSS-update does the trial with 100m Standard error have a 14.347m better accuracy in Northings than the trial with 5m Standard error.

When comparing the accuracy before surfacing the difference in accuracy in Northings has grown to 42.210m.

This result underpins the results from the two previous trials, and is a strong indication that a large standard error is preferble if the distance between the calculated positon and the GNSS-update position is large. If the distance is short, a smaller standard error is preferable.

4.3.5 Summary Variance/Covariance

When comparing the plots regarding variance/covariance the standard error has to be 100m to detect any major differences from a low standard error (5m). A small standard error immediately tends to change to the exact position of the GNSS-receiver. An update of a position with a larger standard error moves towards the position from the GNSS-receiver. It seems like the corrections of the IMU is affected by how much the position moves relative to the last position calculated by the IMU/INS.

The variance can be compared to a “pool of error”. When operating in a narrow fjord it is possible to have an opinion about how large the variance should be, as the shoreline acts as limitations.

The influence of the standard error is larger I North/South direction than in East/West direction.

There are indications that large standard error is preferble if the distance between the calculated positon and the GNSS-update position is large. If the distance is short, a smaller standard error is preferable.

4.4 Introducing Errors

4.4.1 General

The aim for experiment two was to determine what influence a poor GNSS-position update has on the IMU and the accuracy.

The trials were processed in Inertial Explorer with manipulated errors in the GNSS-update positions. The trials were conducted in the same way as in experiment one. The submarine dived at GPS time 122 700 and returned to periscope depth at GPS time 124 830 in order to update the IMU with a GNSS-positon fix.

The update period was 4 seconds, 1Hz for each starting at GPS time 124 830.

In this section, a comparison of trials with a standard error of 5m will be conducted as the impact of standard error is discussed in section 4.3.

A selection of trials is discussed in this section; the complete collection of trials are in Appendix C.

The shapes of the graphs in all trials without trial, A1 and B1 (manipulated errors to the North) are as expected. The position update is at the manipulated error, and the growth of error is typical for a forward IMU-solution.

4.4.2 Trial A1/B1/AA1 – Manipulated error to the North

Trial A1 was processed in Inertial Explorer with a manipulated error of 100m to the north in the GNSS-update positions.

The GNSS-update consisted of four epochs, 1Hz starting at time 123 830.

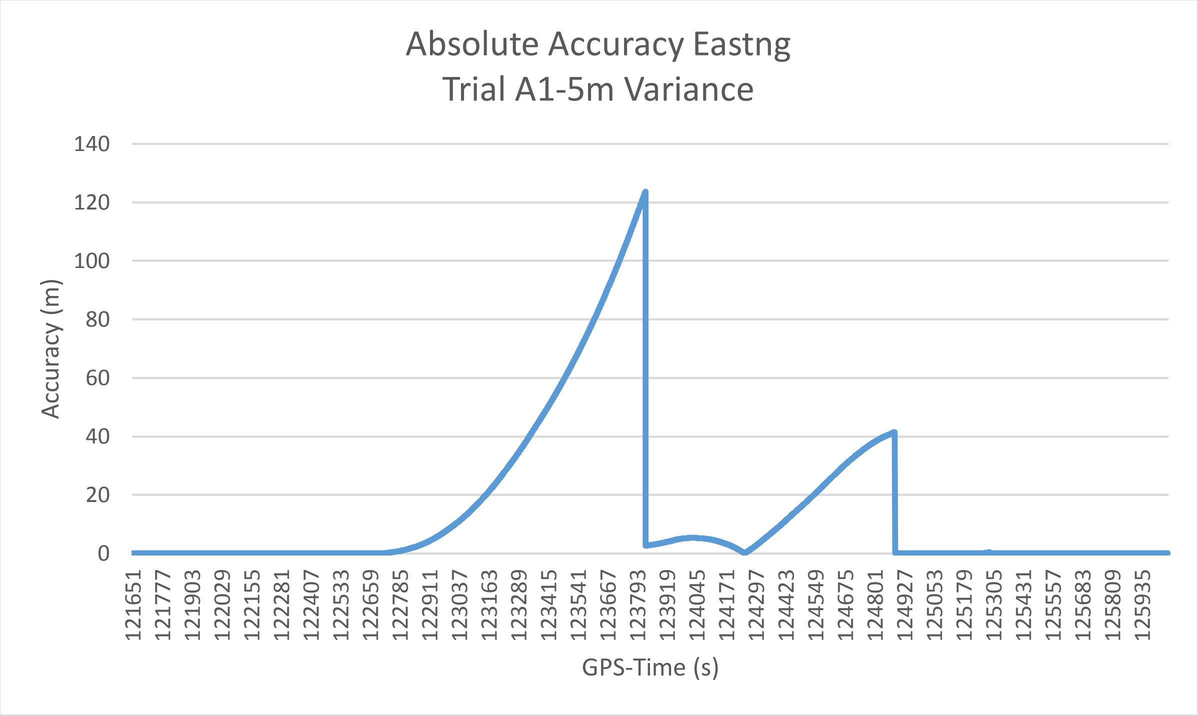

Figure 46: *Trial A1 – Absolute Accuracy easting*

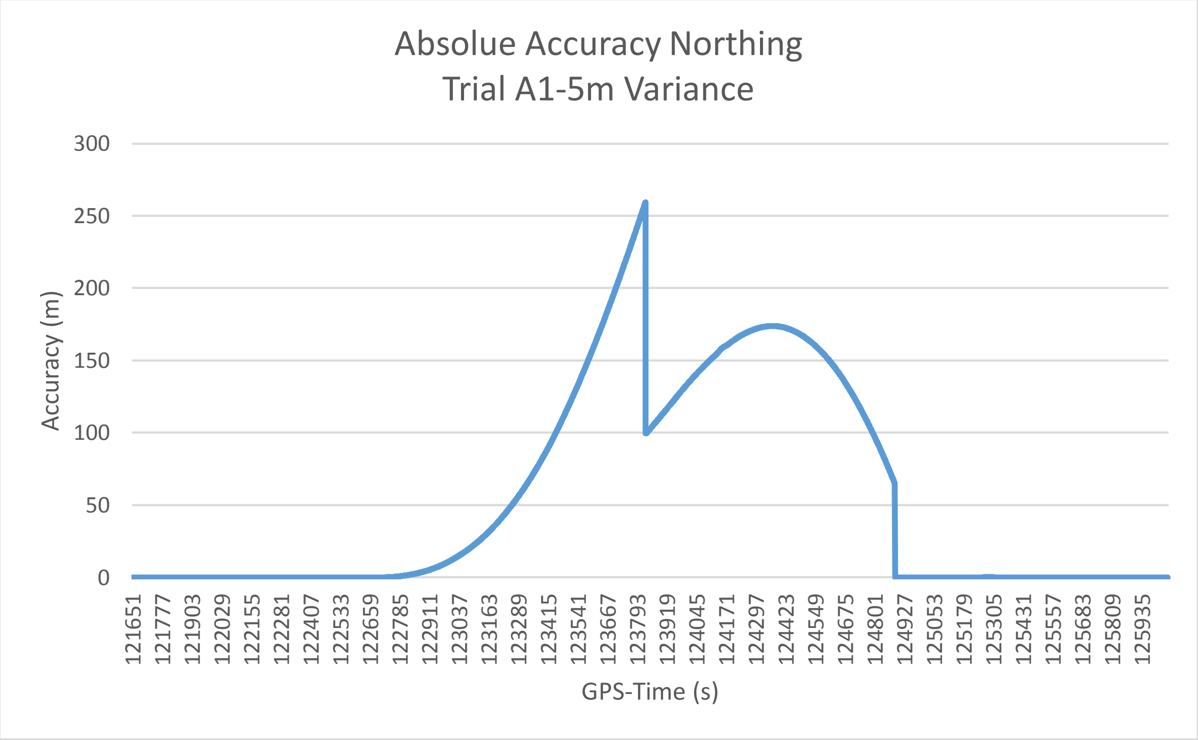

Figure 47: *Trial A1 – Absolute Accuracy northing*

The shape of the graph representing the absolute accuracy northing, for trial A1 with a manipulated error of 100m to the north (Figure 47) is not as expected. When an IMU/INS is processed with a forward solution, it is expected that the error will grow as a factor of time (same shape as the graph before the GNSS update).

In this case, the graph converges, and the accuracy is northings has improved during the trial.

There is also a common pattern in the Easting and Northing graph after the GNSS-update. The Easting is at 0 at the update-epoch before the error slowly increase, than decreases before the surfacing position.

By manipulating the error 100m north the accuracy of the Eastings also will be affected, as the heading of the trajectory continuous to the Northwest after the update. If the trajectory had continued straight North it would not have affected the error in Eastings at all.

At Epoch 123 830 does the trajectory have a heading of 332.8 degrees. This means that the updated positon is 45.71m to the east of "The Truth" trajectory (Calculated by sinus and the manipulated error).

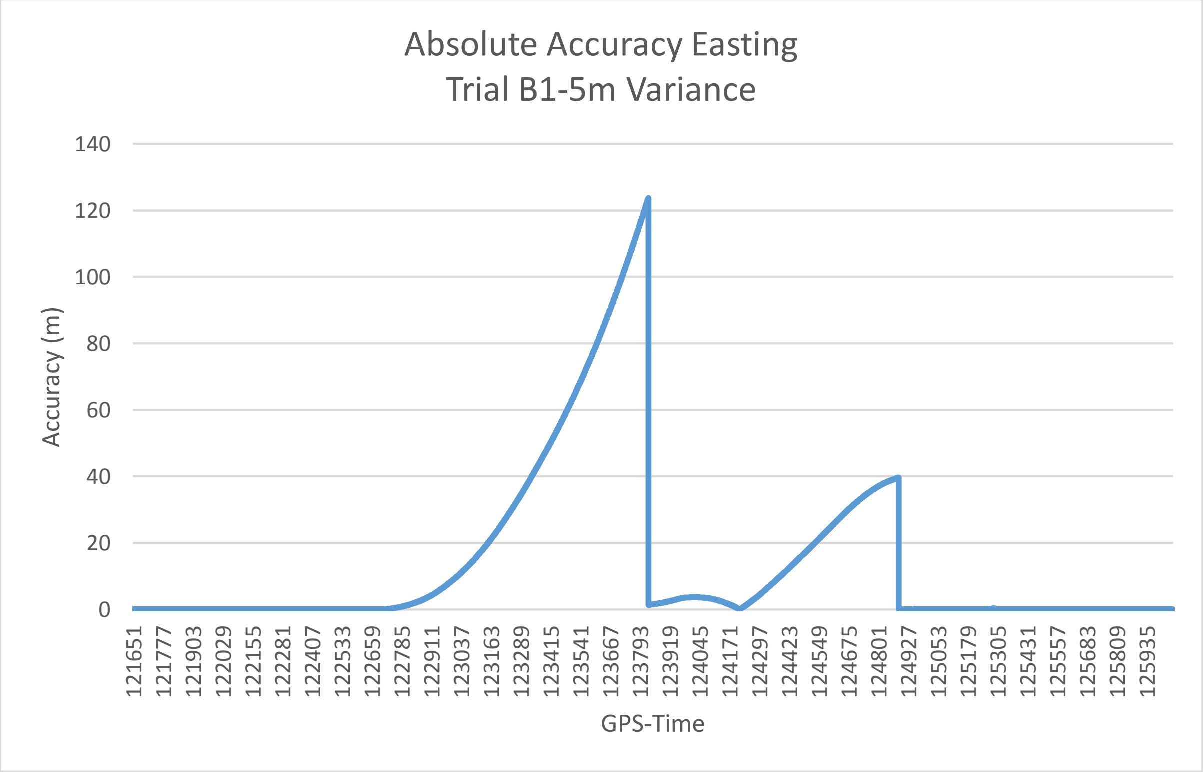

Figure 48: *Trial B1 – Absolute accuracy Easting*

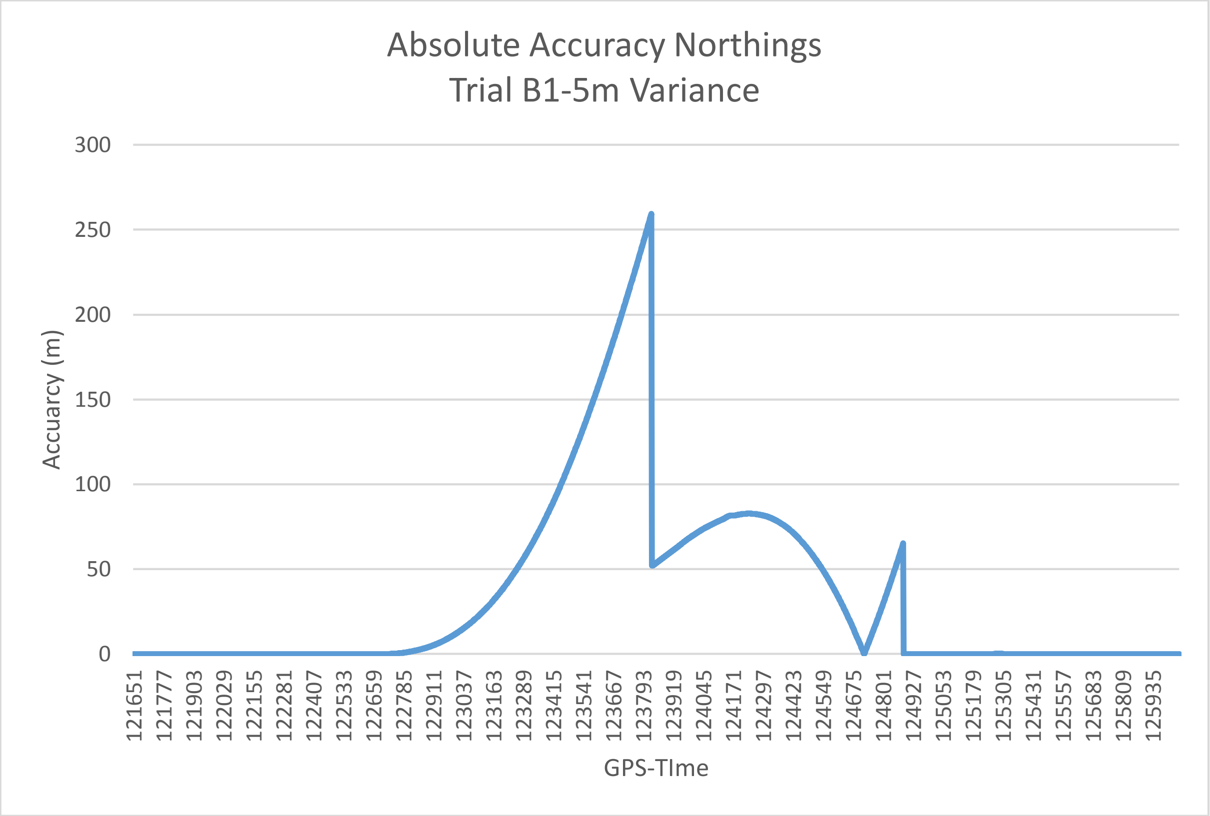

***Figure 49:** Trial B1 – Absolute accuracy Northing*

Trial B1 is processed as a forward IMU/INS in Inertial Explorer with a manipulated error of 50m to the north. The result is similar to trial A1, with the same path on the graph. The main difference is that Trial B1 reaches an absolute accuracy of 0m, before it increase again. The reason for this is that the error switches from the northern side to the southern side of "The Truth". As the graph only shows the absolute accuracy, it does not say what direction the error occurs.

The phenomenon only occurs on the trials with a manipulated northern error, where the manipulated position is between "The Truth" and the last calculated IMU/INS position. A new trial was processed, forcing the manipulated position North of the last calculated INS/IMU position.

Trial AA1 is processed in Inertial Explorer with a manipulated positon error 300m to the north.

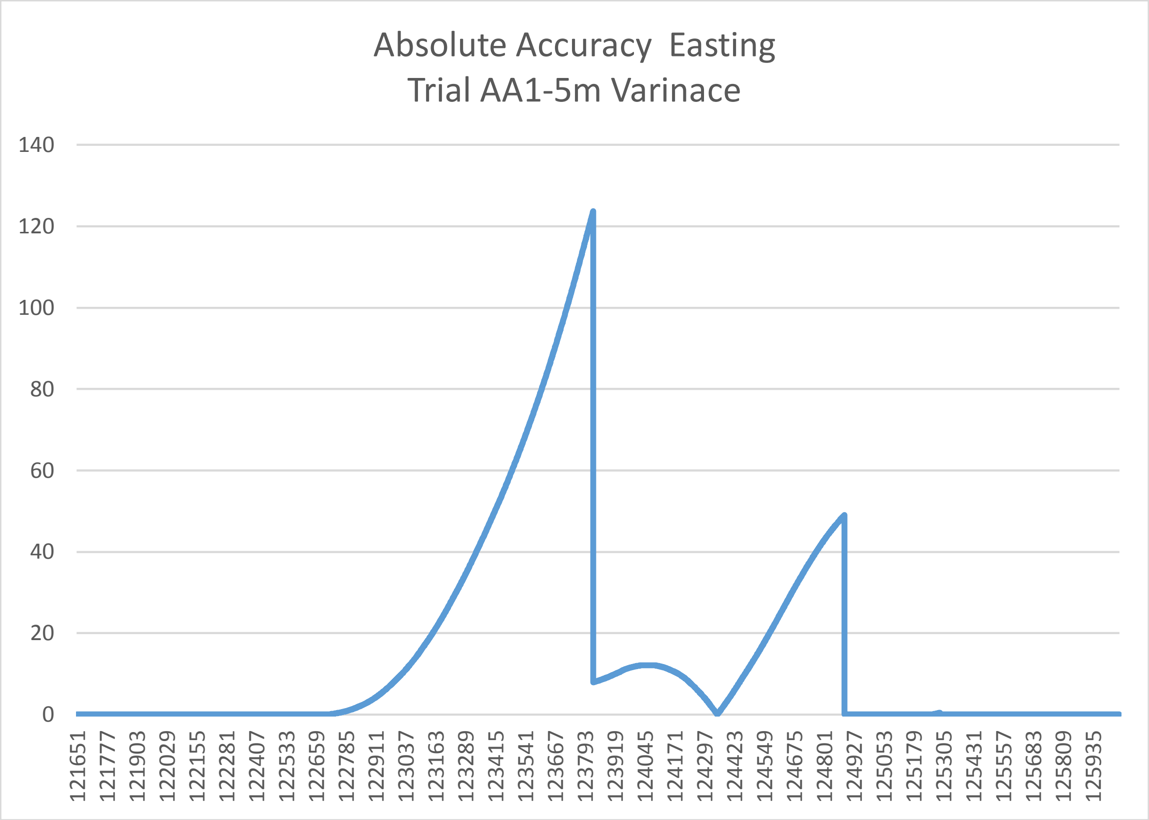

***Figure 50:** Trial AA1 – Absolute accuracy Easting*

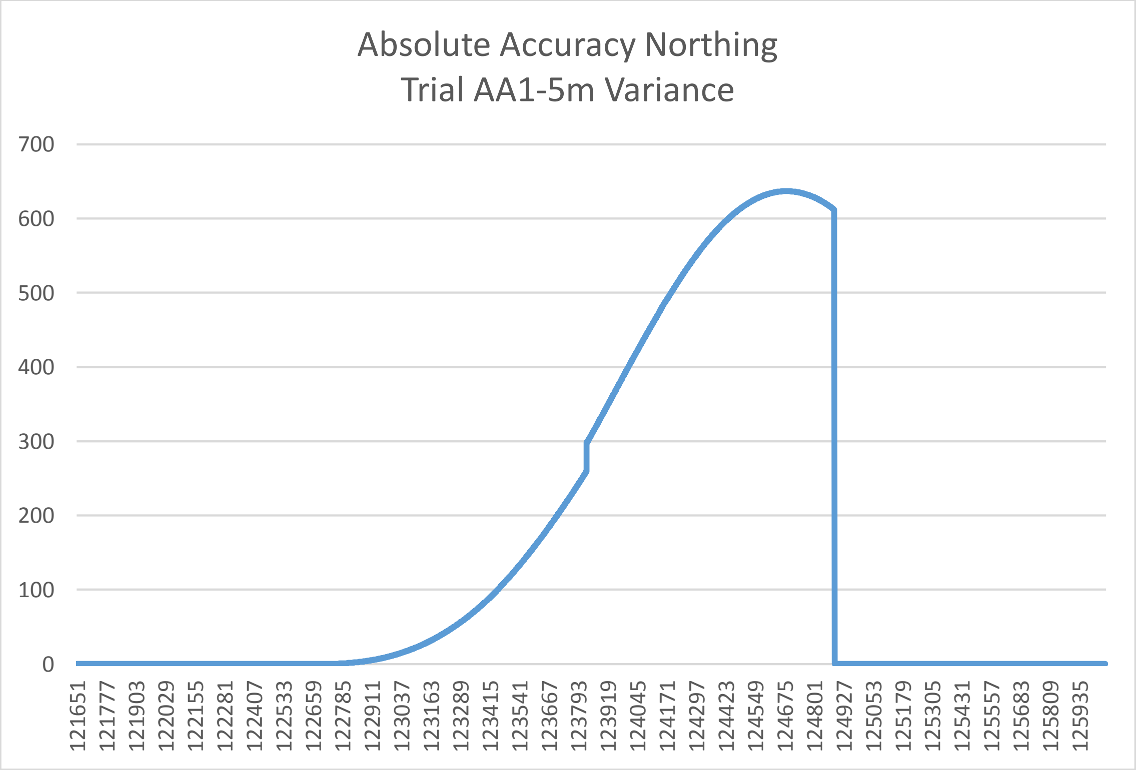

***Figure 51:** Trial AA1 – Absolute accuracy Northing*

Figure 50 and Figure 51 show the results for trial AA1 with a manipulated error of 300m to the North. The influence of the accuracy in East/West direction due to an error to the north is shown in Figure 50. The pattern of the graph in Figure 50 is similar to the pattern of the graph in Figure 46 and Figure 48.

The graph in Figure 51 does also converge before the surfacing position. Due to the large position error, and the updated position at the surfacing position it is unknown if the absolute accuracy will continue to improve, or if the graph will converge again.

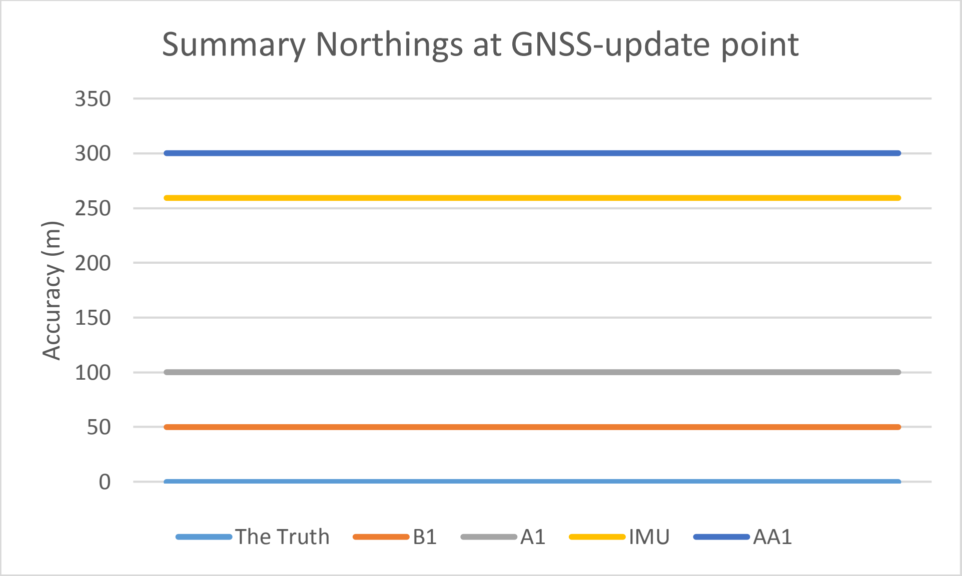

Figure 52: *Summary of Northings at GNSS-update position*

Figure 52 illustrates the difference in Northing value for the different trials. Trial A1 (100m) and trial B1 (50m) are both between the last calculated IMU/INS position solution and "The Truth". The figure illustrates that the manipulated position actually reduces the gap between the last calculated INS/IMU position and "The Truth".

Trial A1 and B1 gives the best results at the surfacing position compared to the other trials.

The result is similar to what happened when introducing a large standard error, where the position did not coincided the GNSS-position. One explanation might be that the corrections to the IMU is more balanced, and gives a more accurate

IMU after the position update if the difference between the calculated and update position is smaller.

One issue by using the GNSS for updating the INS/IMU for the submarine operating submerged for several hours is that the GNSS do not know the movements the submarine has done when submerge. That might cause problems when calculating the corrections, especially during a few epochs. There is a huge difference by going in a straight course between two GNSS-updates compered to lot of maneuvering and velocity changes.

5 Conclusion and further work

5.1 General

This project is an analysis of the affect the environment have on the accuracy of an IMU/INS combined with long periods with GNSS-outage in a Submarine context.

The study focuses on the effects of the following parameters have on an IMU:

- Long periods with GNSS-Outage
- Environment (Multipath)
- Standard error
- Impact of velocity on the accelerometers

The experiment is conducted by simulating the different effects in Inertial Explorer. The data has been collected by car.

All results are based on a scenario where the Submarine is forced deep after only 4 epochs, 1Hz GNSS-updates.

5.2 Summary of results

The accuracy of the INS/IMU is relatively well when operating in autonomous without any GNSS-input. This is based on an INS/IMU with a proper alignment and available GNSS before the Submarine is diving. The error growth is larger during accelerations/de-accelerations and during abrupt turns.

The result show that the accuracy is better cross-track than long-track. That indicates that the gyro is very accurate.

Based on the trials with different time-intervals of GNSS-availability, the duration of GNSS-access does not seem to have a large impact of the accuracy for short intervals (up to 120 seconds). The quality of the GNSS-solutions are more important than the quantity.

The results shows that the absolute accuracy is better with a high standard error, both in Easting and Northing in all trials except two (A1 and B3). The size of the standard error has the greatest influence in North/South direction.

When there is a huge difference between the last calculated- and the updated positon, a large standard error gives the best results. Likewise, if there is a small difference between the last calculated- and update position, a small standard error gives the best result.

The INS/IMU acts as expected when position errors are added (simulated Multipath). In most cases does the error growth as a factor of time until another GNSS-update is added. However, in two of the trials (A1 and B1) does the multipath has a positive effect on the INS/IMU, and the most precise absolute accuracy is from trial B1, where a manipulated error of 50m to the north was added.

Operating in the littorals will affect the accuracy and the performance of an integrated GNSS/INS. The user should be very diligent if the update period is short. The multipath might improve the performance of the IMU, but it is hard for the user determine the accuracy of the update (beside DOP-values).

5.3 Recommendations for further research

This study is based on data collected by a van in Nottingham. The trial is realistic, but the velocity and dynamics are not typical for a Submarine. The following recommendations for further research are as follows:

- Retake the trial for dataset two in order to determine if the velocity affects the performance of the accelerometers.
- A data collection inside a fjord with the antenna 10-30 cm above the sea surface should been conducted in order to get a realistic multipath environment.
- Define the "Time to first fix" in a realistic environment.
- Verify the influence by depth-changes to the IMU.
- Conduct the trials with both tightly coupled and loosely coupled INS/GNSS.
- Deeper study of the influence of the standard error.
- A deeper study of Trial A1 and B1.
- Develop a tool for the user as makes it easy to determine the quality of the INS/IMU update.

The study would be more precise by conducting the experiment on a Submarine. However, there are some challenges both from the Navy and the industry. It would also be difficult to define "The truth", as the submarine is without any GNSS-access.

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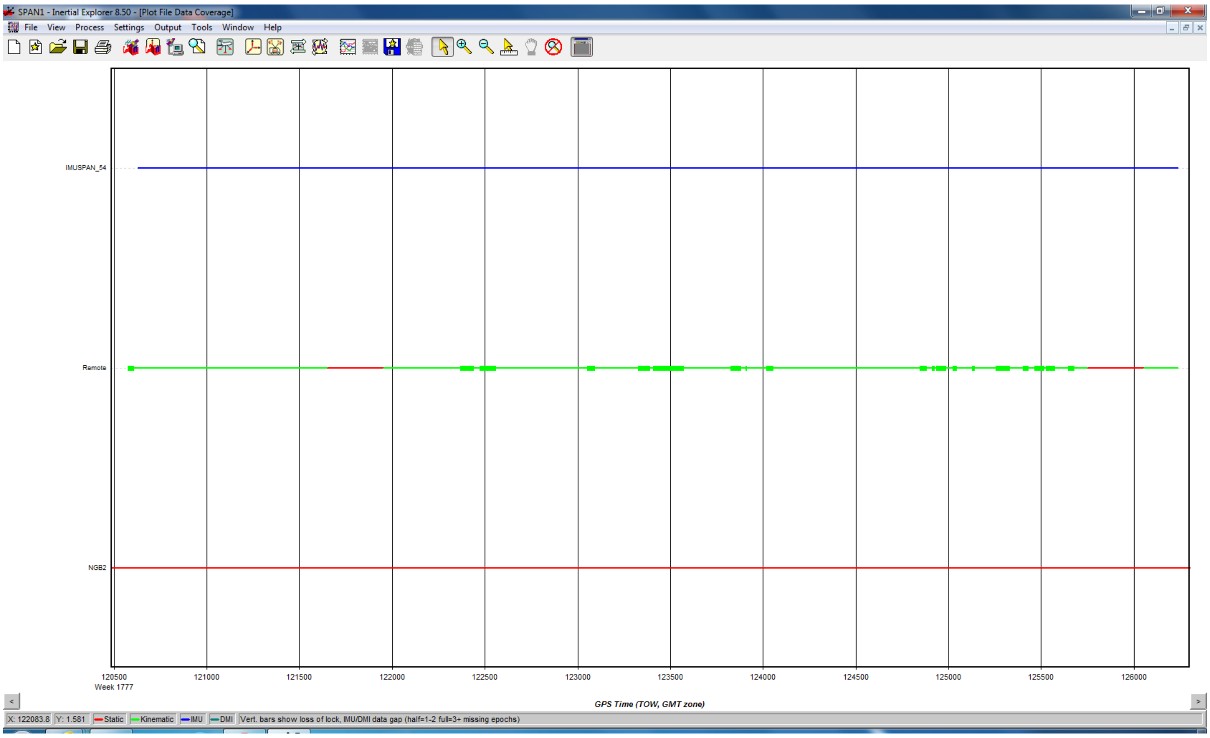
Appendix A: Workflow

A1: Processing SPAN GNSS Data

The outcome of the project is only interested in the performance of the IMU component. Therefore the same GNSS data set will be used for each IMU.

The “File Data Coverage Plot” shows the data overlap, and determines if the Rover-, IMU- and Base station data overlaps.

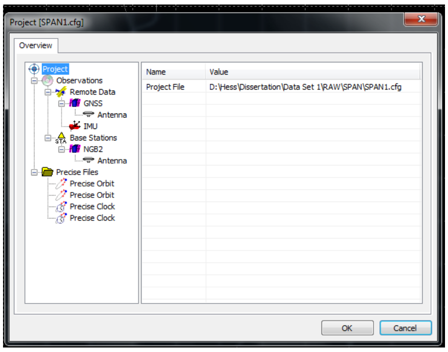


Vert. bars show loss of lock, IMU/DMI data gap (half=1-2 full=3+ missing epochs)

Figure A1.1: *File Data Coverage Plot (Inertial Explorer)*

<u>Precise Clocks/orbits:</u>

The precise orbits/clocks latency is 12-18 days and the GPS rapid is less than a day. The precise clocks and orbits are uploaded from the actual day (27.01.2014).



Name	Value
Project File	D:\Hess\Dissertation\Data Set 1\RAW\SPAN\SPAN1.cfg

Figure A1.2: *Project overview (Inertial Explorer)*

Processing GNSS data:

Processing method: *Differential GNSS*

Processing Direction: *Both*

Profile GNSS: *Ground Vehicle* (No changes to the *Advanced* options made)

The overview of the quality of the position is available in “Processing summary”.

Plots:

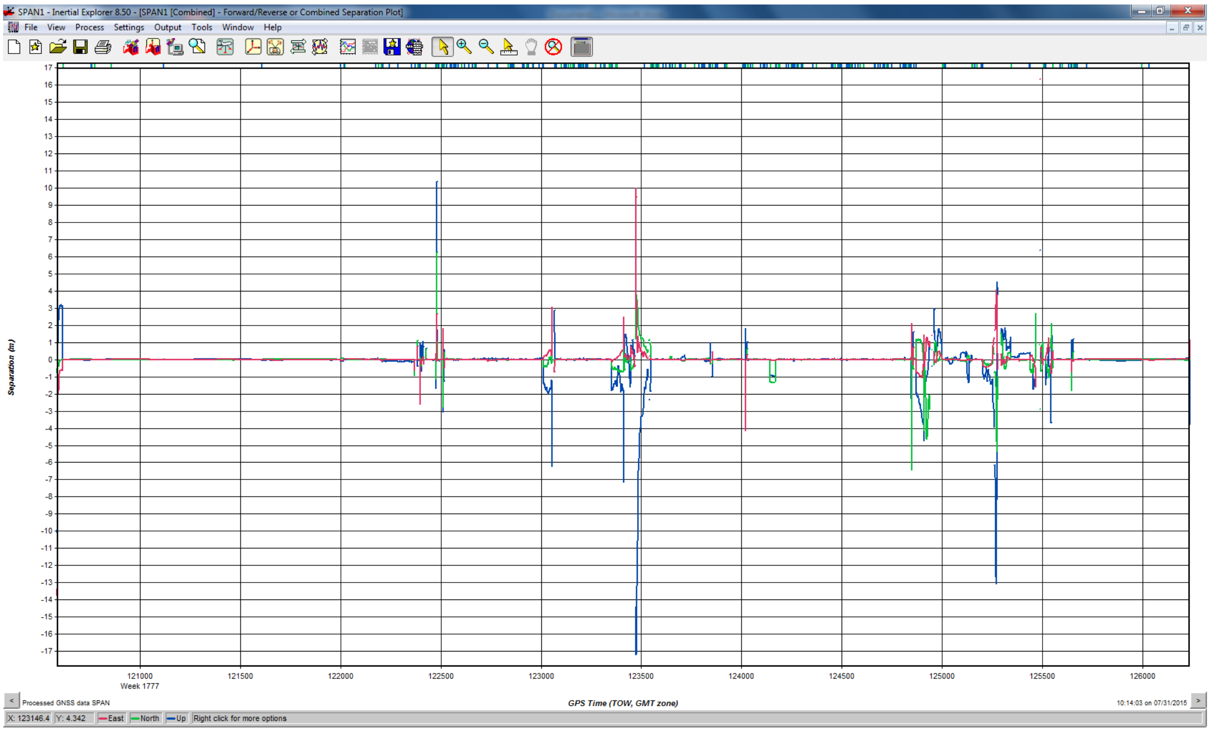


Figure A1.3: *Combined Separation Plot (Inertial Explorer)*

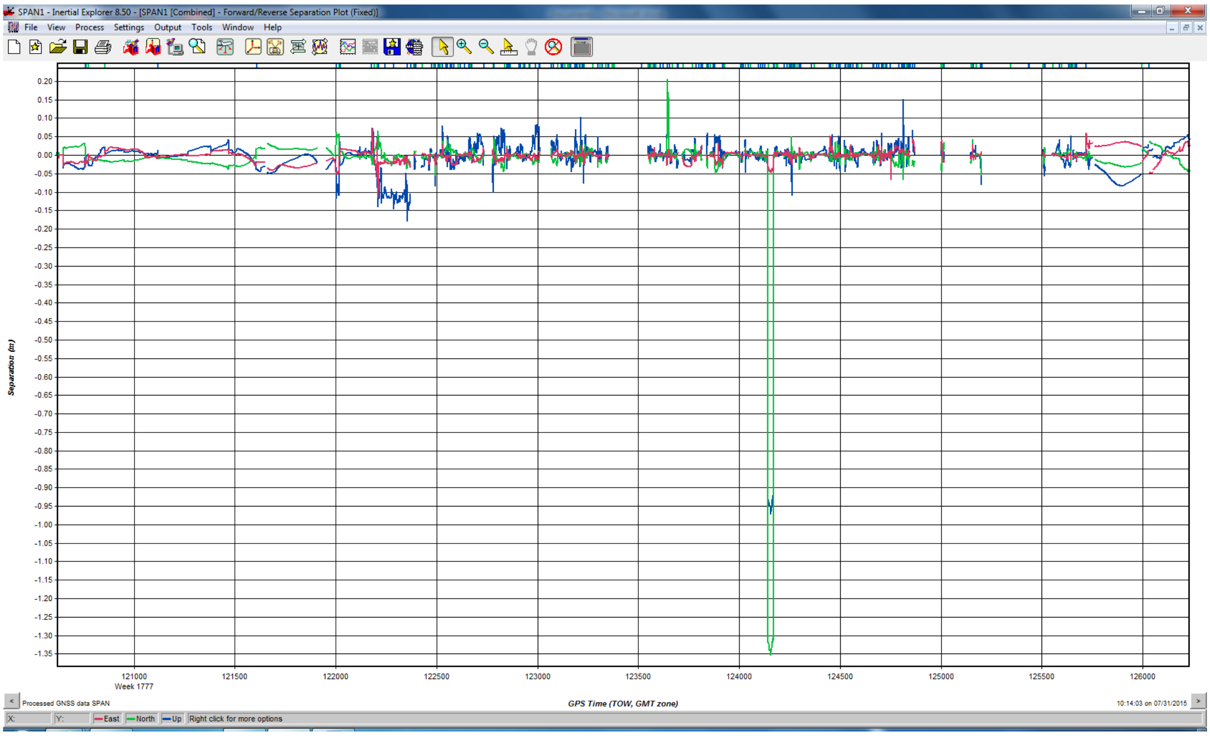


Figure A1.4: *Combined Separation with Fixed Ambiguity (Inertial Explorer)*

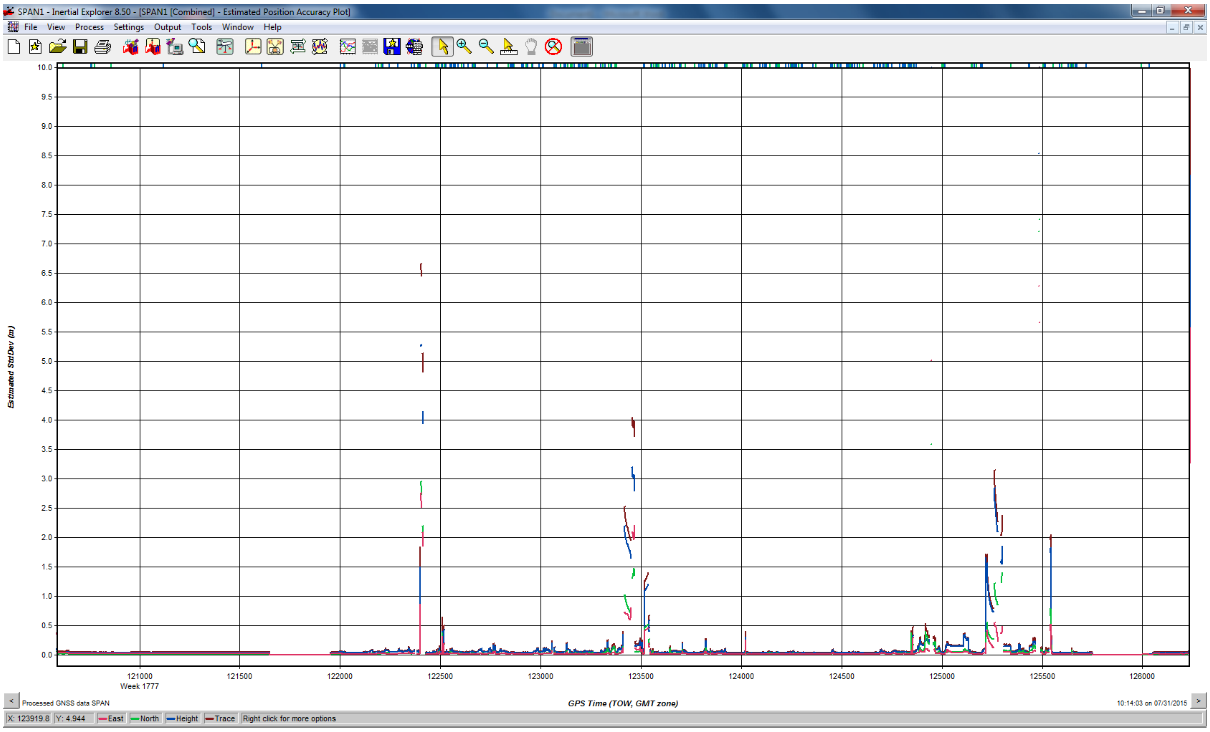


Figure A1.5: *Estimated Position Accuracy (Inertial Explorer)*

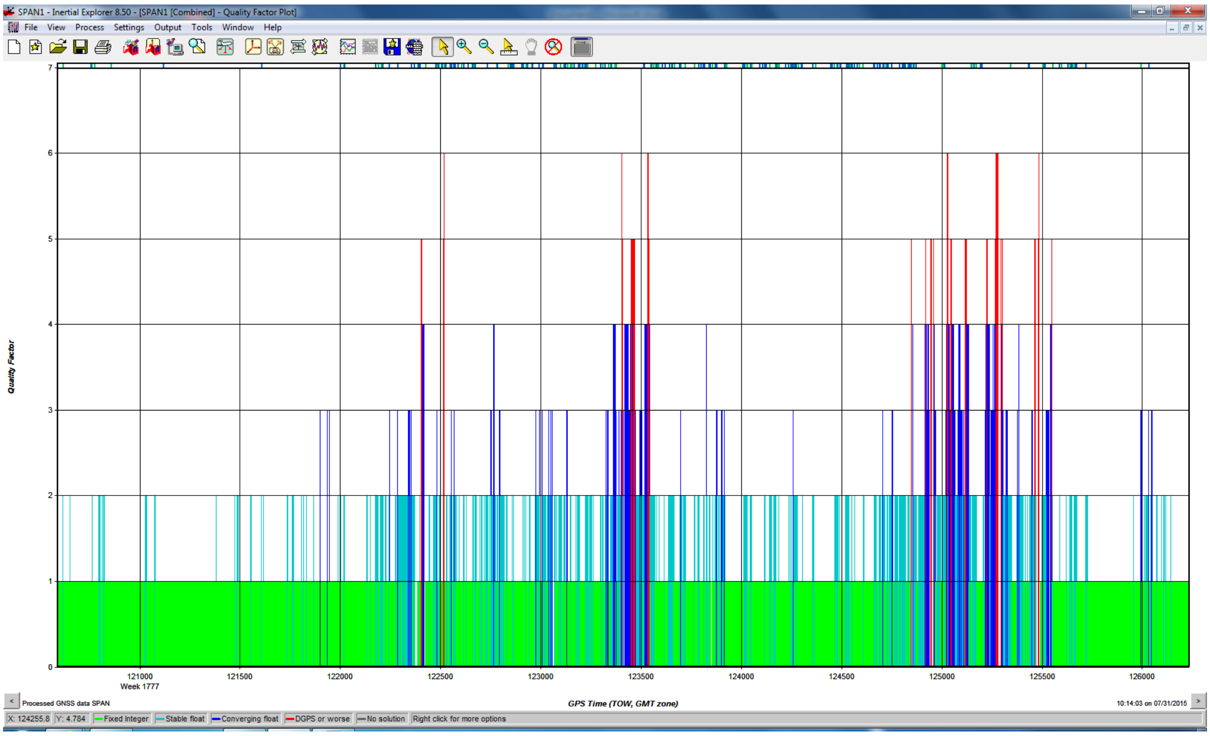


Figure A1.6: *Quality Factor (Inertial Explorer)*

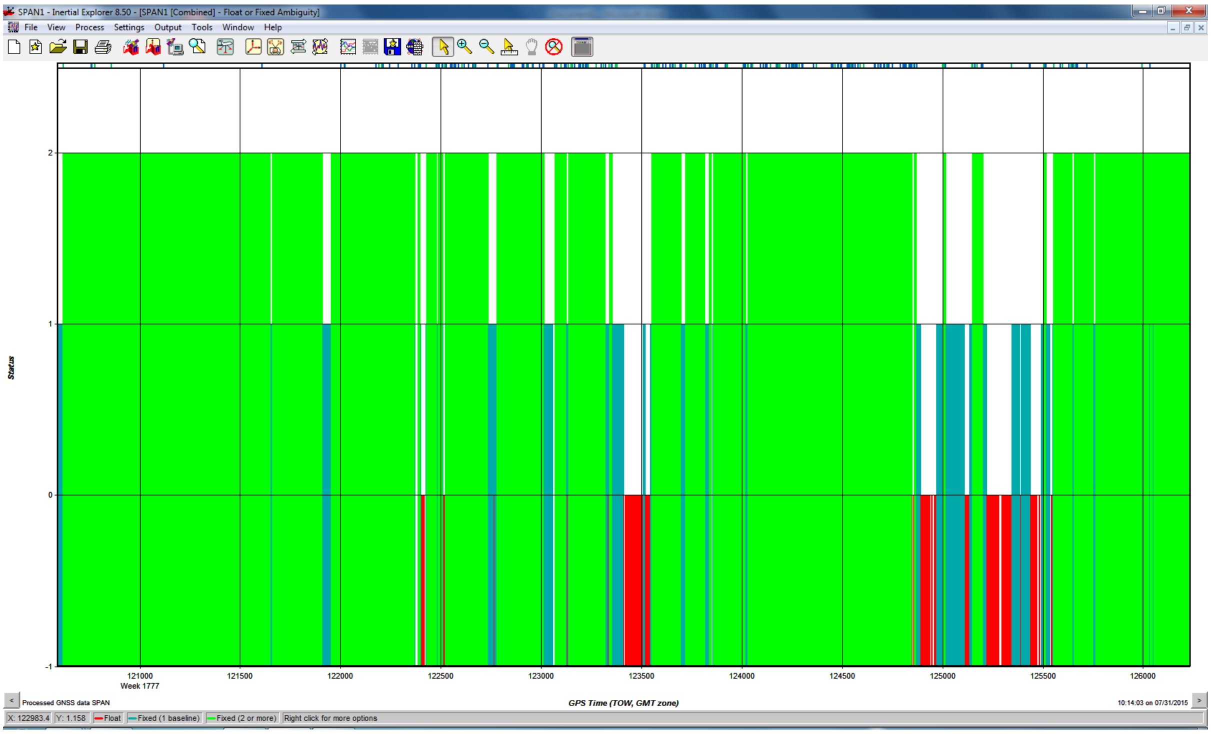


Figure A1.7: *Float Fix Ambiguity (Inertial Explorer)*

To check Baseline data the following plots have been used:

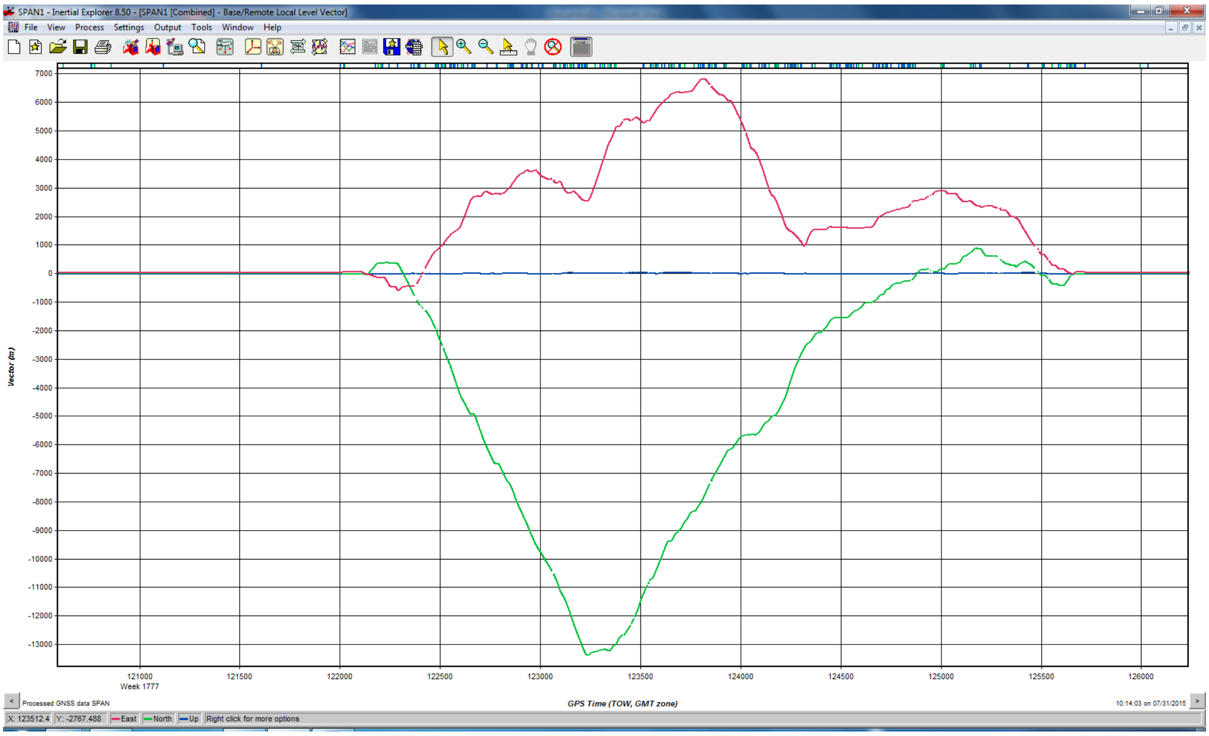


Figure A1.8: *Local Level Vector (Inertial Explorer)*

A2: Introducing Static data in GNSS processing

For the initialisation of IMU it is needed to identify static periods at the beginning and the end of the dataset. It is possible to use the GNSS data only, but it is preferred to try to improve the accuracy.

Acceleration profile and Velocity profile plots were used to identify static periods.

A3: Processing POSRS

Processing Loosely Coupled IMU:

Update data: External Trajectory (GNSS.cmb) – GNSS.cmb is based on the combination of GNSS and SPAN IMU, and are defined as “The Truth”.

POSRS Profile: SPAN Airborne (uIRS)

Lever Arm offset: In accordance with data from NGI-van.

Body to IMU Rotation: 180, 0, 90

It is preferable to only process the data from the beginning of the static period to the end of the static period.

“Acceleration Profile” and “Velocity Profile” is used to confirm that only data within the static periods have been processed.

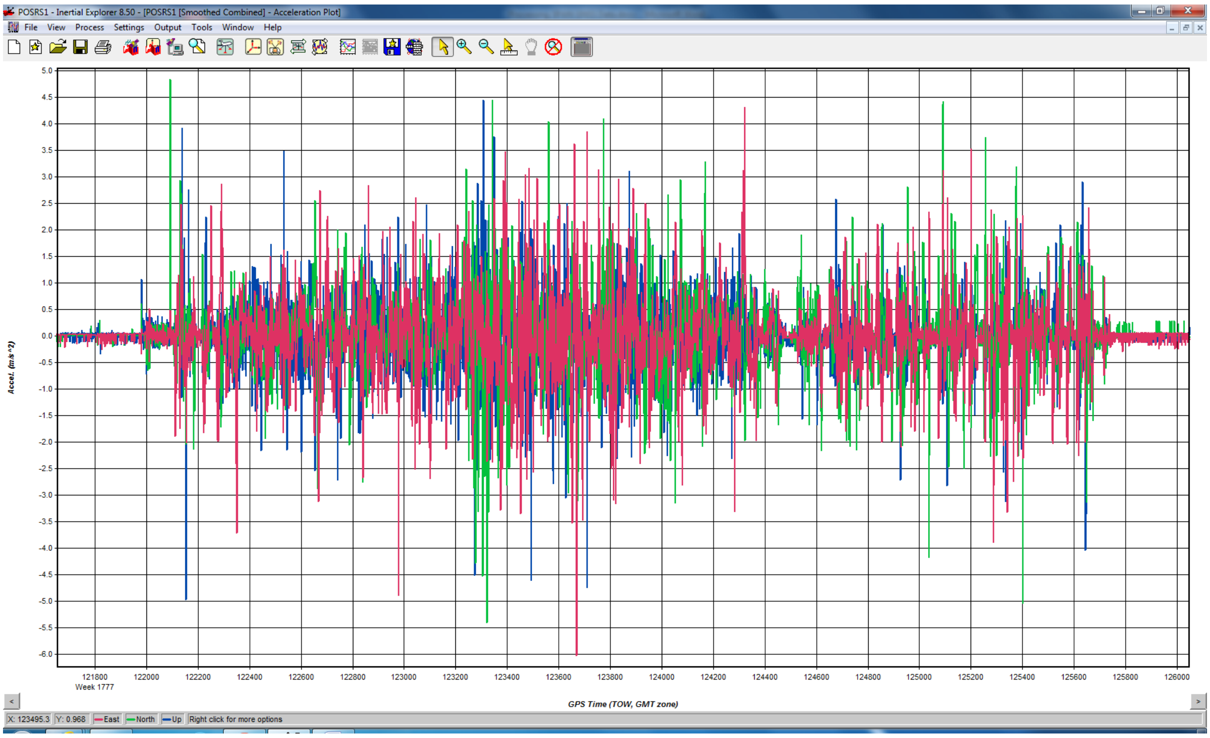


***Figure A3.1:** Acceleration Profile (Inertial Explorer)*

Figure A3.1 shows that only data within the beginning and the end of the static period is processed.

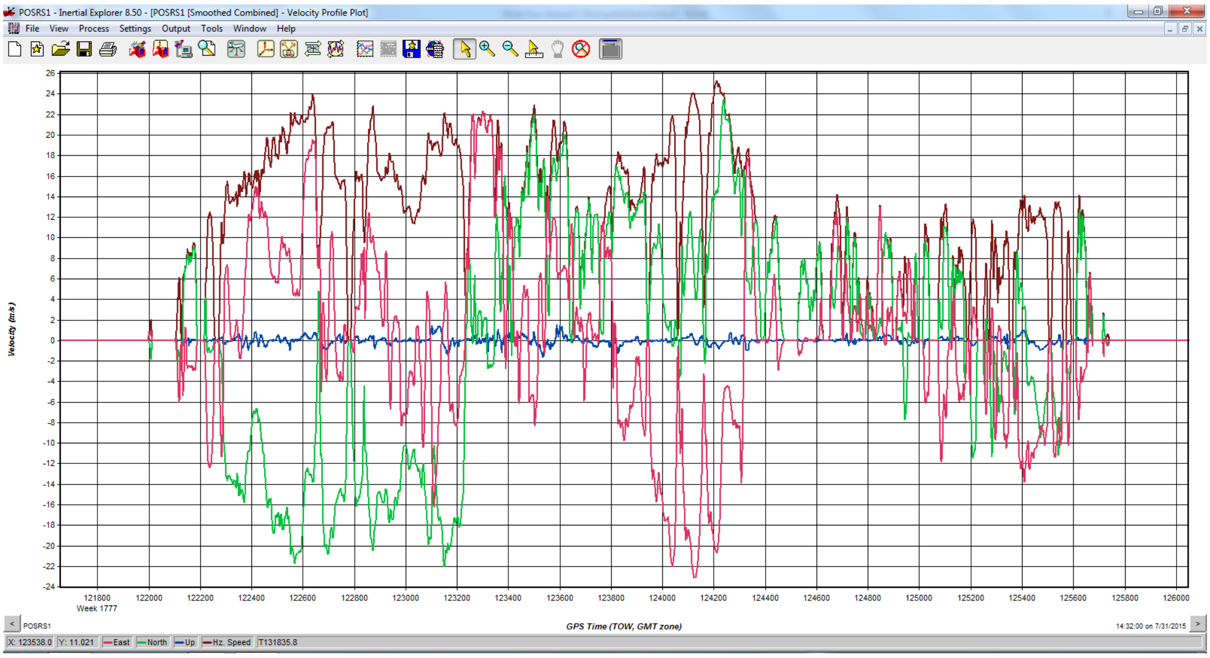


***Figure A3.2:** Velocity Profile (Inertial Explorer)*

Figure A3.2 shows the defined static periods both at the beginning and the end of the dataset. And only data inside the static period is processed

A4: Static Coarse + Fine alignment

Within 5-10 minutes, GNSS updates enable the IMU to provide attitude information consistent with the accuracy level achievable by the accelerometer/gyro triad (Inertial Explorer user guide p.17)

A staic period of 300 seconds (5 minutes) at the beginning and the end of the trial was used in this experiment, giving sufficent time to aligne the IMU.

Settings:

Coarse: 80s

Fine: 110s

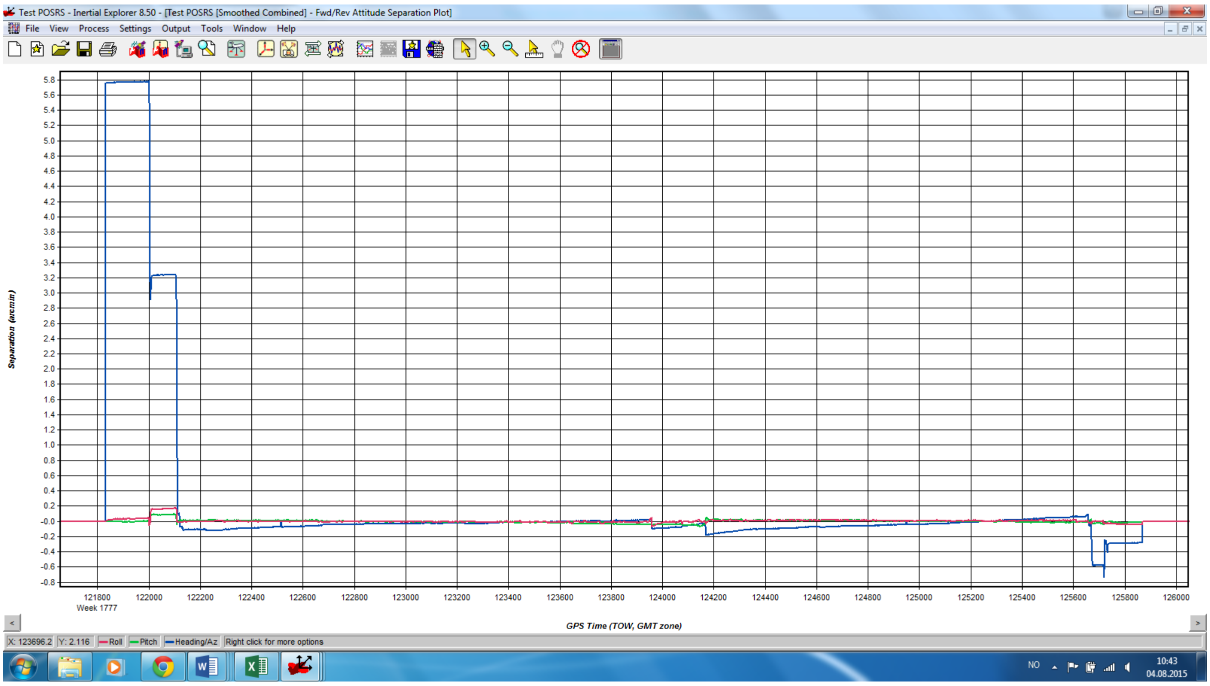


Figure A4.1: *Attitude Separation plot after Static Coarse+Fine Alignment (Inertial Explorer)*

The Attitude Separation plot (Figure A4.1) shows the difference between the forward and reverse solutions in terms of roll, pitch and heading.

Static coarse and Fine alignment was conducted both forward and reverse. The result is within the recommended 10 arc-sec. As expected, there are large jumps in heading in the beginning and the end of the dataset.

A5: Transfer Alignment:

When Transfer Alignment is conducted, the data from both the forward and reverse solution from Coars+Fine alignment is used for a smoothed solution.

Checking Results:

IMU lever Arms:

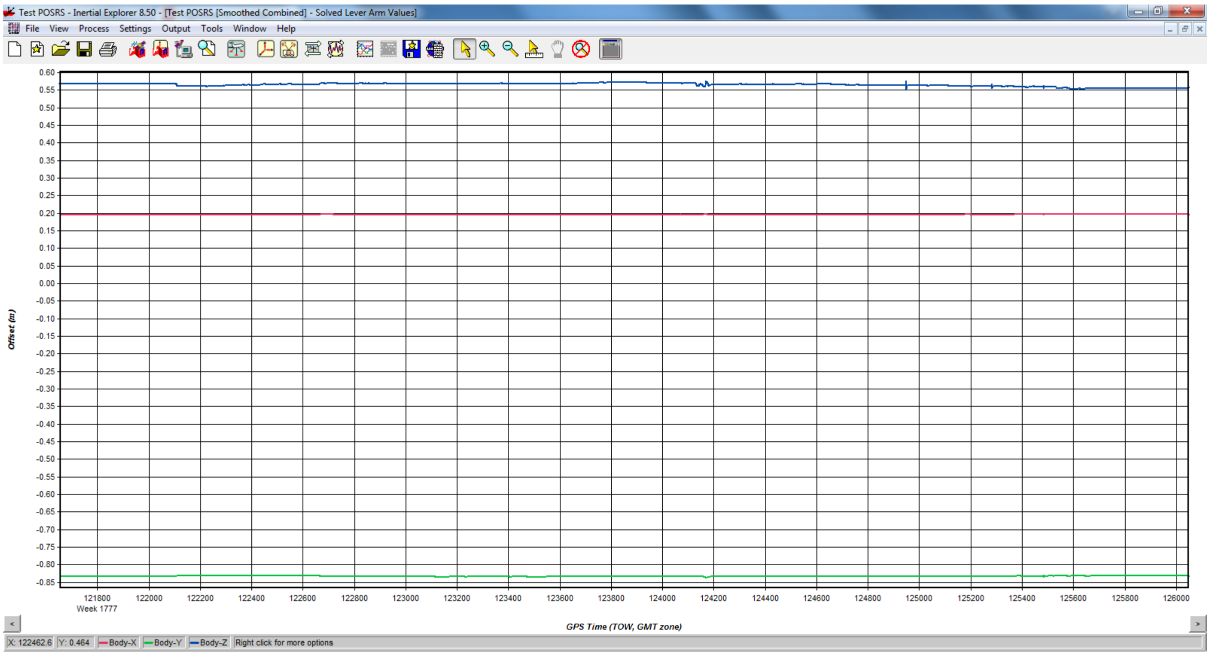


Figure A5.1: *IMU-GPS Lever Arms (Inertial Explorer)*

The IMU-GPS Lever Armos plot (Figure A5.1)presents the body-frame components of the lever arm offset between the IMU and GNSS antenna.

Transfer Alignment:

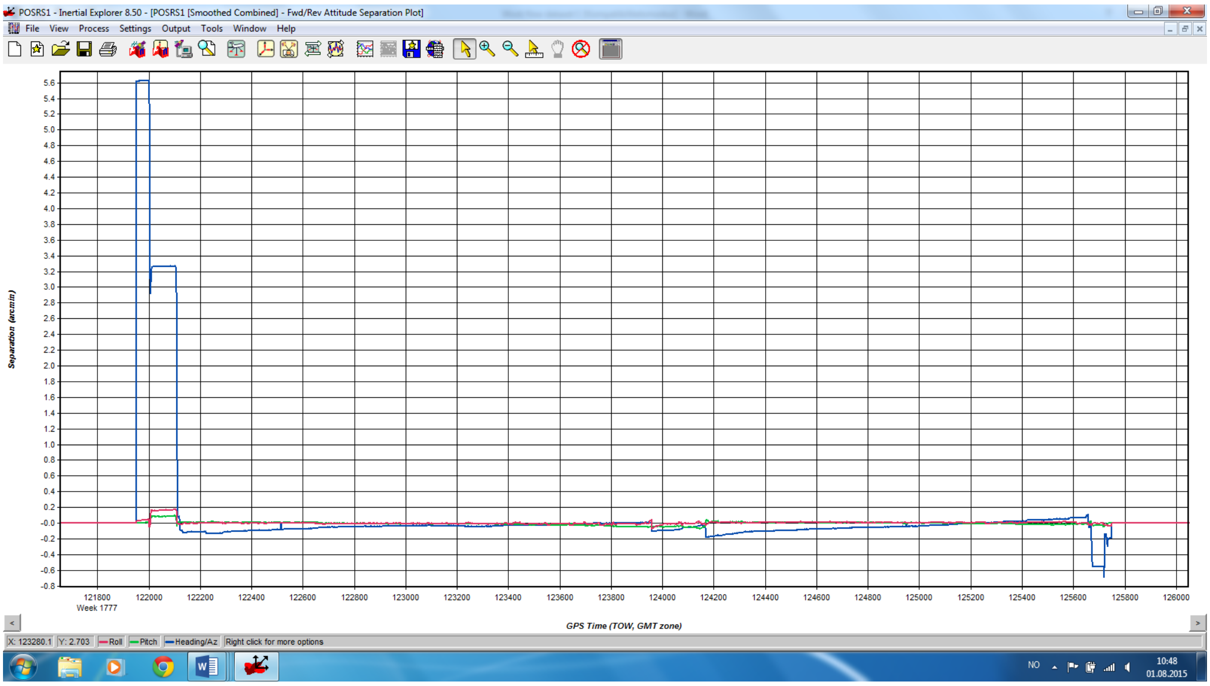


Figure A5.2: *Transfer Alignment (Inertial Explorer)*

Checking Results:

Lever ARM POSRS:

X: 0.198

Y: -0.830

Z: 0.552

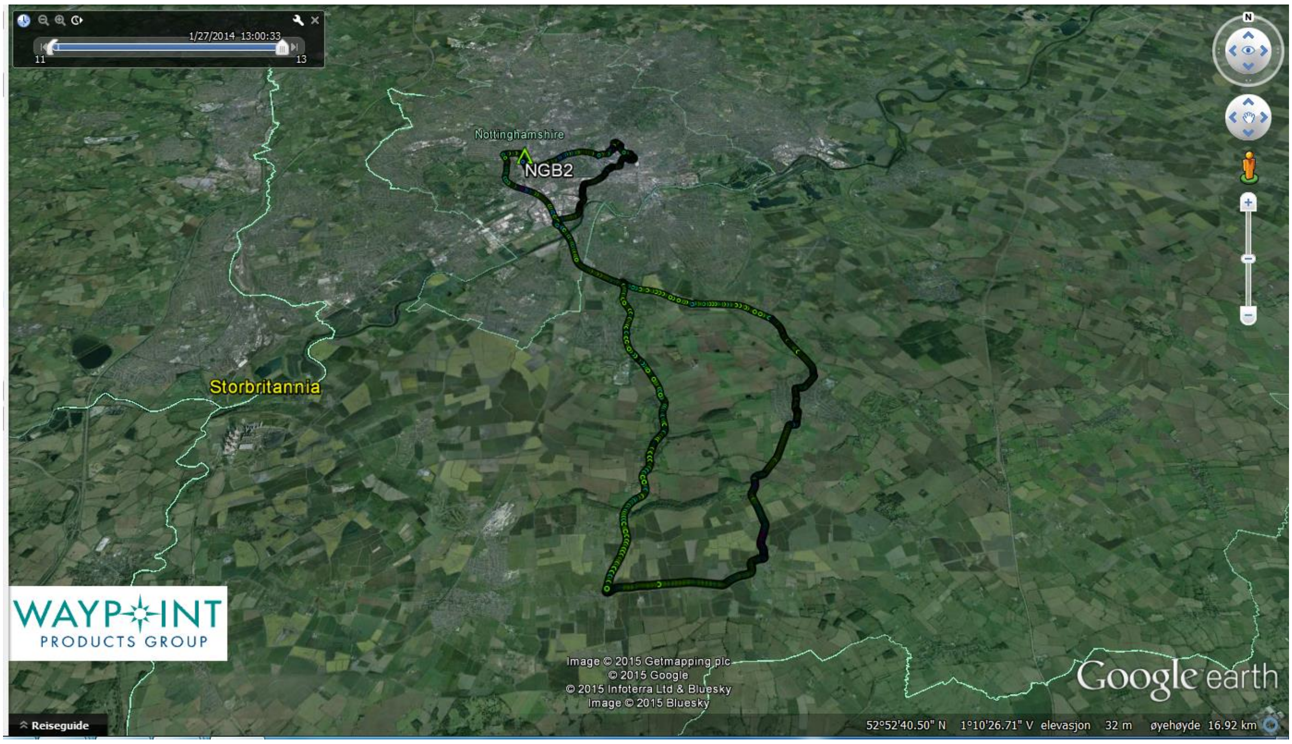


Figure A5.4: *"The Truth" – Complete route (Google Earth)*

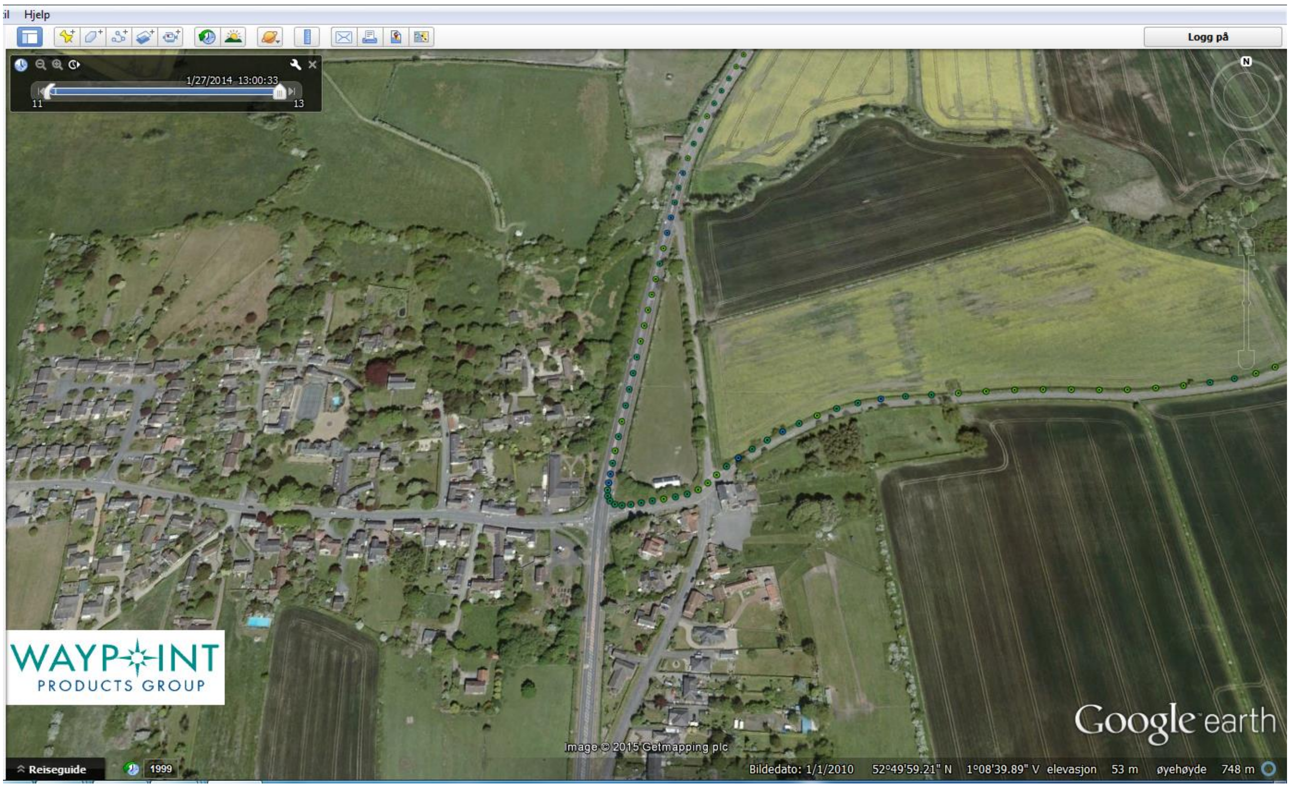
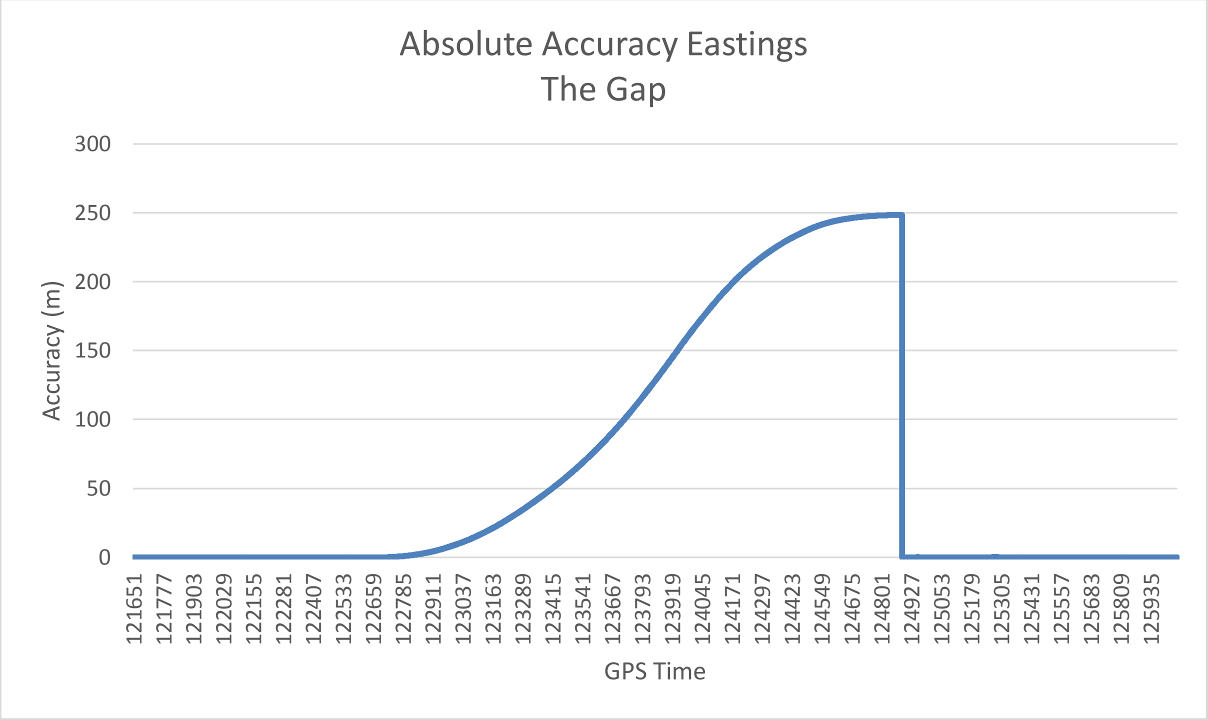
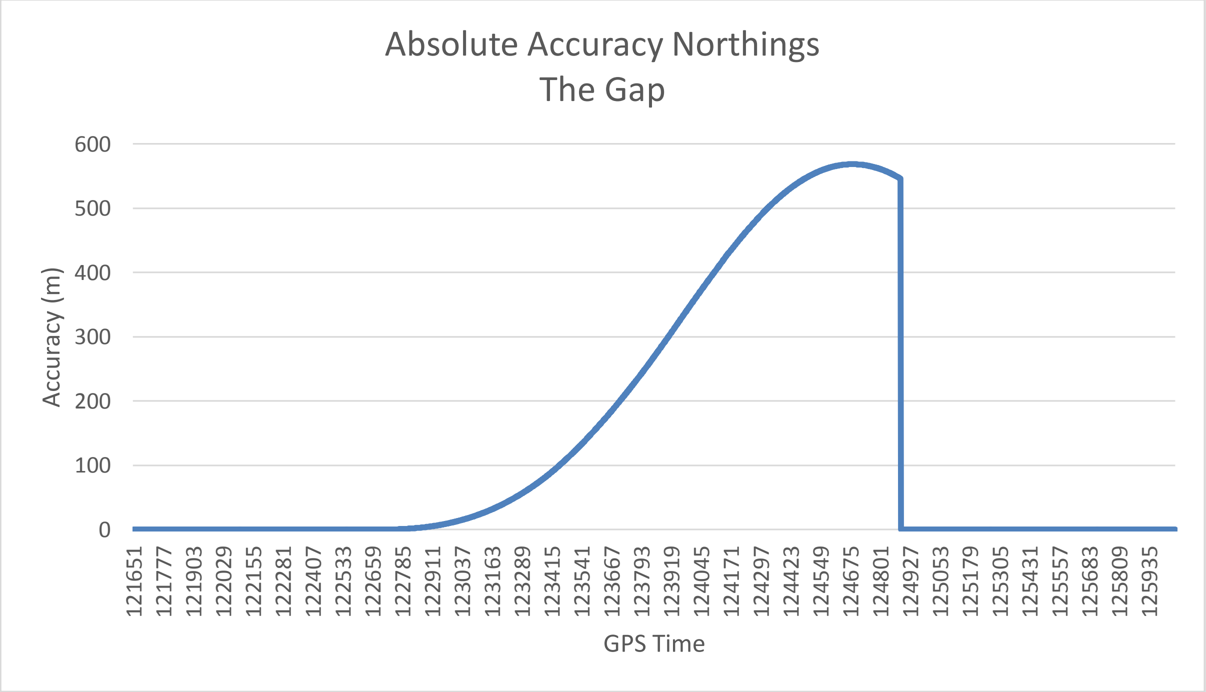


Figure A5.5: *"The Truth" – Turn (Google Earth)*

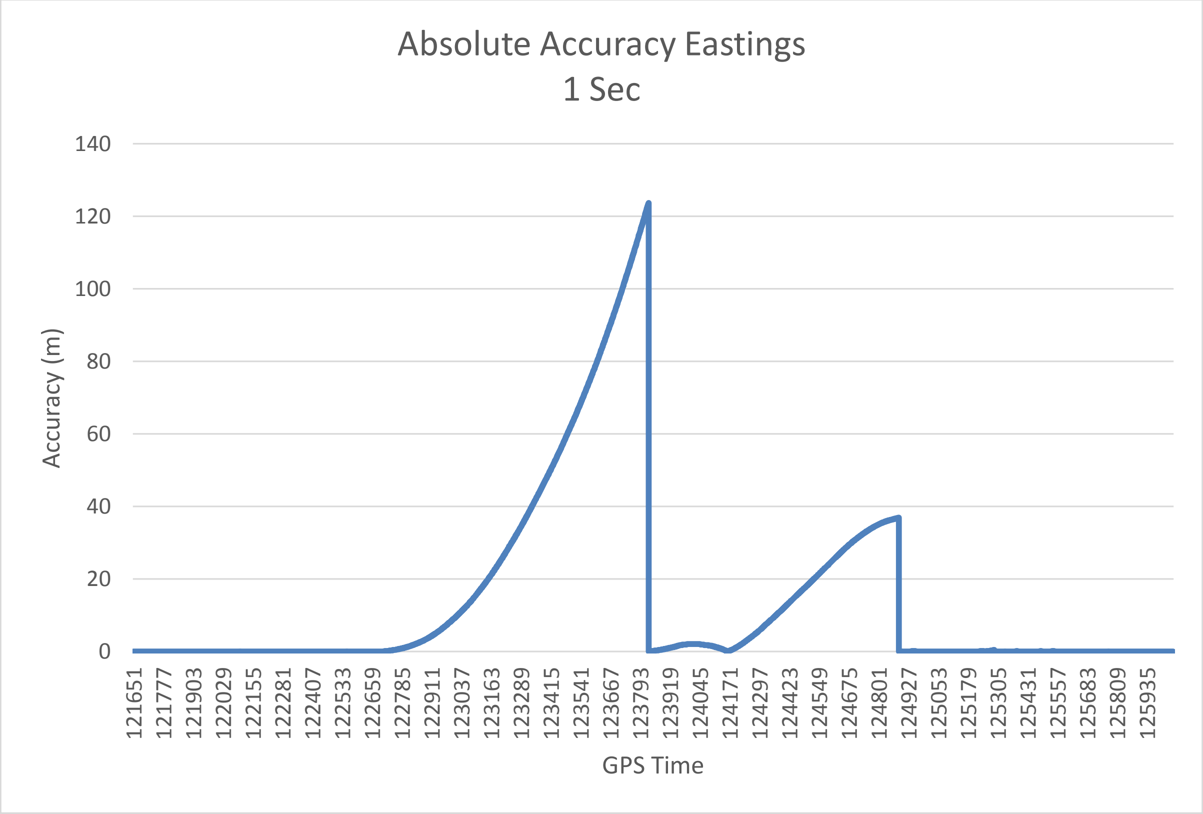
Appendix B: Absolute Accuracy with different GNSS-update intervals from "The Truth)

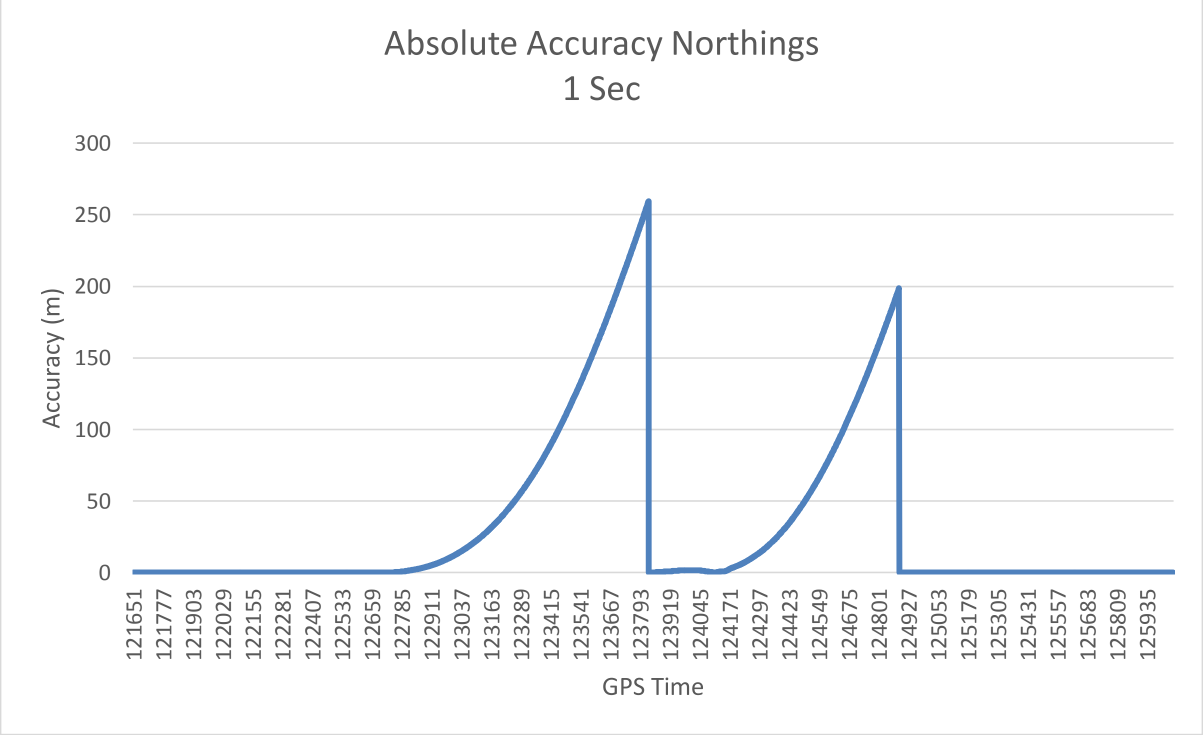
B1: The Gap



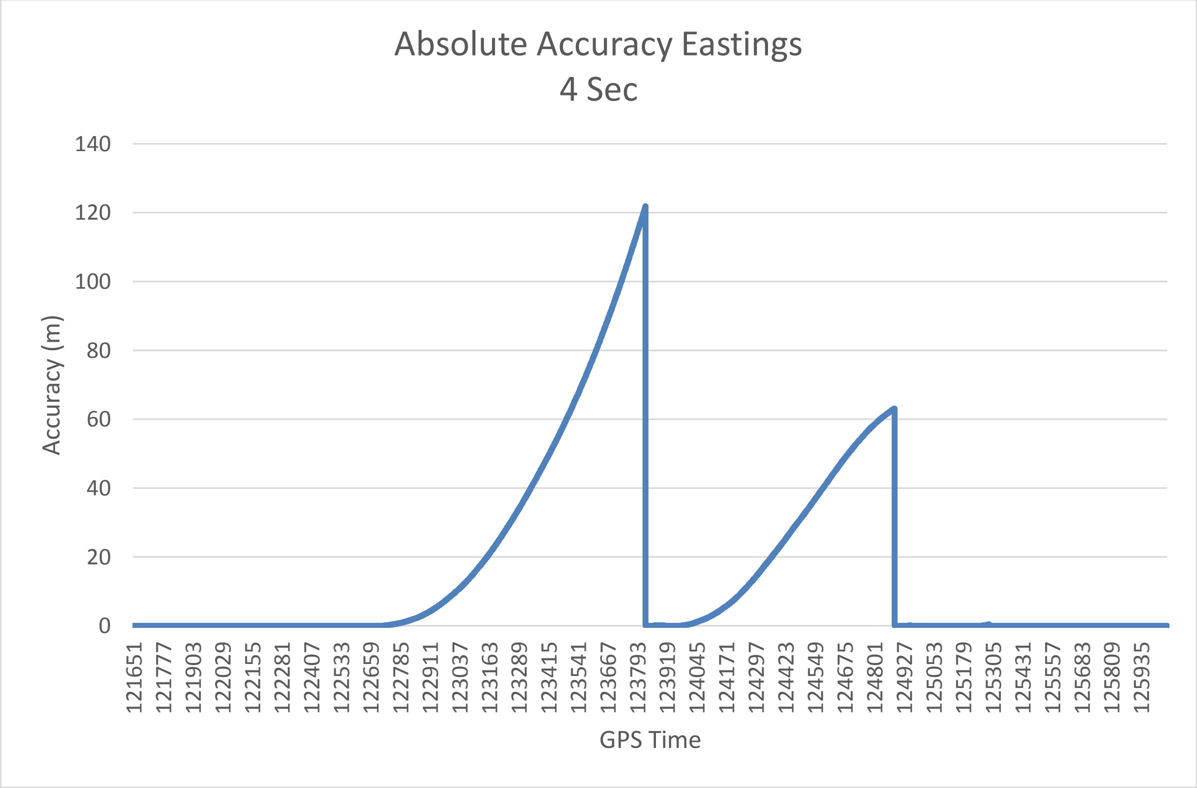


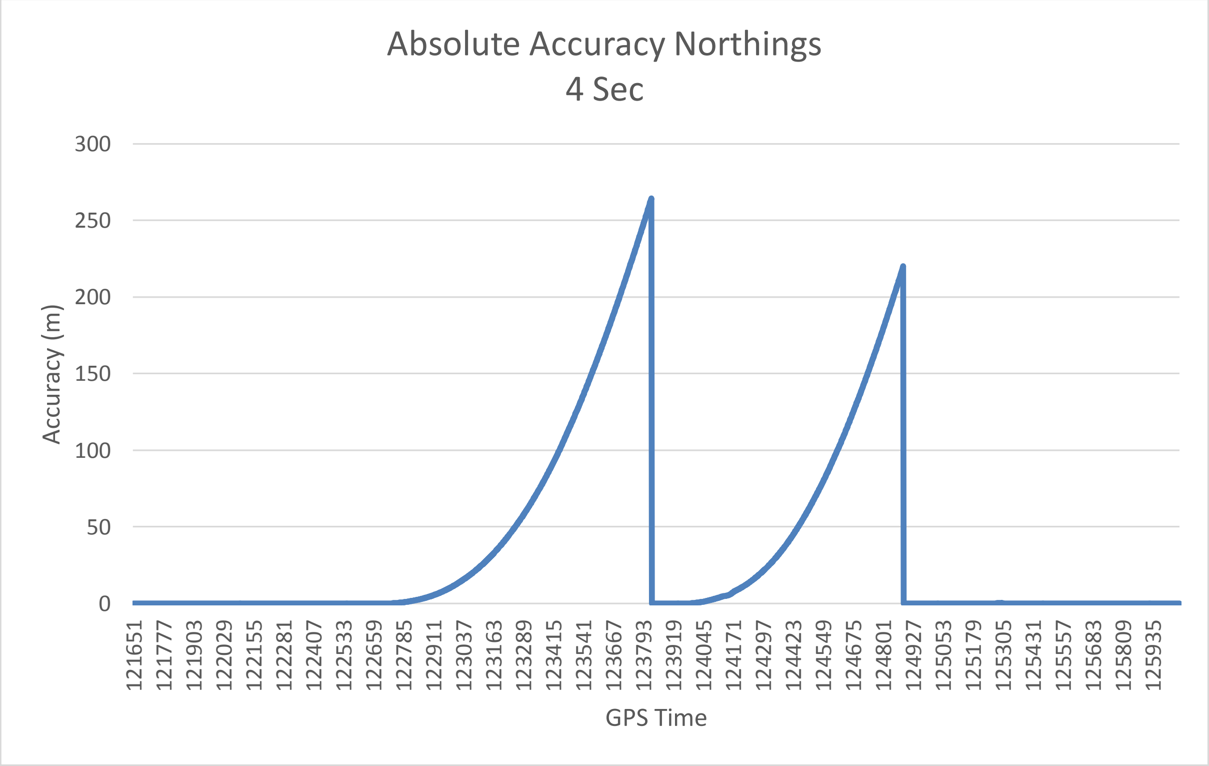
B2: 1 Second Update



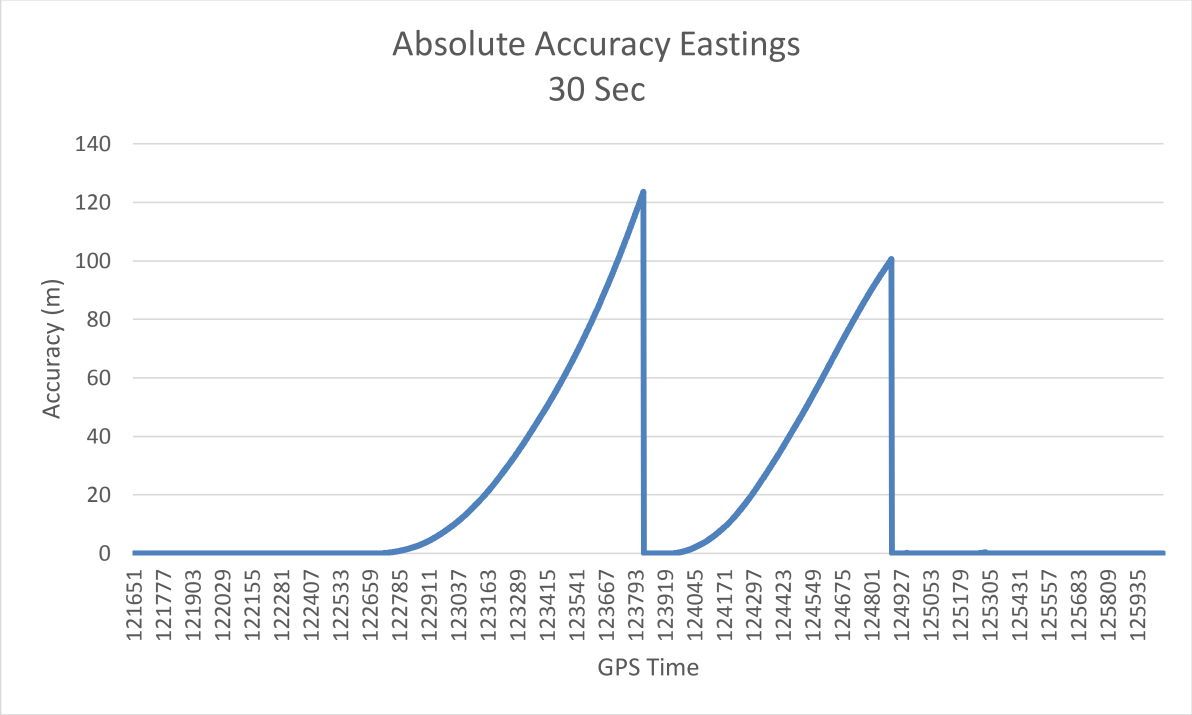


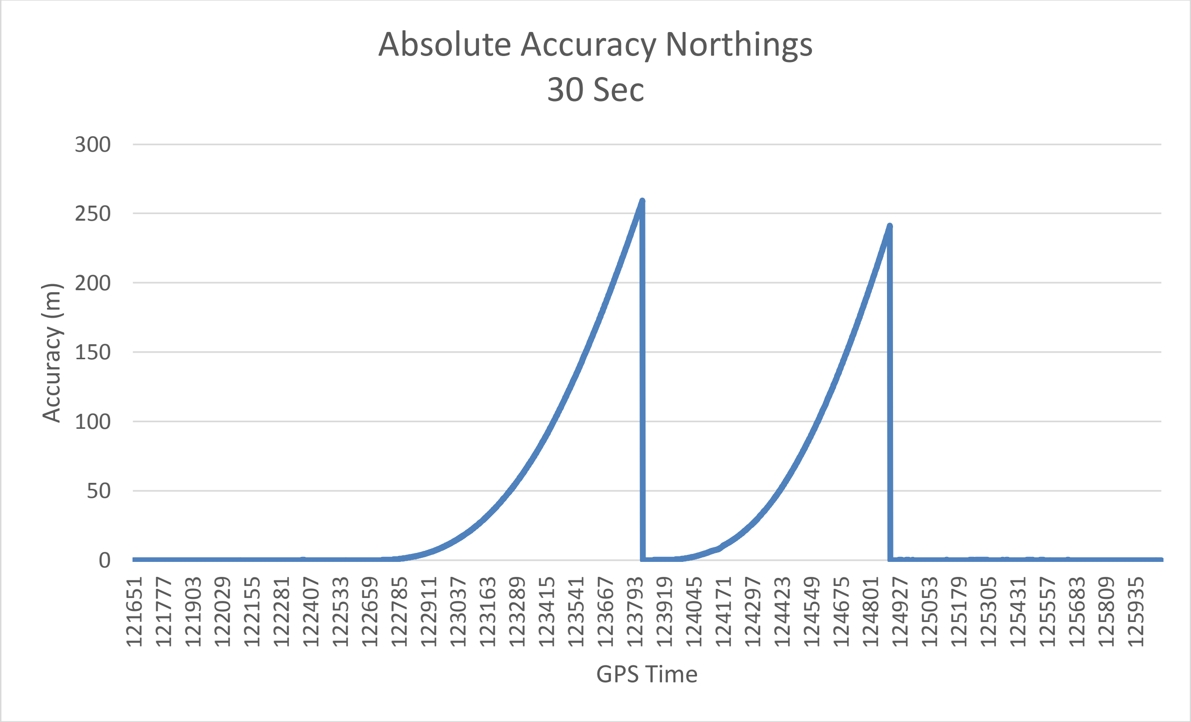
B3: 4 Seconds update



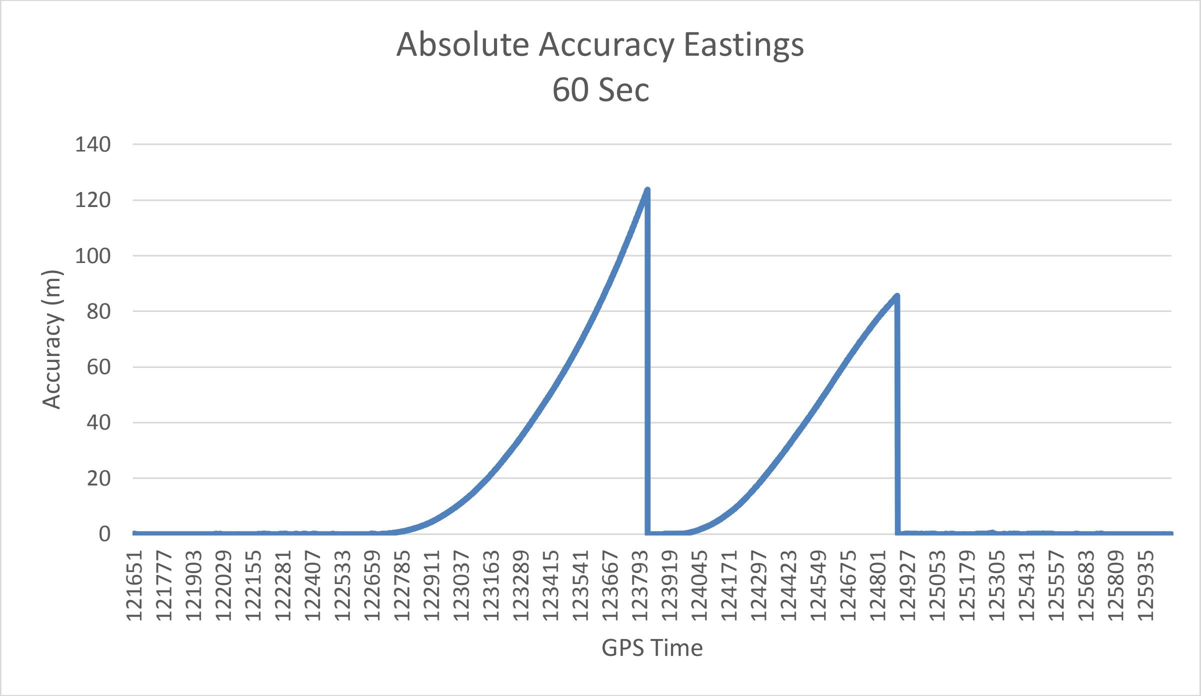


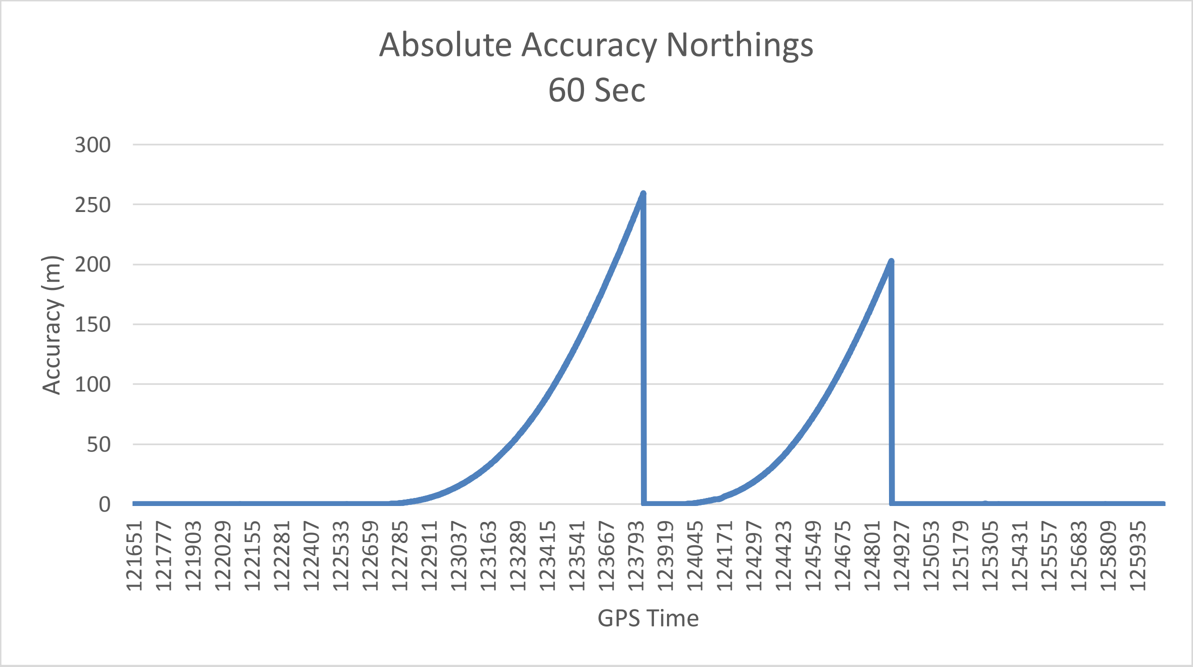
B4: 30 Seconds update



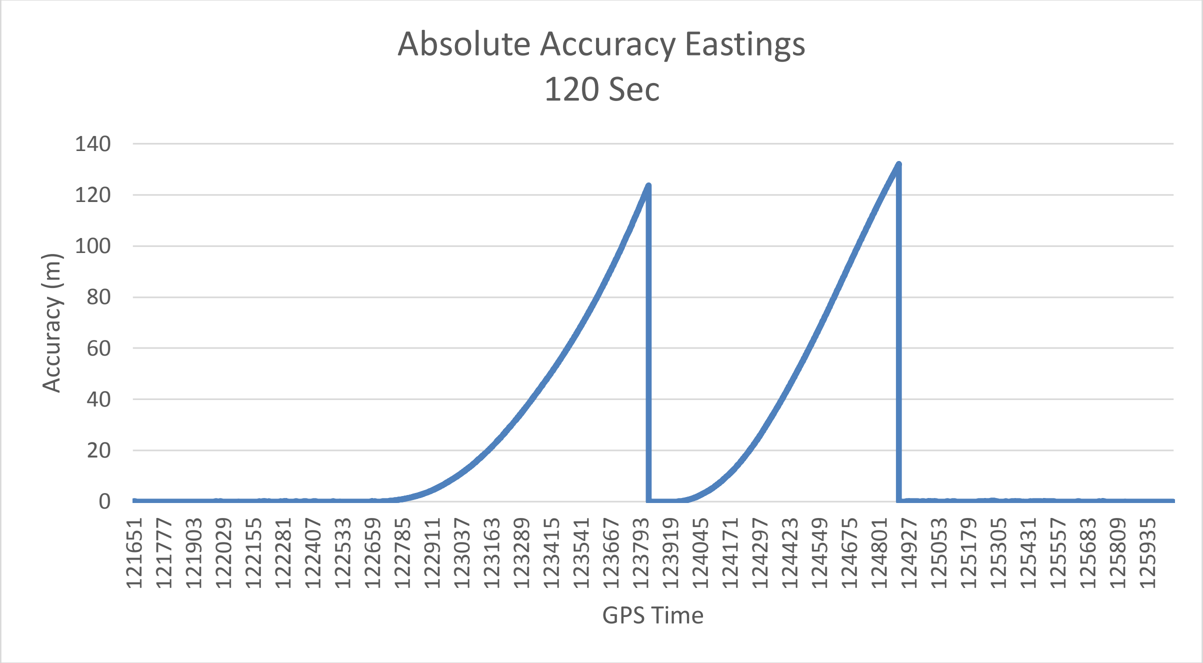


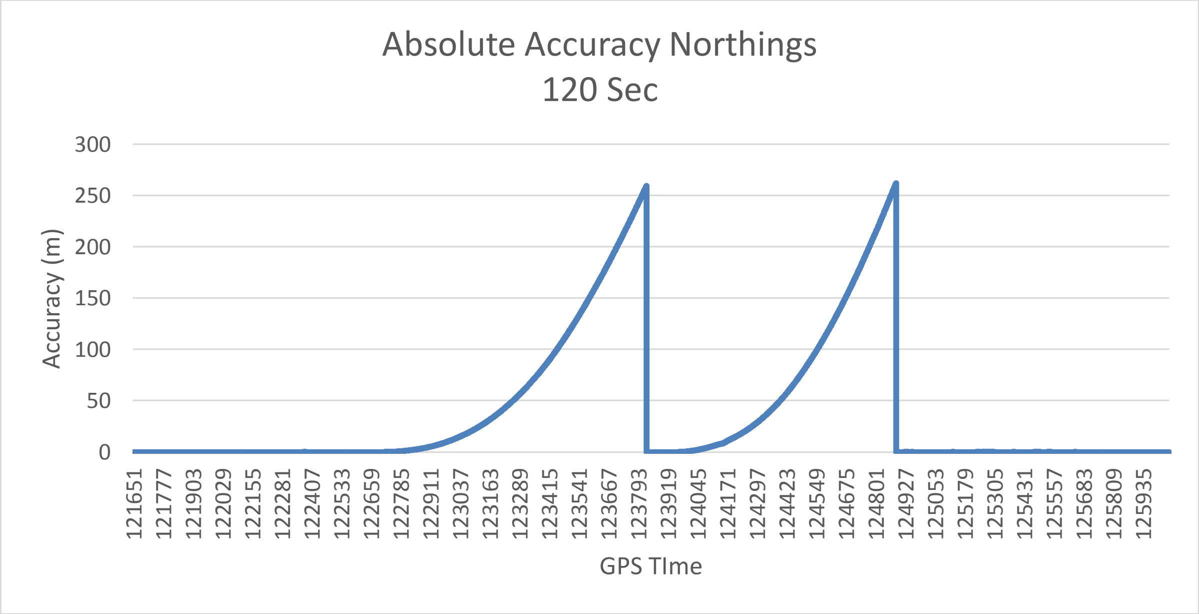
B5: 60 Seconds update:



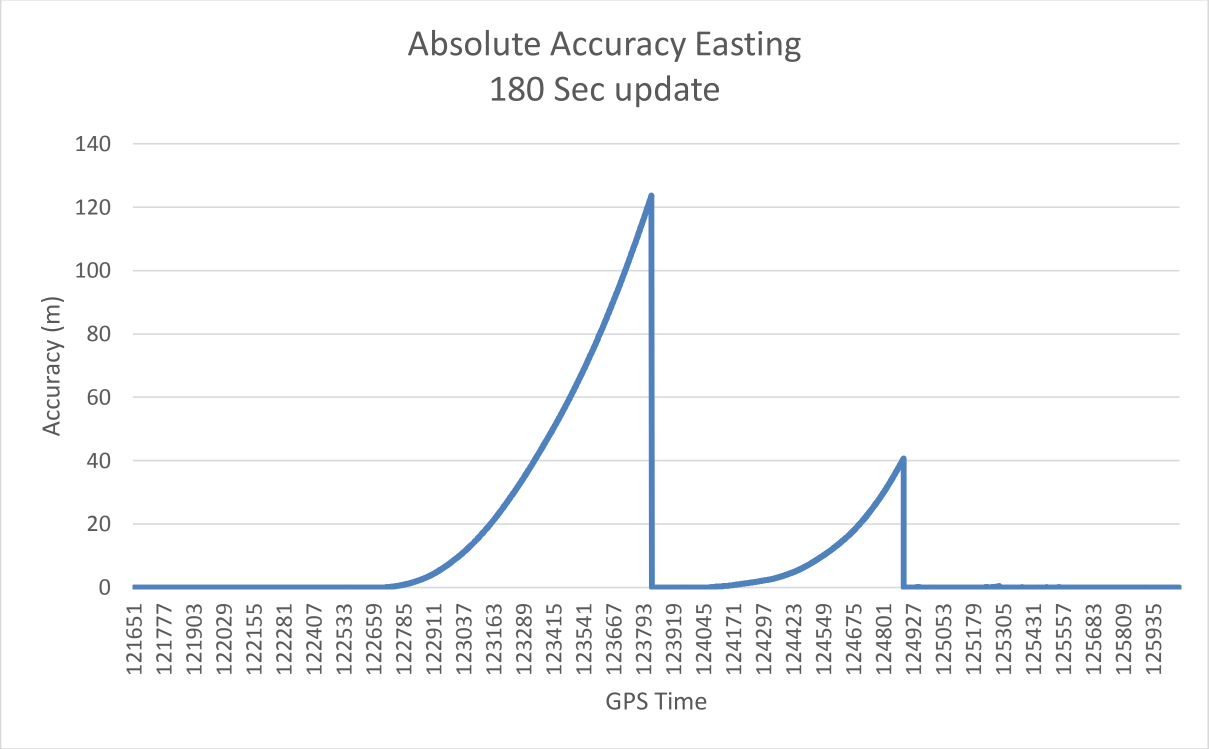


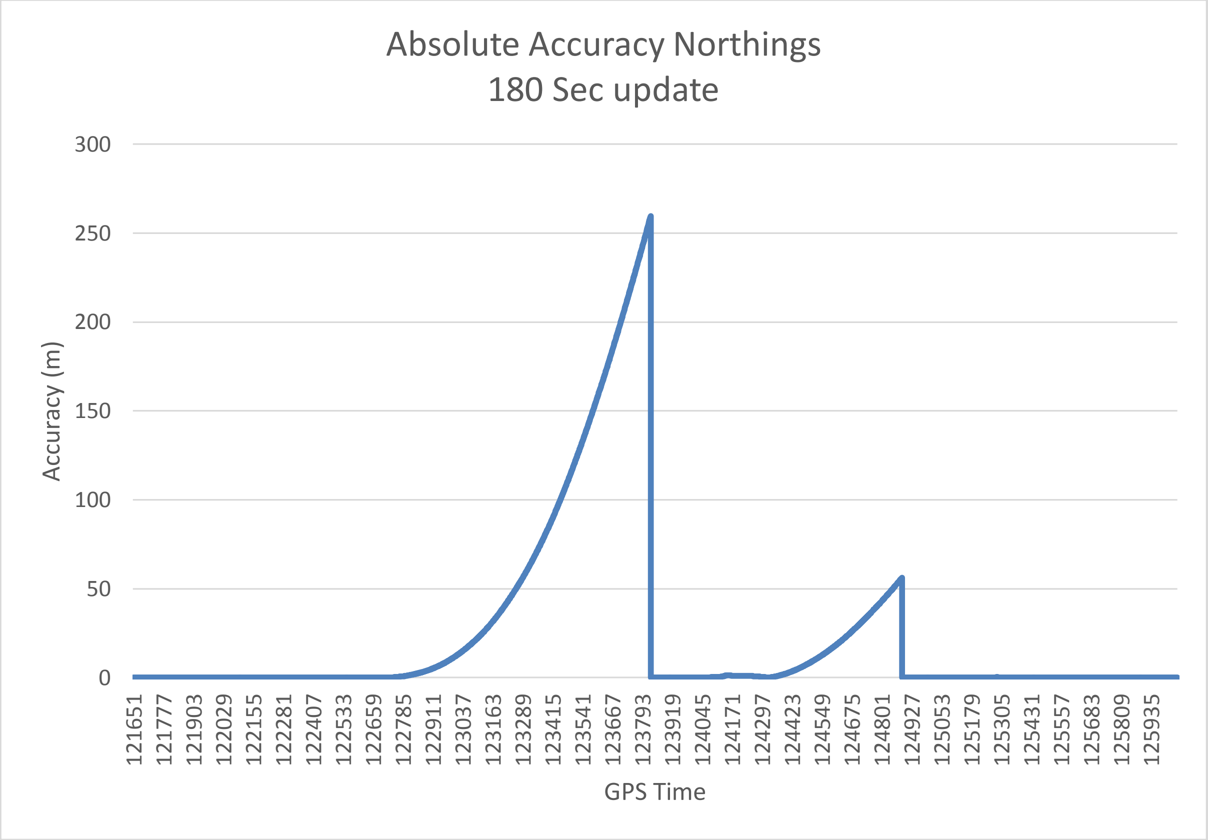
B6: 120 Seconds update





B7: 180 Seconds update

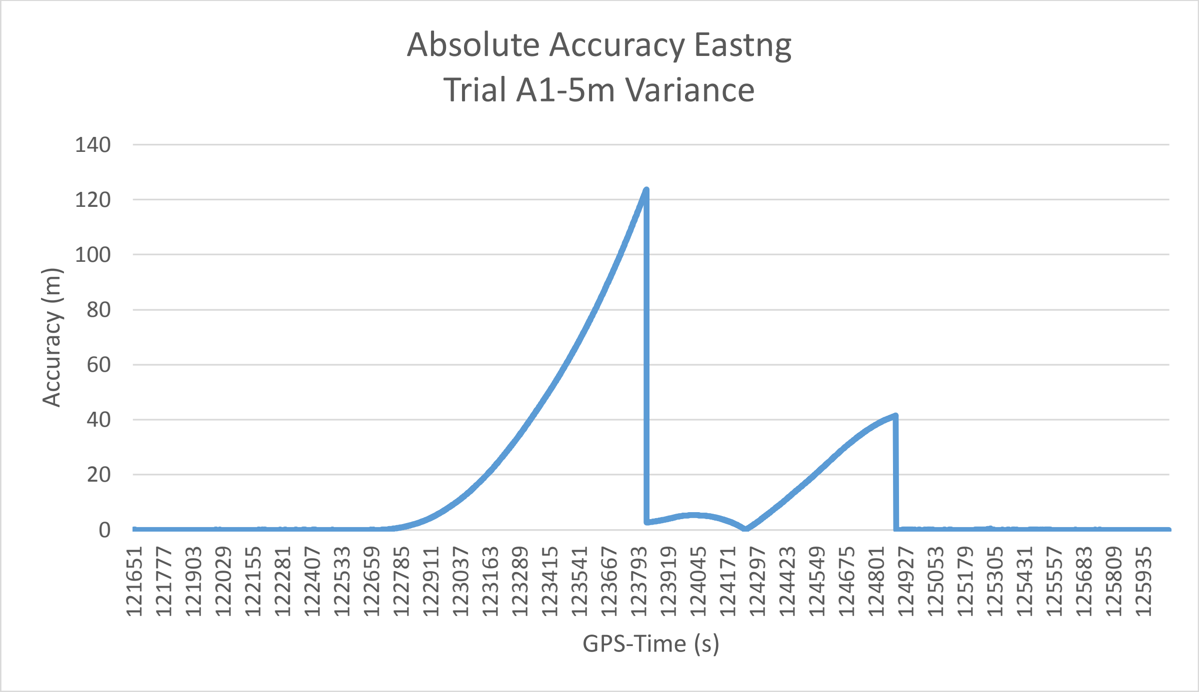


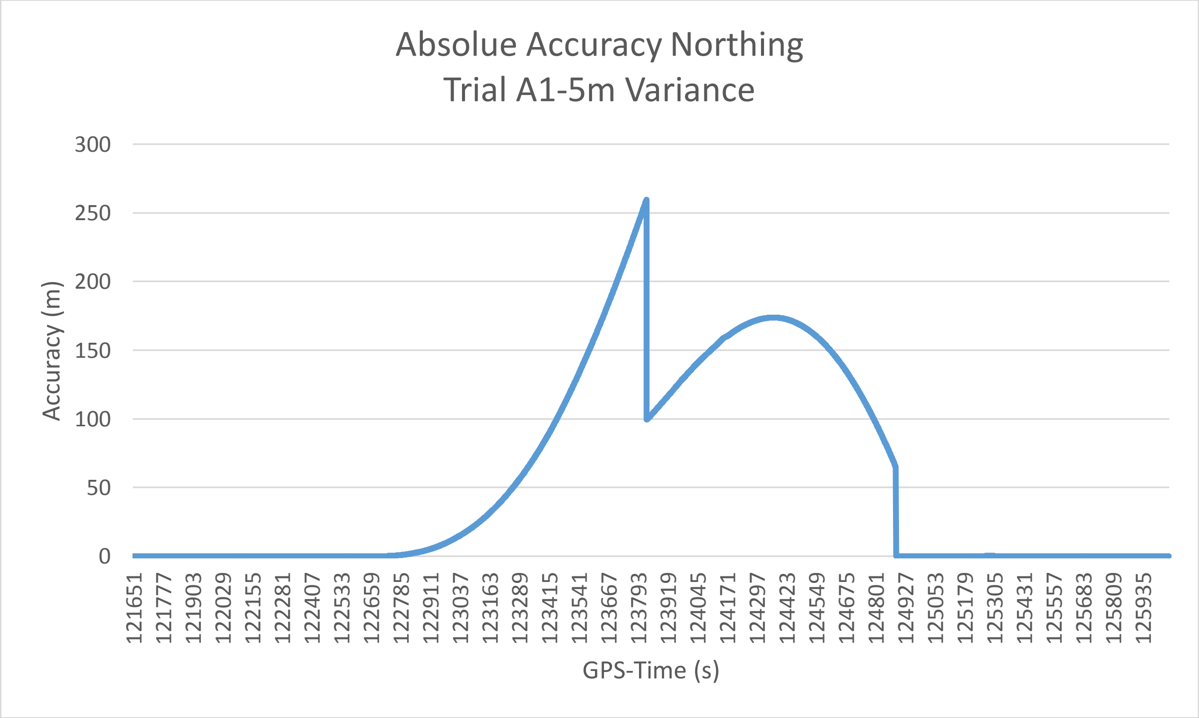


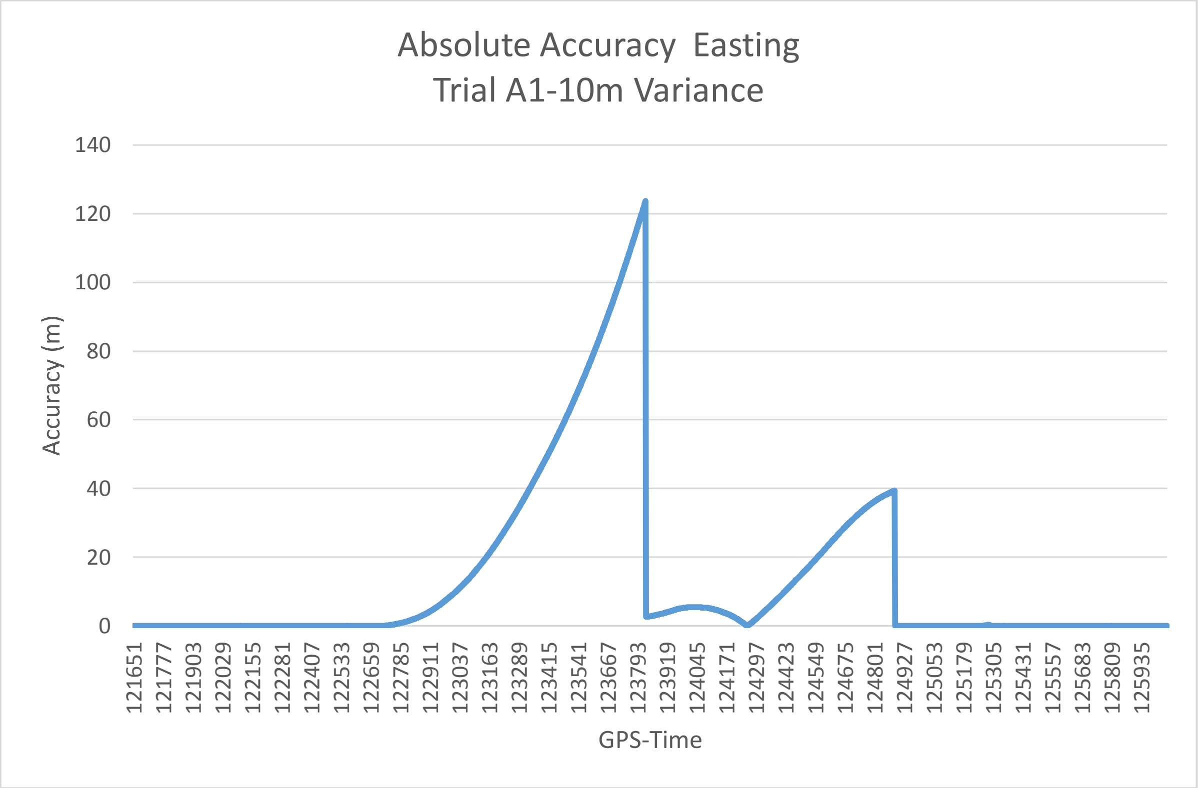
Appendix C: Introducing Errors

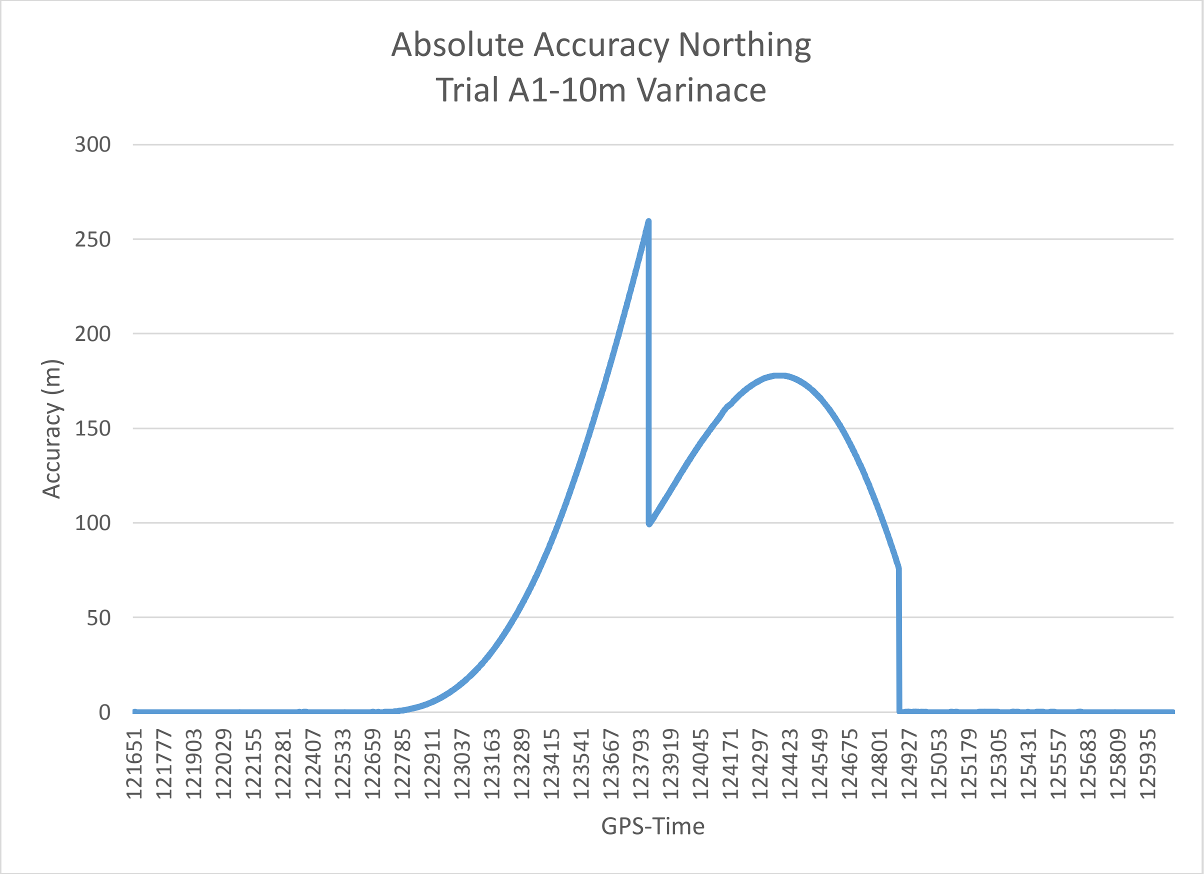
C1: Trial A1

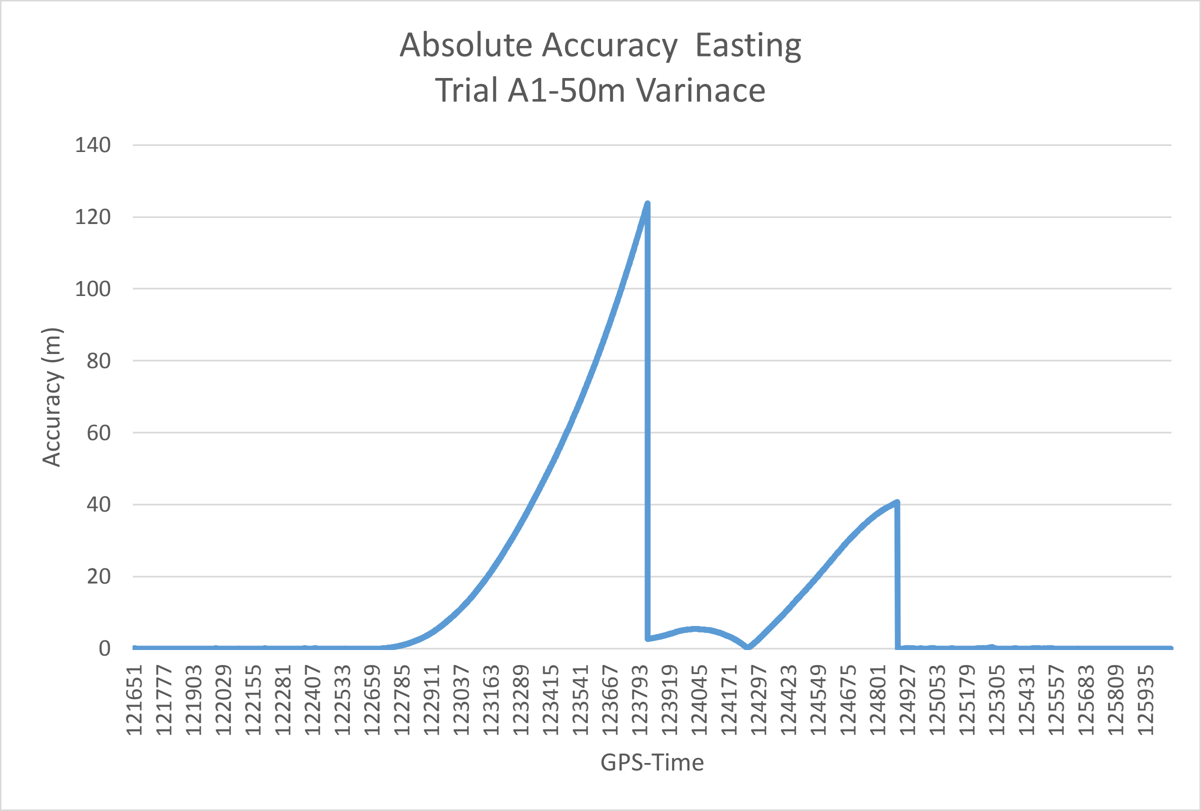
Manipulated Error: 100m North

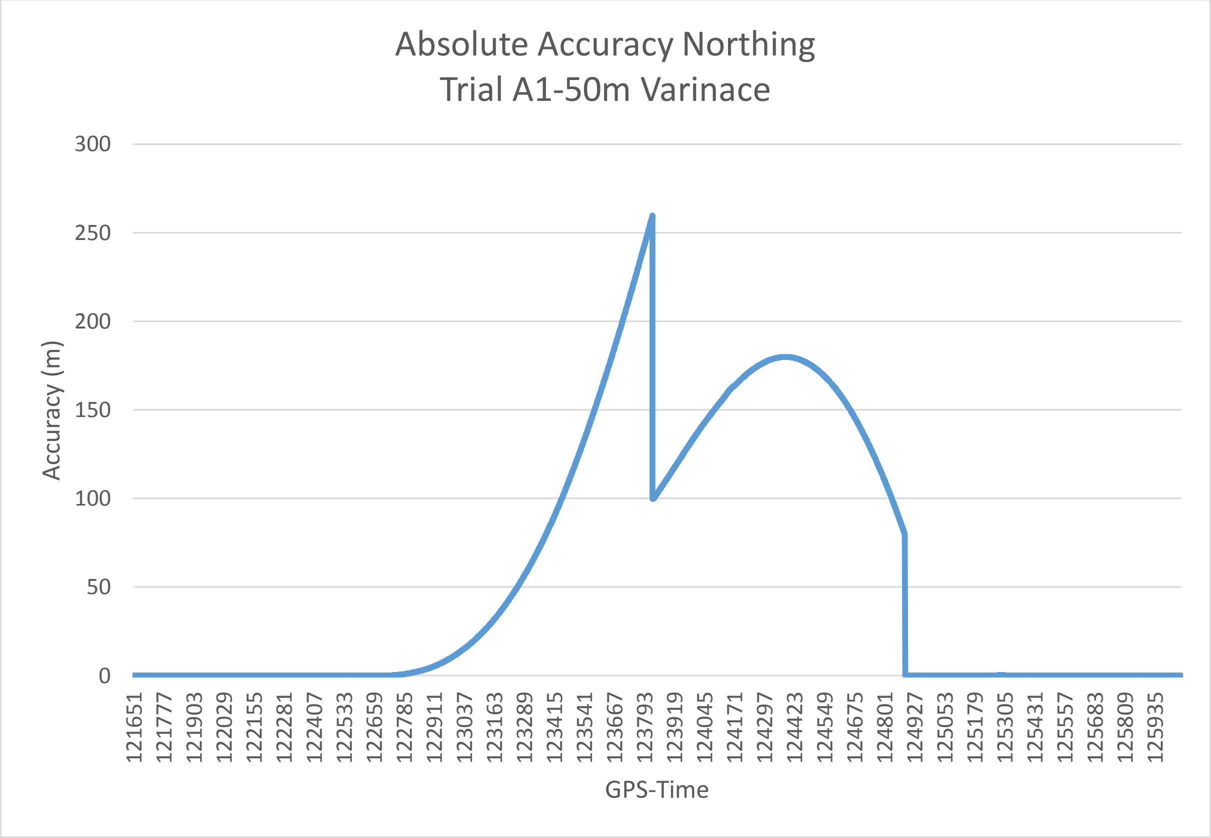


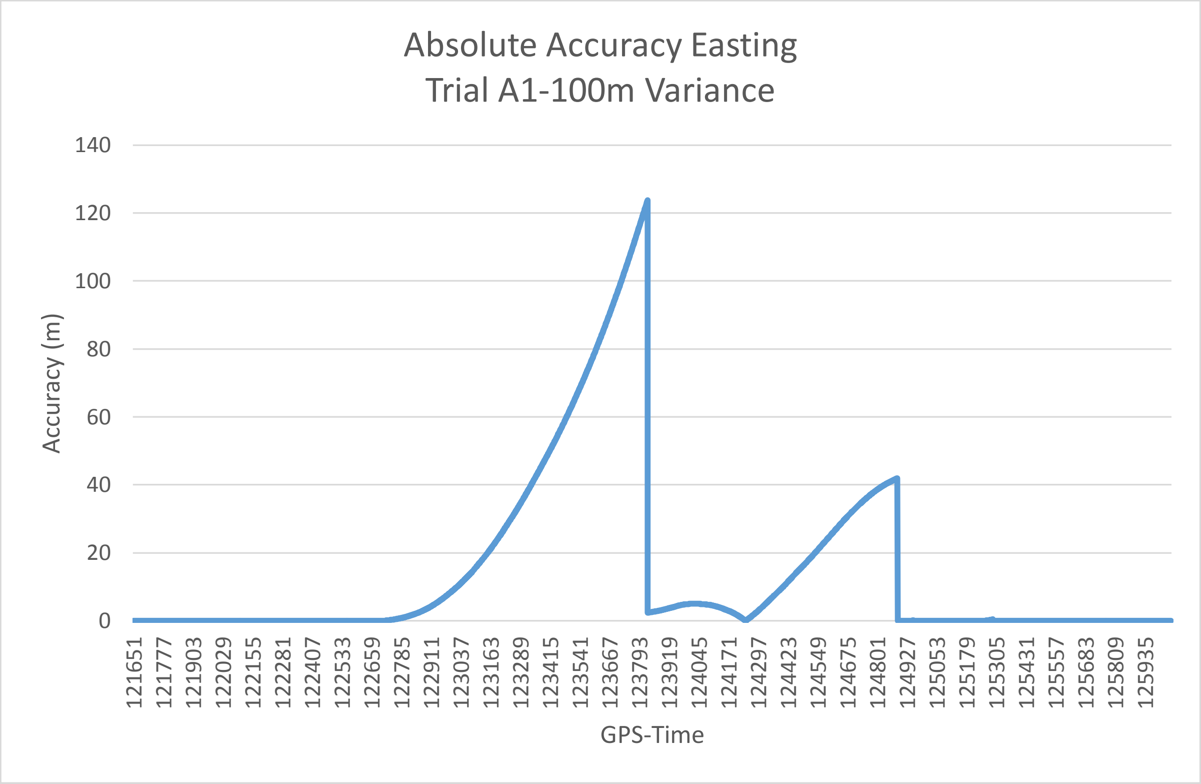


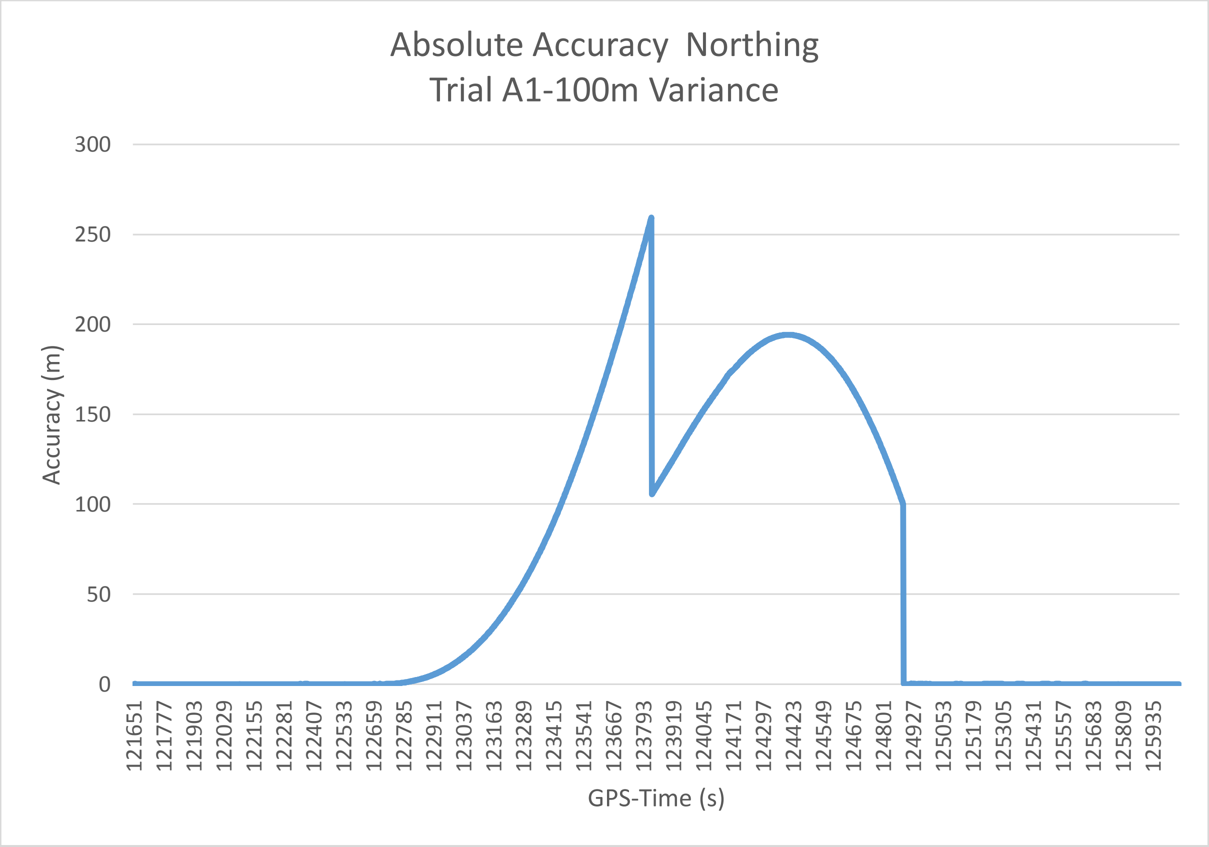

Absolute Accuracy Easting
Trial A1-10m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
124549
124675
124801
124927
125053
125179
125305
125431
125557
125683
125809
125935
GPS-Time


Absolute Accuracy Northing
Trial A1-10m Varinace
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
124549
124675
124801
124927
125053
125179
125305
125431
125557
125683
125809
125935
GPS-Time


Absolute Accuracy Easting
Trial A1-50m Varinace
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
124549
124675
124801
124927
125053
125179
125305
125431
125557
125683
125809
125935
GPS-Time

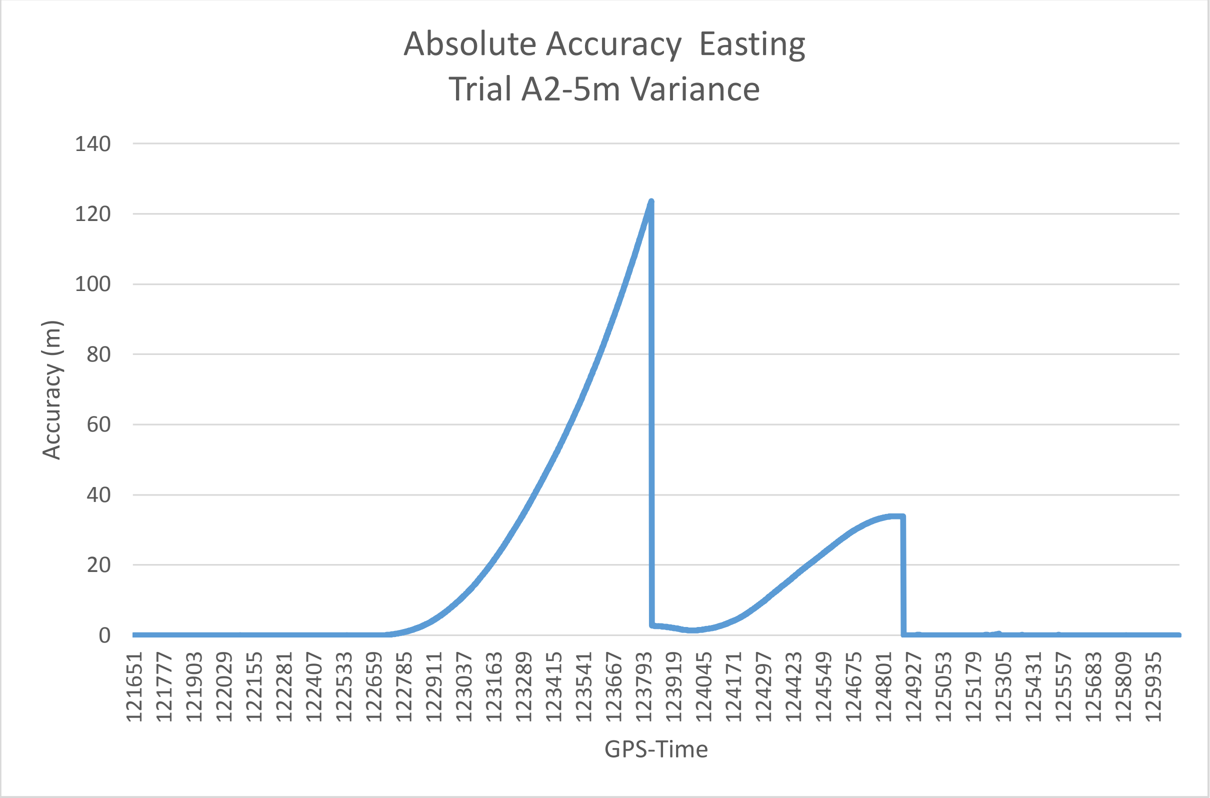

Absolute Accuracy Northing
Trial A1-50m Varinace
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
124549
124675
124801
124927
125053
125179
125305
125431
125557
125683
125809
125935
GPS-Time

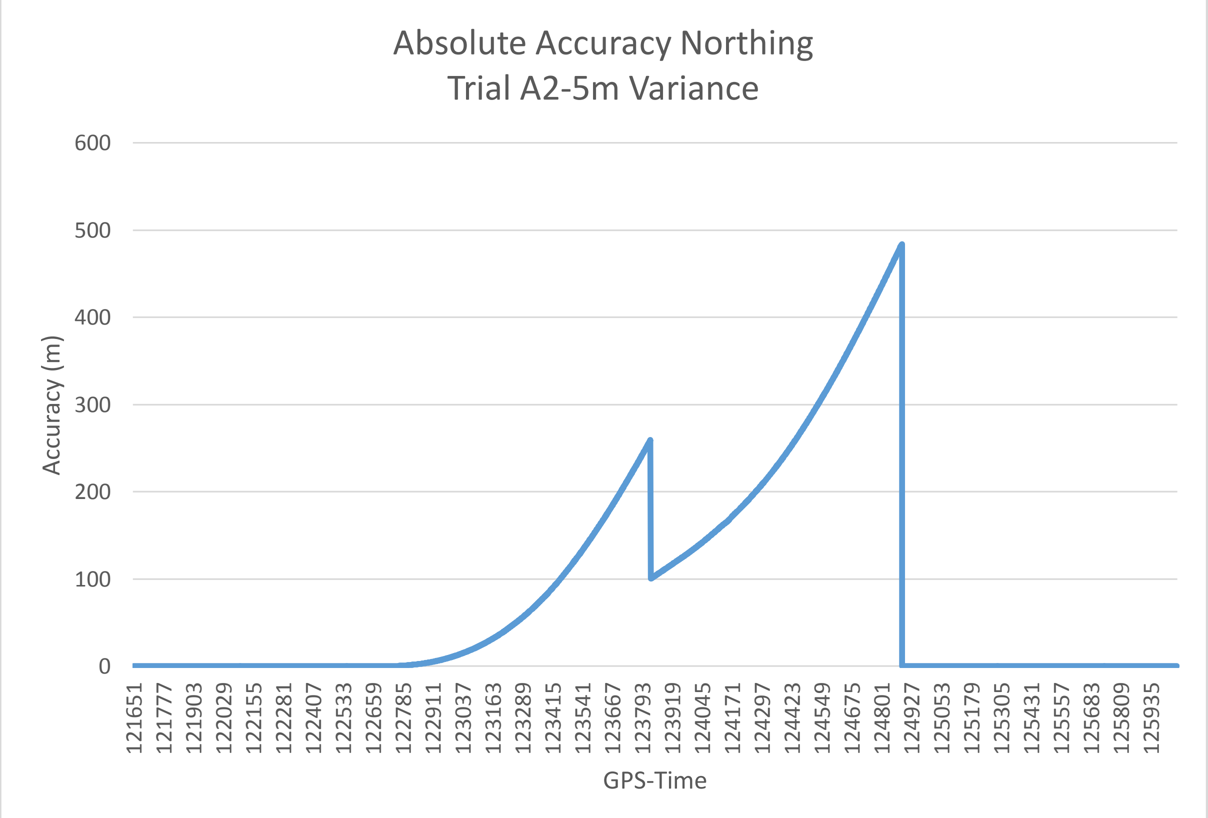

Absolute Accuracy Easting
Trial A1-100m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
124549
124675
124801
124927
125053
125179
125305
125431
125557
125683
125809
125935
GPS-Time

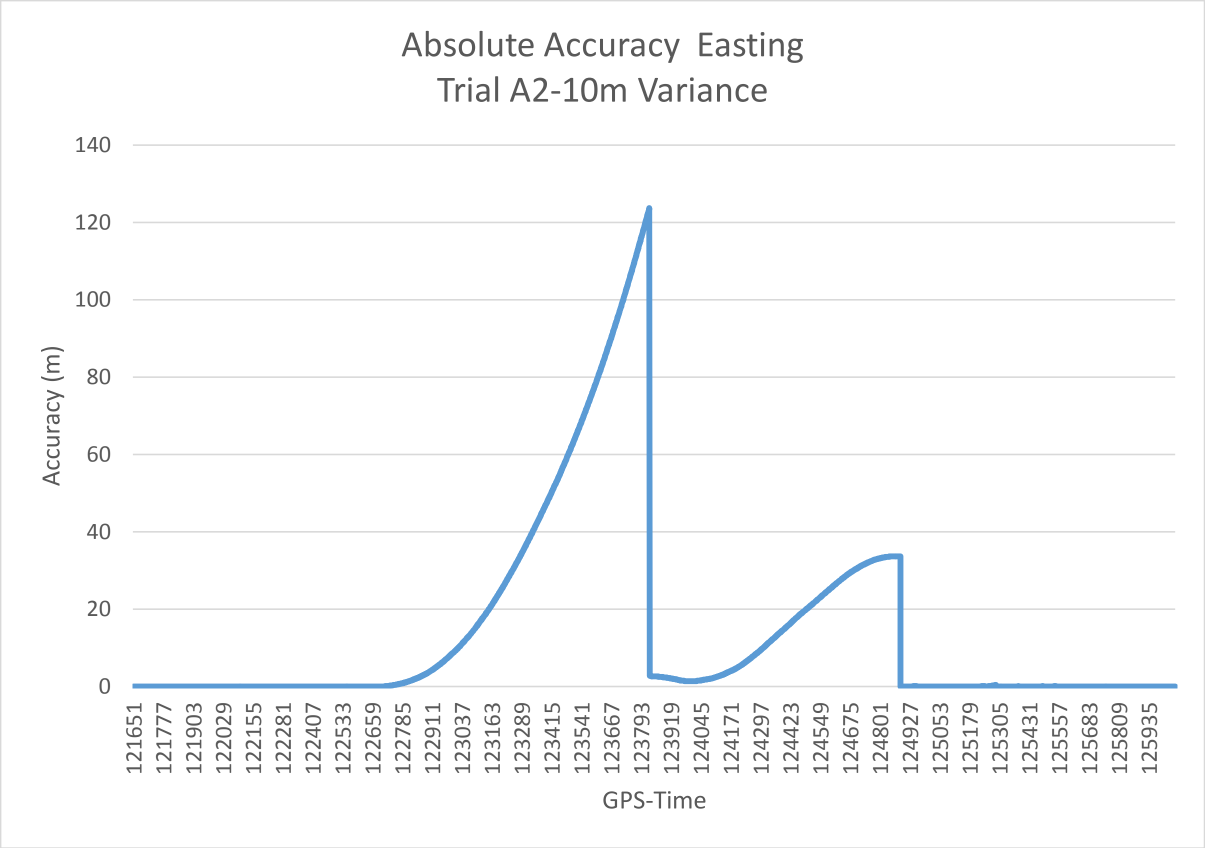

Absolute Accuracy Northing
Trial A1-100m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
124549
124675
124801
124927
125053
125179
125305
125431
125557
125683
125809
125935
GPS-Time (s)

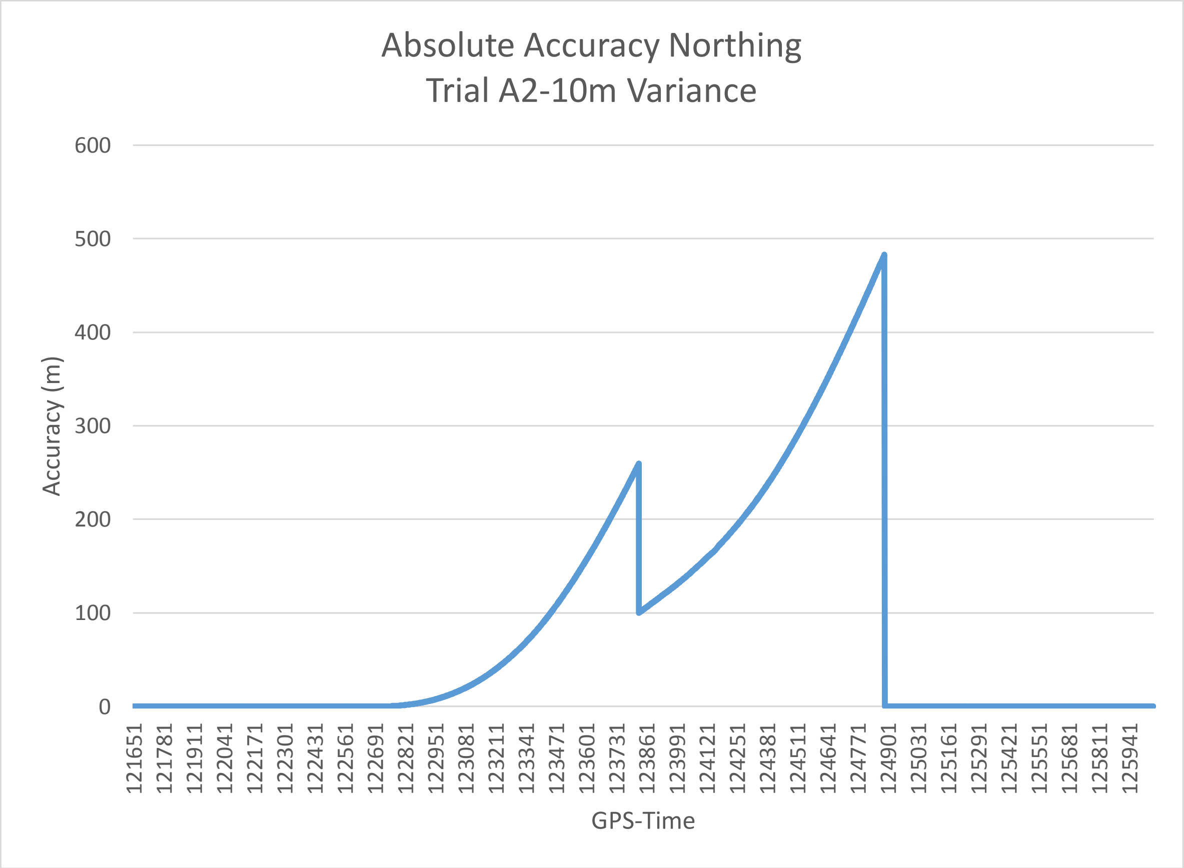
C2: Trial A2

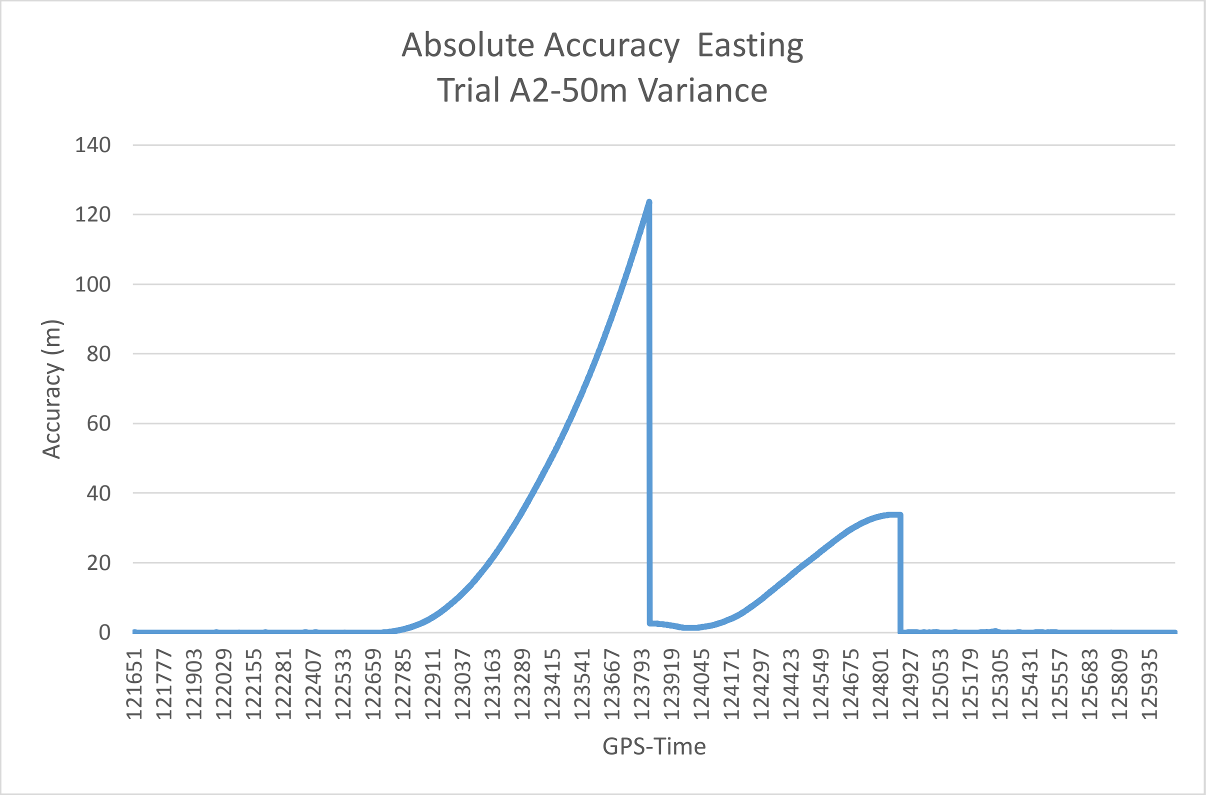
Manipulated Error: 100m South

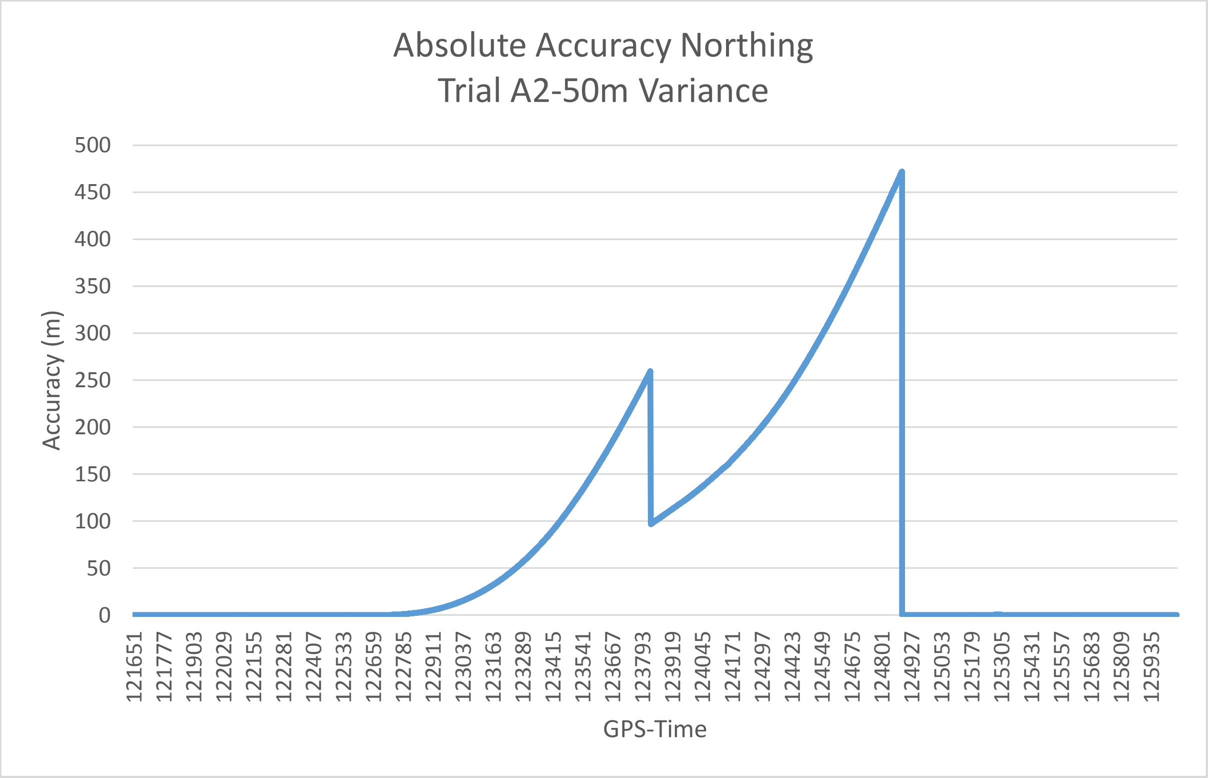


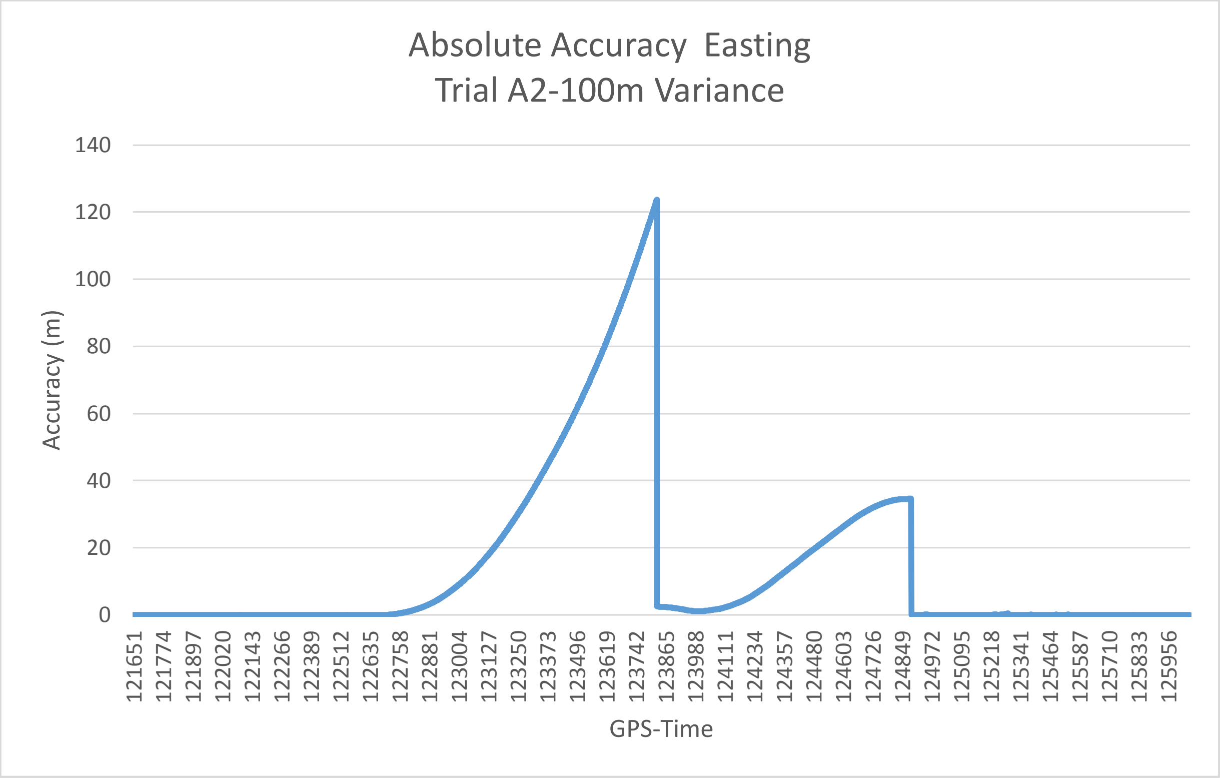


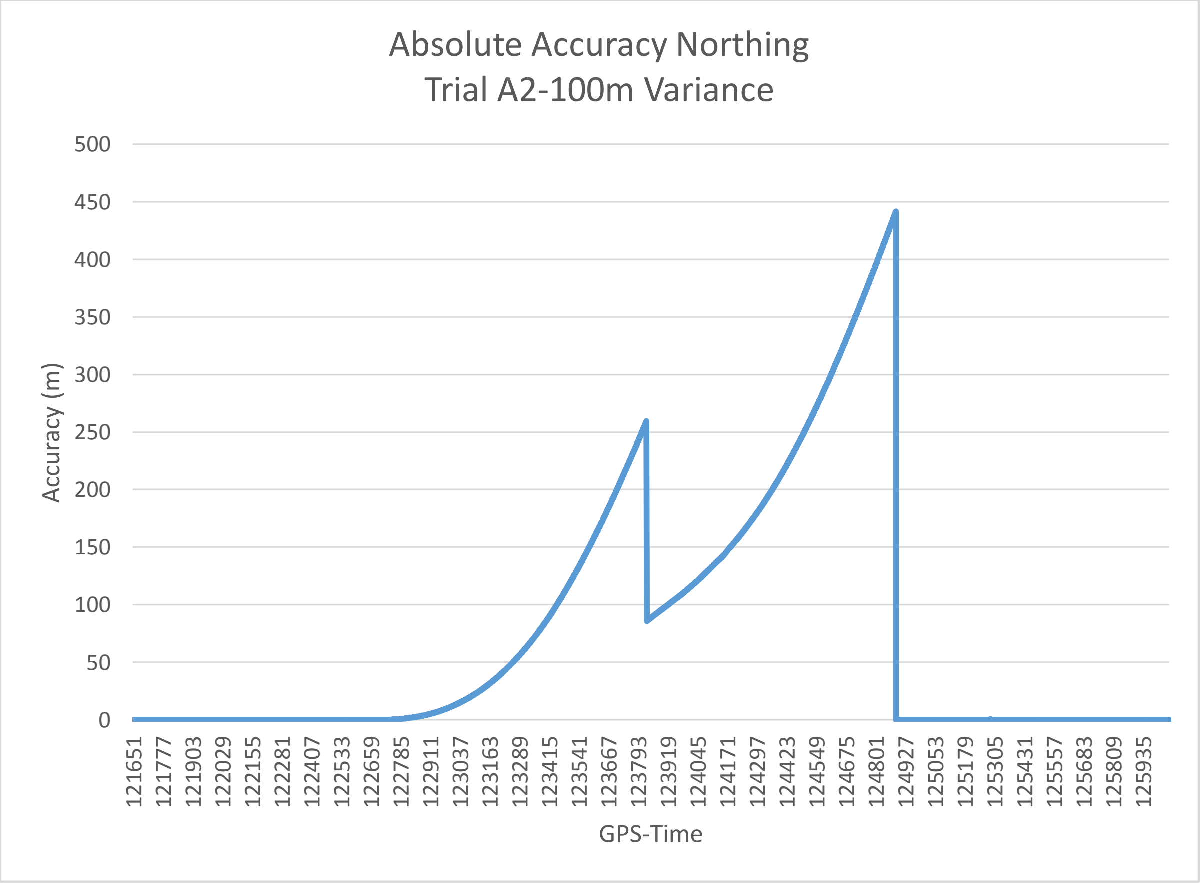

Absolute Accuracy Easting
Trial A2-10m Variance
Accuracy (m)
0
20
40
60
80
100
120
140
GPS-Time
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
124549
124675
124801
124927
125053
125179
125305
125431
125557
125683
125809
125935


Absolute Accuracy Northing
Trial A2-10m Variance
Accuracy (m)
0
100
200
300
400
500
600
GPS-Time
121651
121781
121911
122041
122171
122301
122431
122561
122691
122821
122951
123081
123211
123341
123471
123601
123731
123861
123991
124121
124251
124381
124511
124641
124771
124901
125031
125161
125291
125421
125551
125681
125811
125941


Absolute Accuracy Easting
Trial A2-50m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
124549
124675
124801
124927
125053
125179
125305
125431
125557
125683
125809
125935
GPS-Time

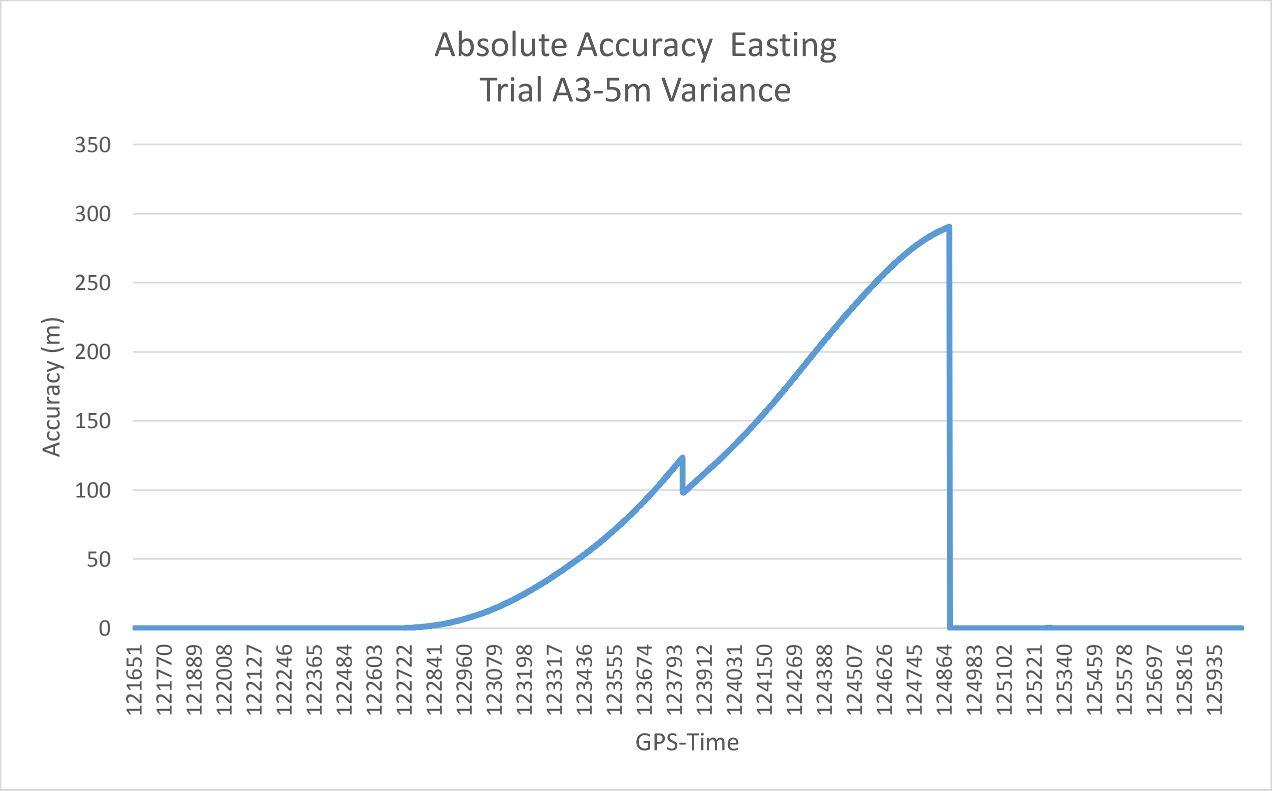

Absolute Accuracy Northing
Trial A2-50m Variance
Accuracy (m)
500
450
400
350
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
124549
124675
124801
124927
125053
125179
125305
125431
125557
125683
125809
125935
GPS-Time

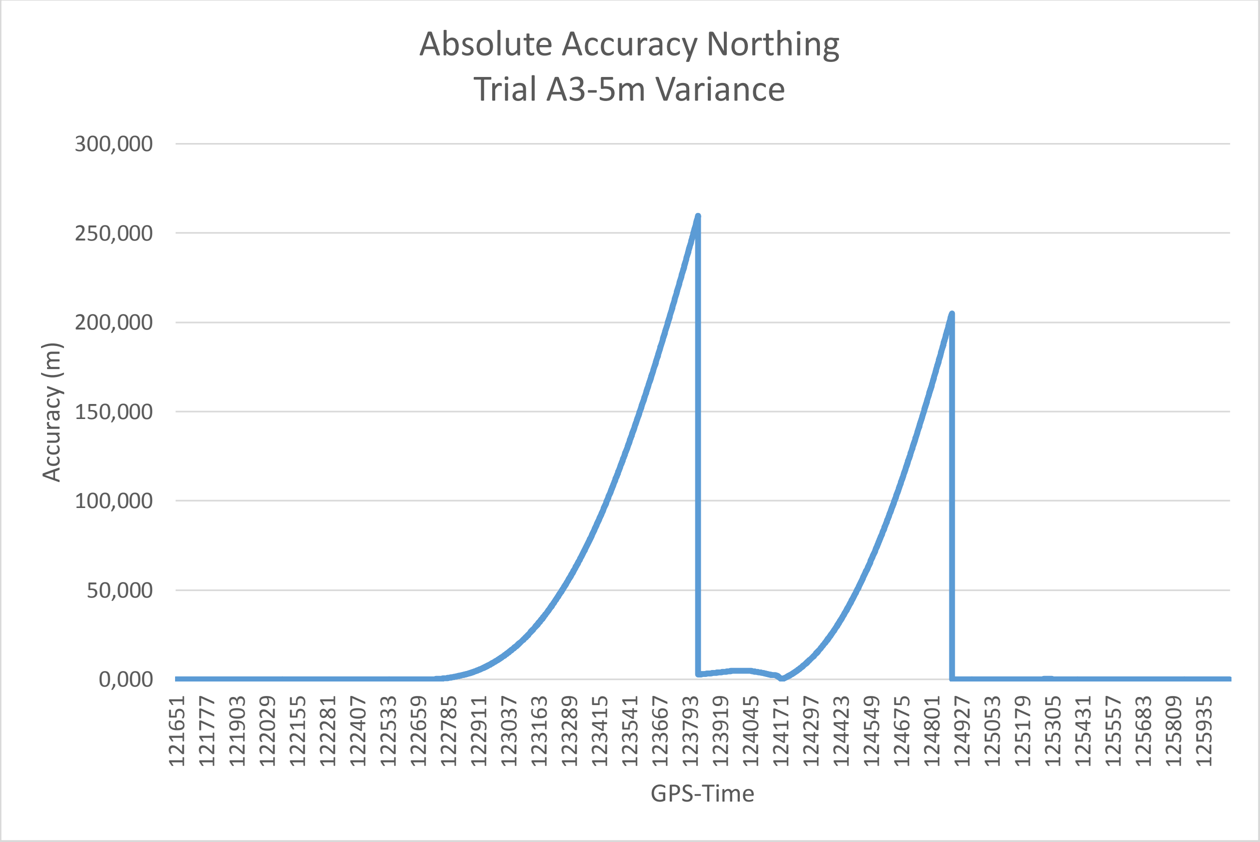

Absolute Accuracy Easting
Trial A2-100m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121774
121897
122020
122143
122266
122389
122512
122635
122758
122881
123004
123127
123250
123373
123496
123619
123742
123865
123988
124111
124234
124357
124480
124603
124726
124849
124972
125095
125218
125341
125464
125587
125710
125833
125956
GPS-Time

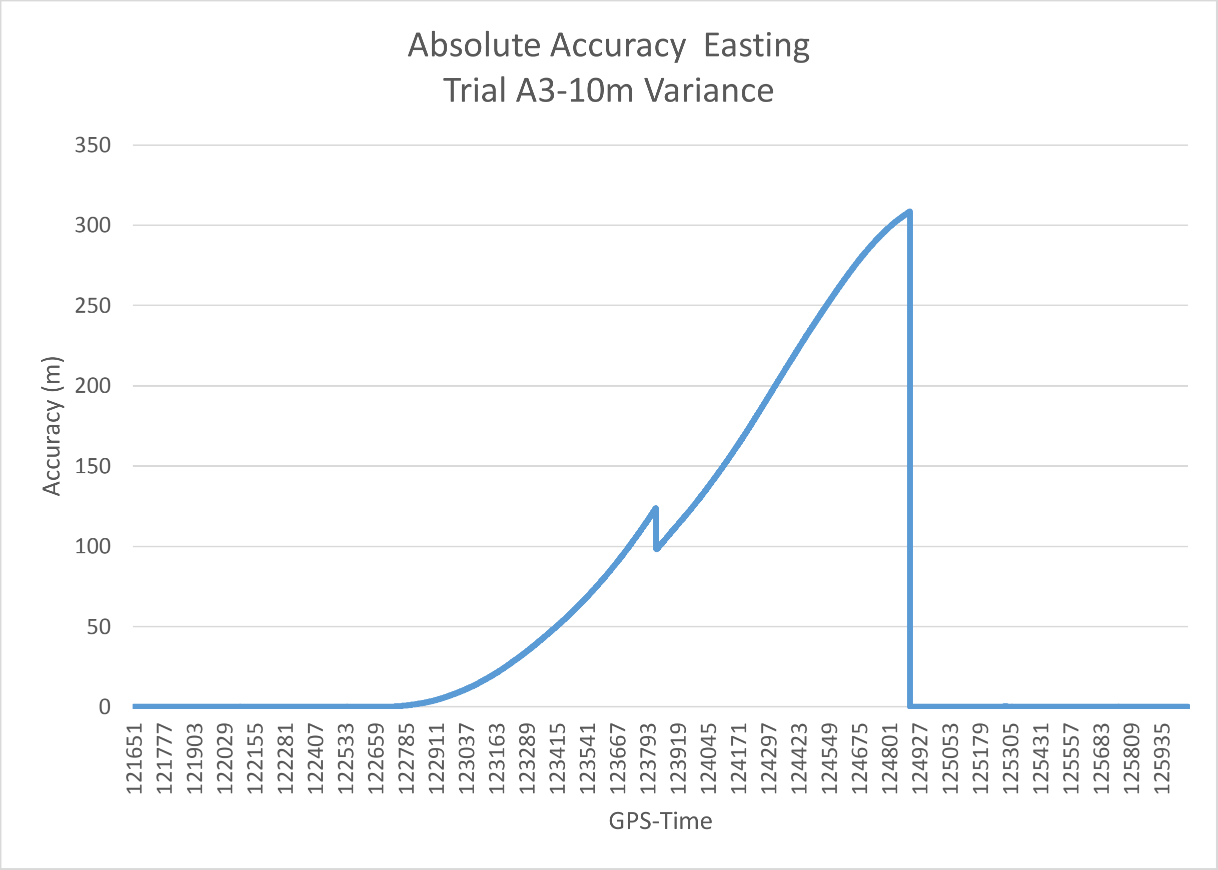

Absolute Accuracy Northing
Trial A2-100m Variance
Accuracy (m)
500
450
400
350
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
124549
124675
124801
124927
125053
125179
125305
125431
125557
125683
125809
125935
GPS-Time

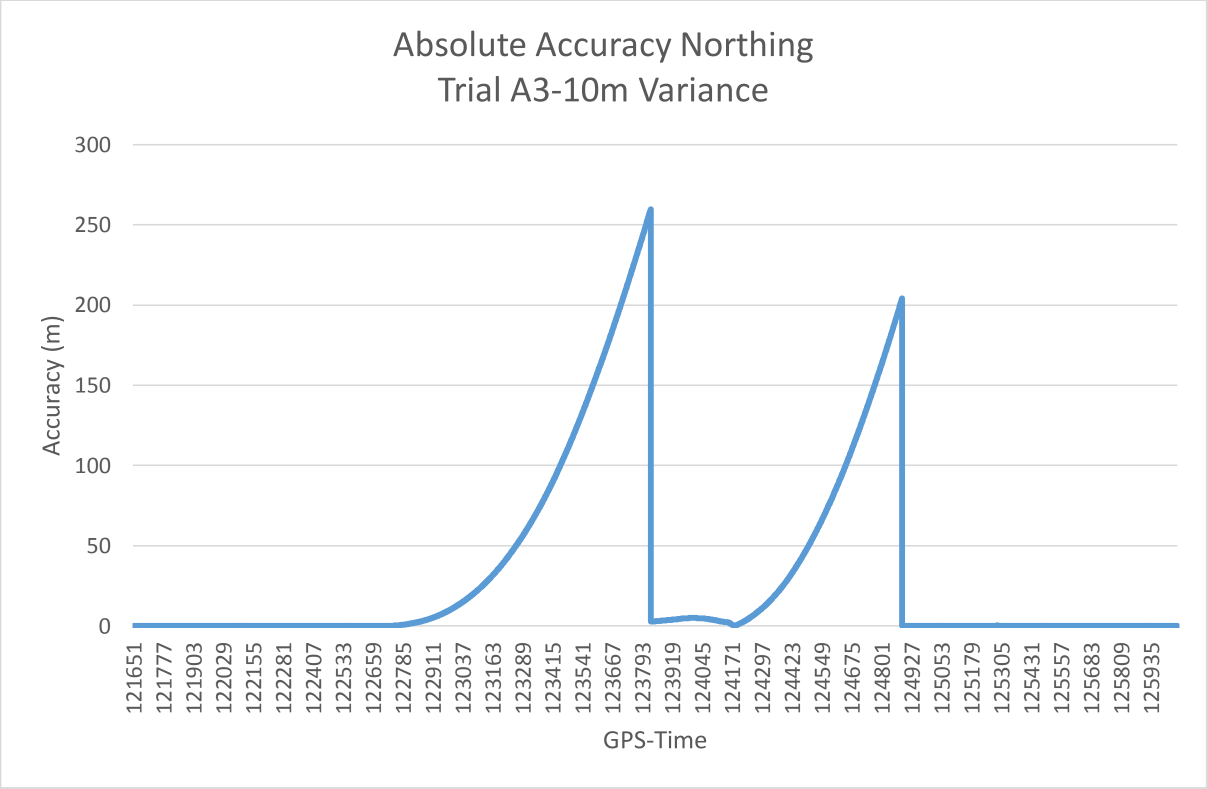
C3: Trial A3

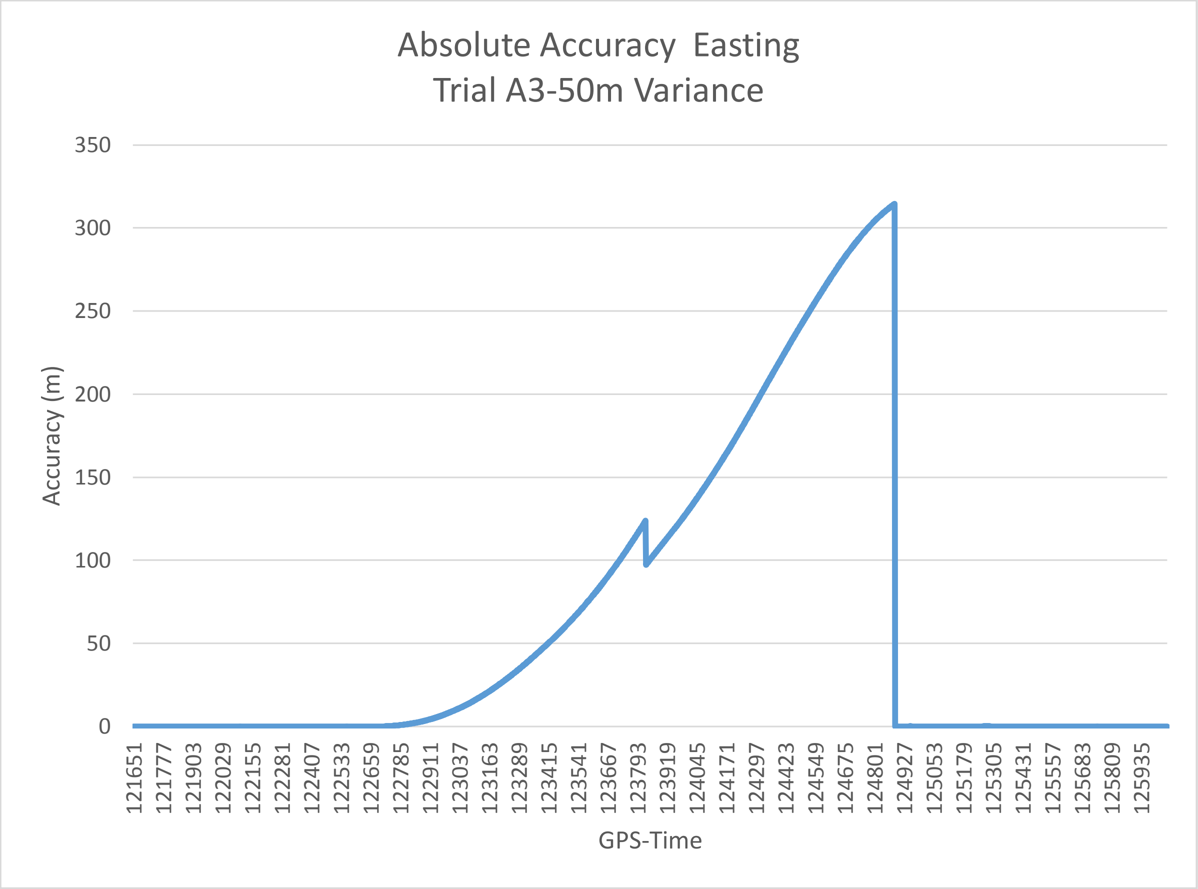
Manipulated Error: 100m East

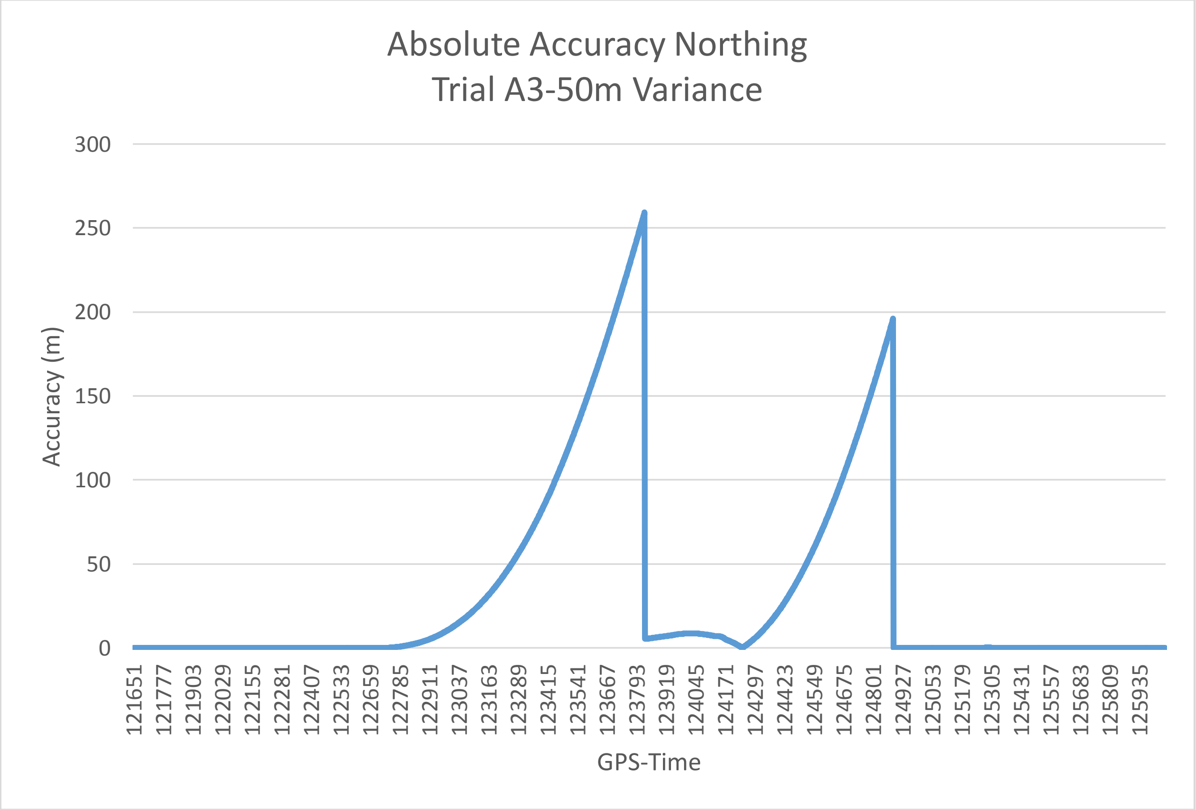


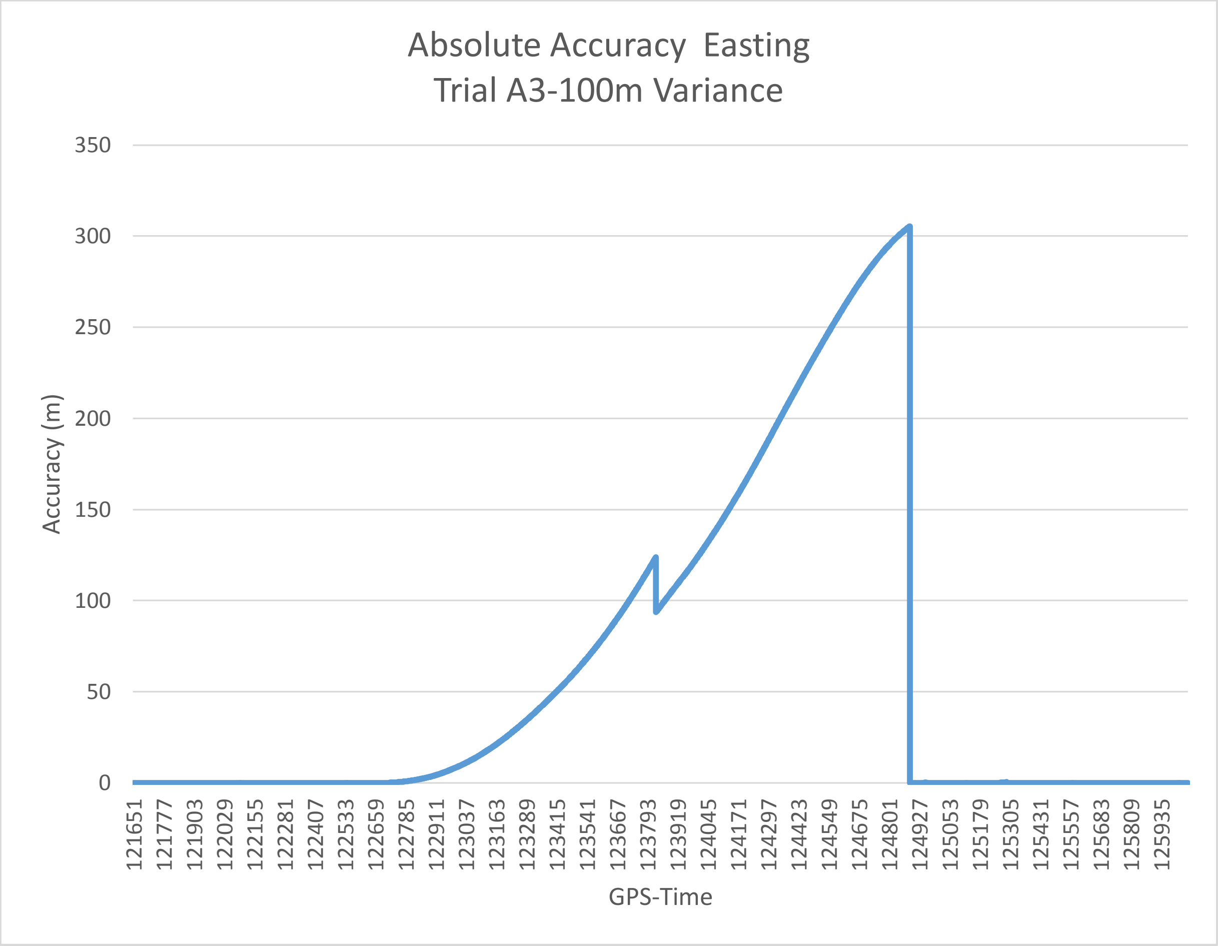



Absolute Accuracy Easting
Trial A3-10m Variance
Accuracy (m)
350
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
124549
124675
124801
124927
125053
125179
125305
125431
125557
125683
125809
125935
GPS-Time


Absolute Accuracy Northing
Trial A3-10m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
124549
124675
124801
124927
125053
125179
125305
125431
125557
125683
125809
125935
GPS-Time


Absolute Accuracy Easting
Trial A3-50m Variance
Accuracy (m)
350
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
124549
124675
124801
124927
125053
125179
125305
125431
125557
125683
125809
125935
GPS-Time


Absolute Accuracy Northing
Trial A3-50m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
124549
124675
124801
124927
125053
125179
125305
125431
125557
125683
125809
125935
GPS-Time


Absolute Accuracy Easting
Trial A3-100m Variance
Accuracy (m)
350
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
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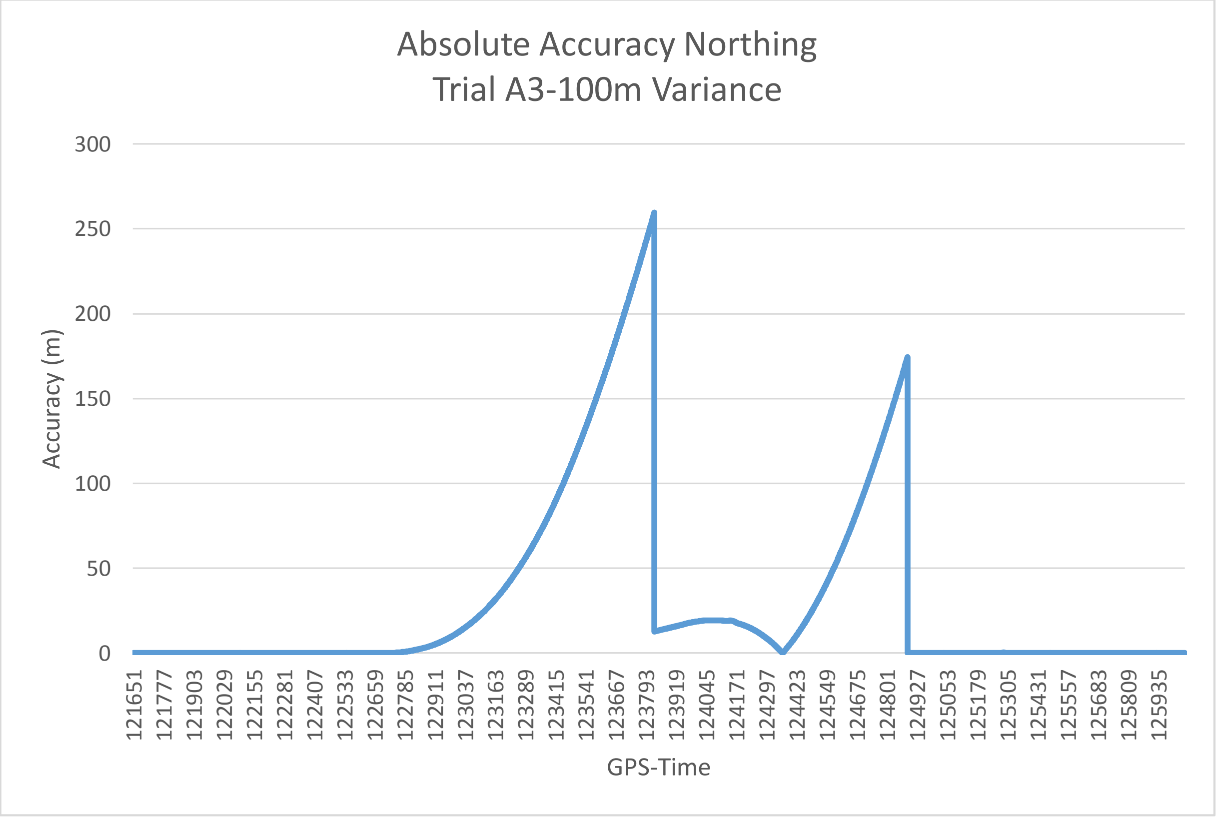
125557
125683
125809
125935
GPS-Time


Absolute Accuracy Northing
Trial A3-100m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
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124045
124171
124297
124423
124549
124675
124801
124927
125053
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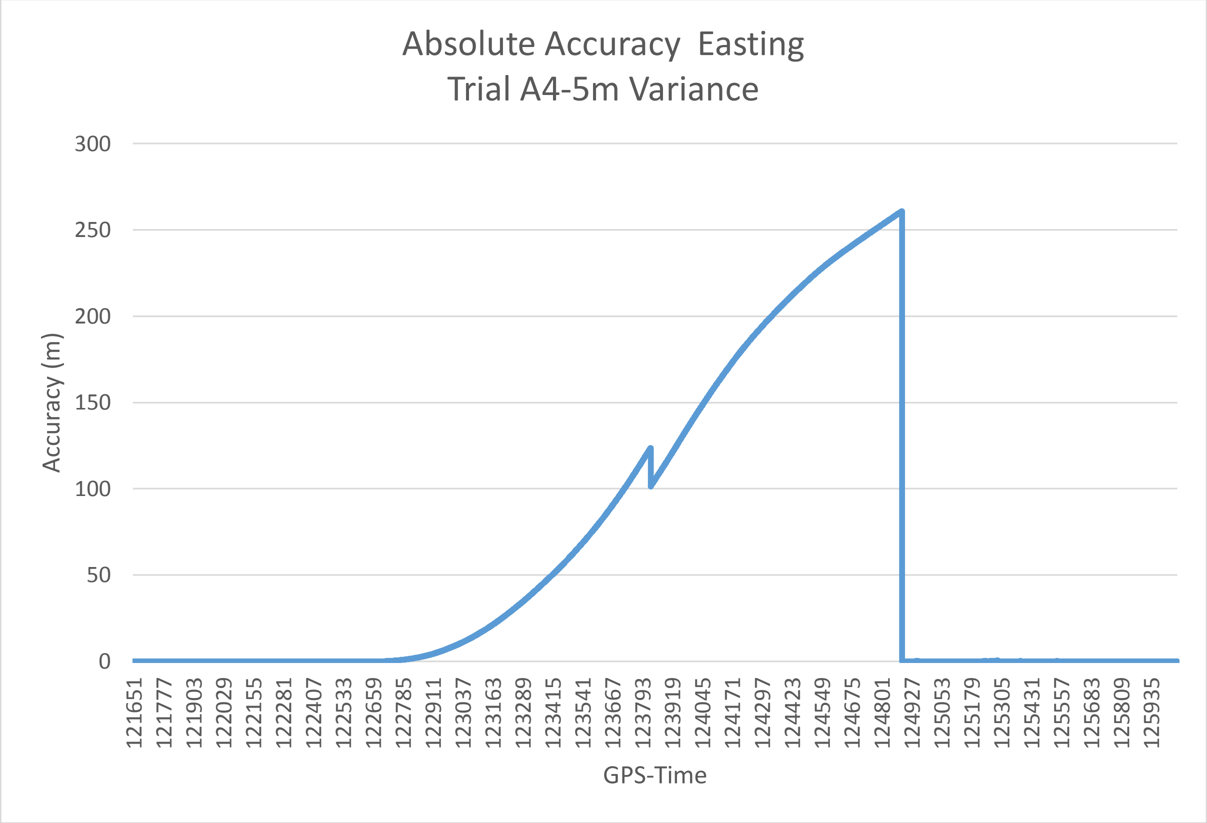
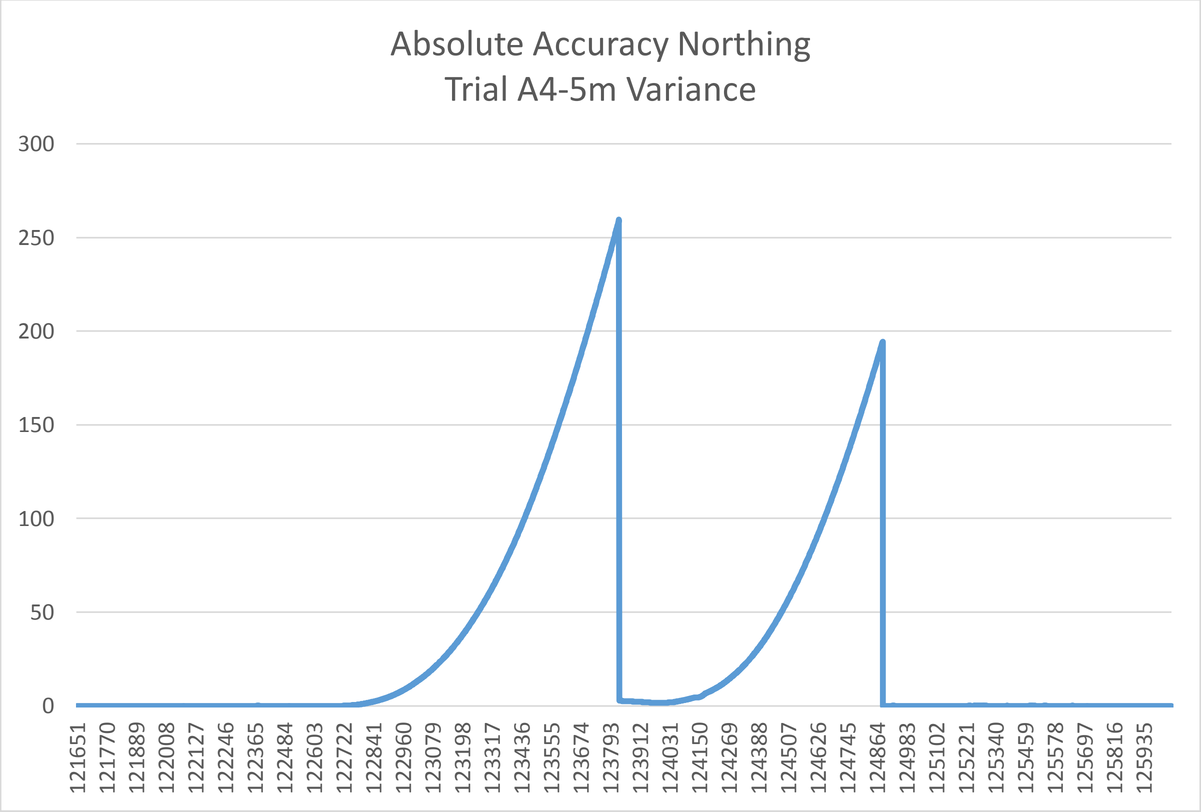
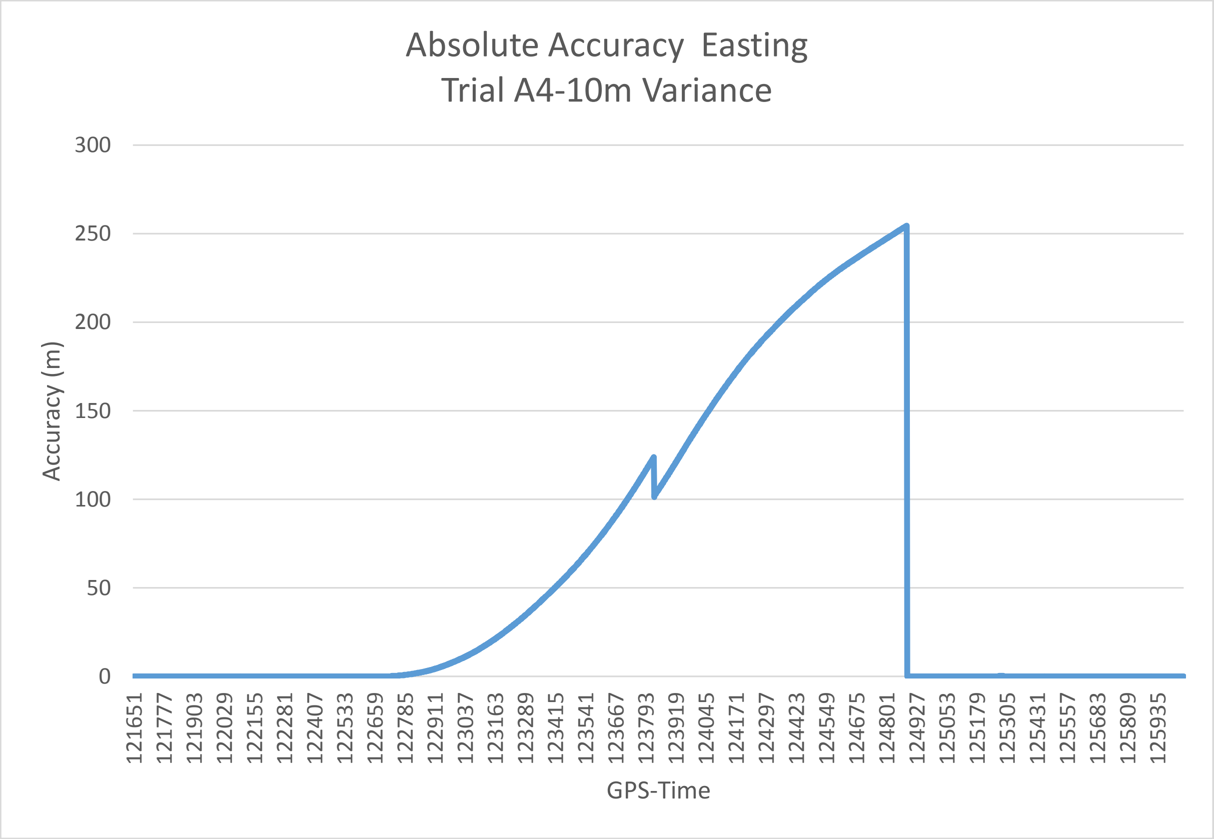
125431
125557
125683
125809
125935
GPS-Time

C4: Trial A4

Manipulated Error: 100m West






Absolute Accuracy Easting
Trial A4-10m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
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124045
124171
124297
124423
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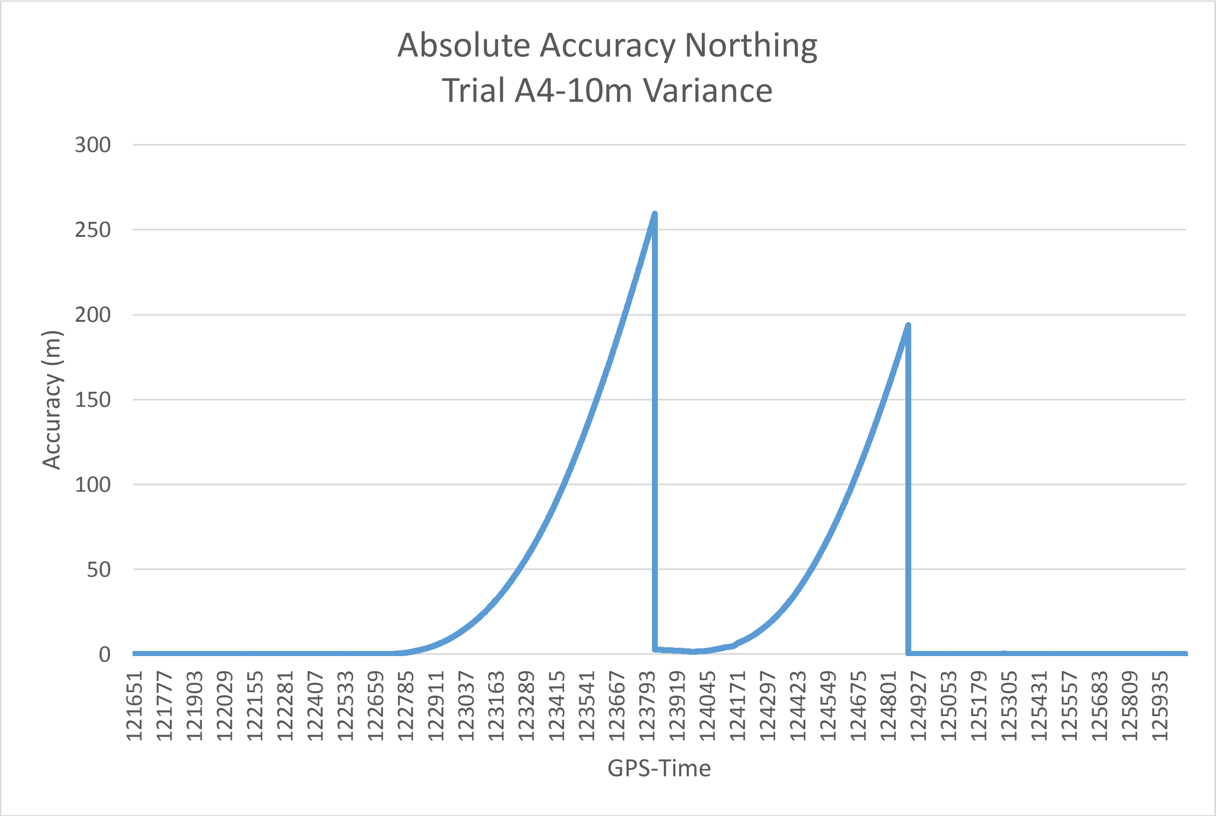
125557
125683
125809
125935
GPS-Time


Absolute Accuracy Northing
Trial A4-10m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
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124045
124171
124297
124423
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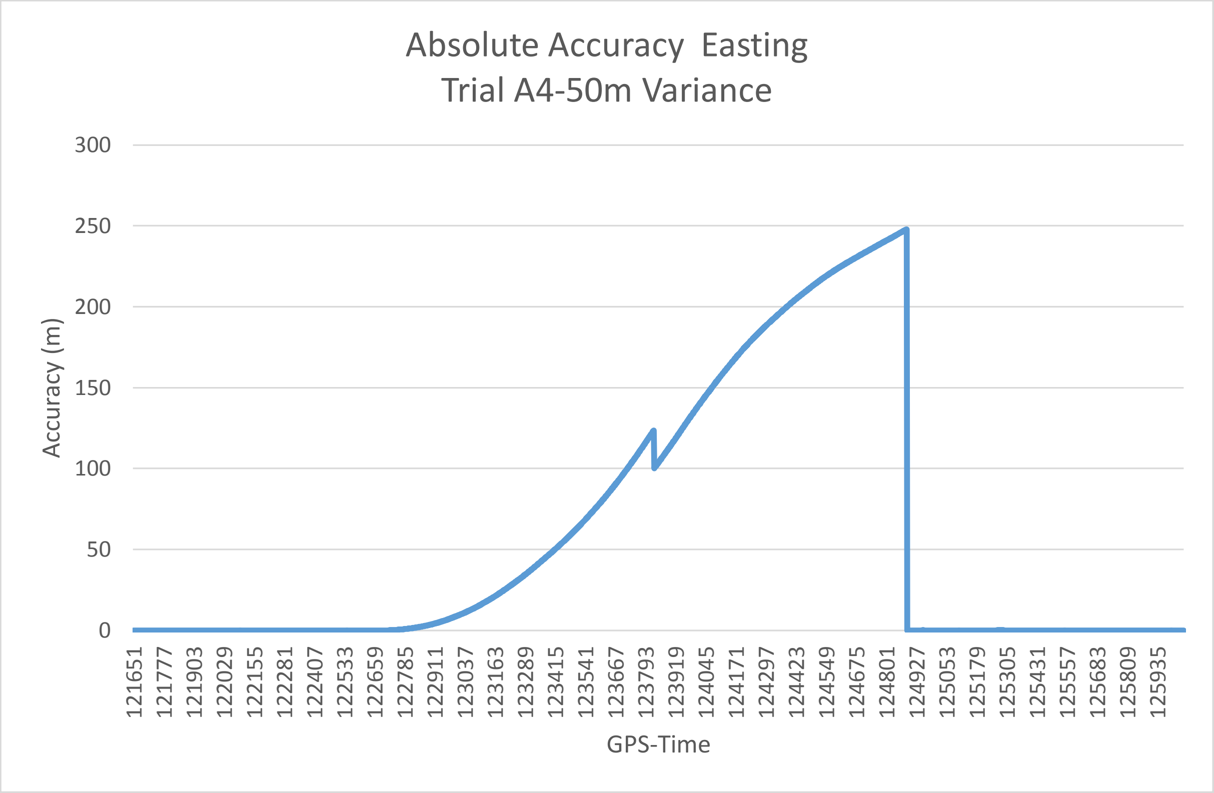
125683
125809
125935
GPS-Time


Absolute Accuracy Easting
Trial A4-50m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
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125053
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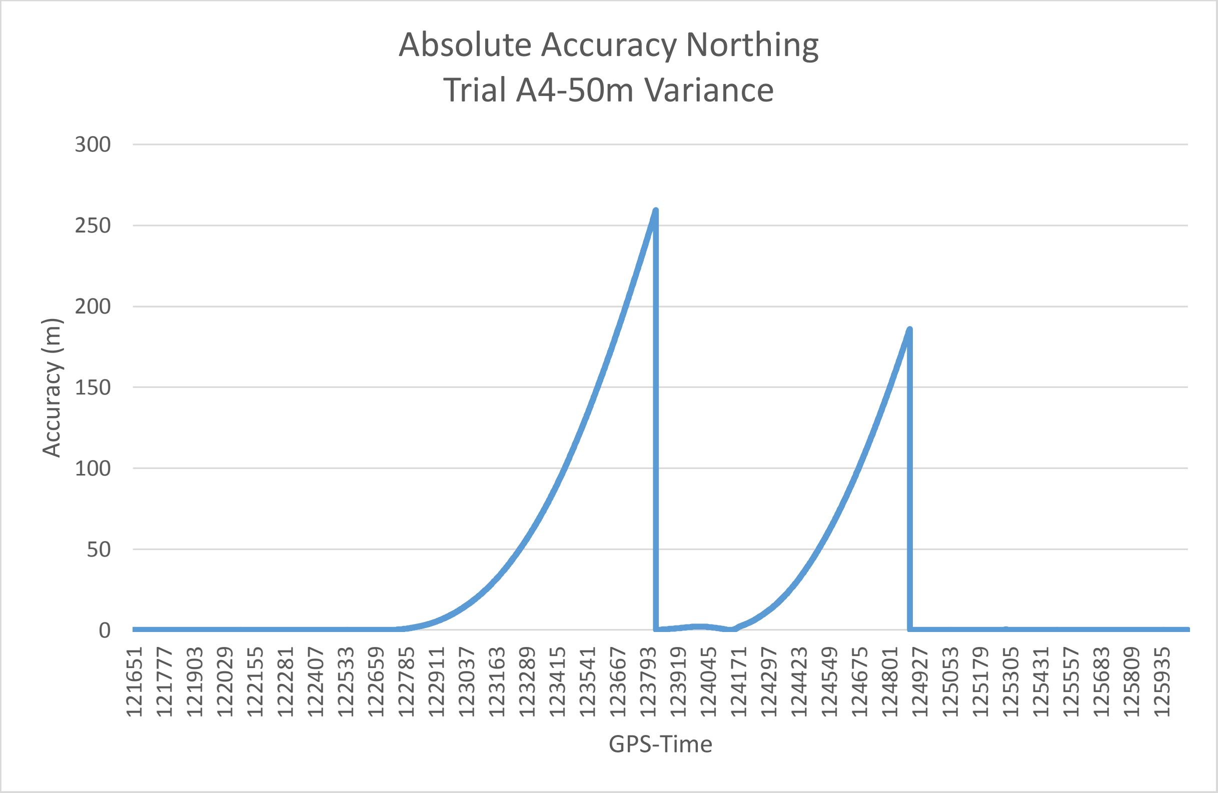
125557
125683
125809
125935
GPS-Time


Absolute Accuracy Northing
Trial A4-50m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
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124045
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124297
124423
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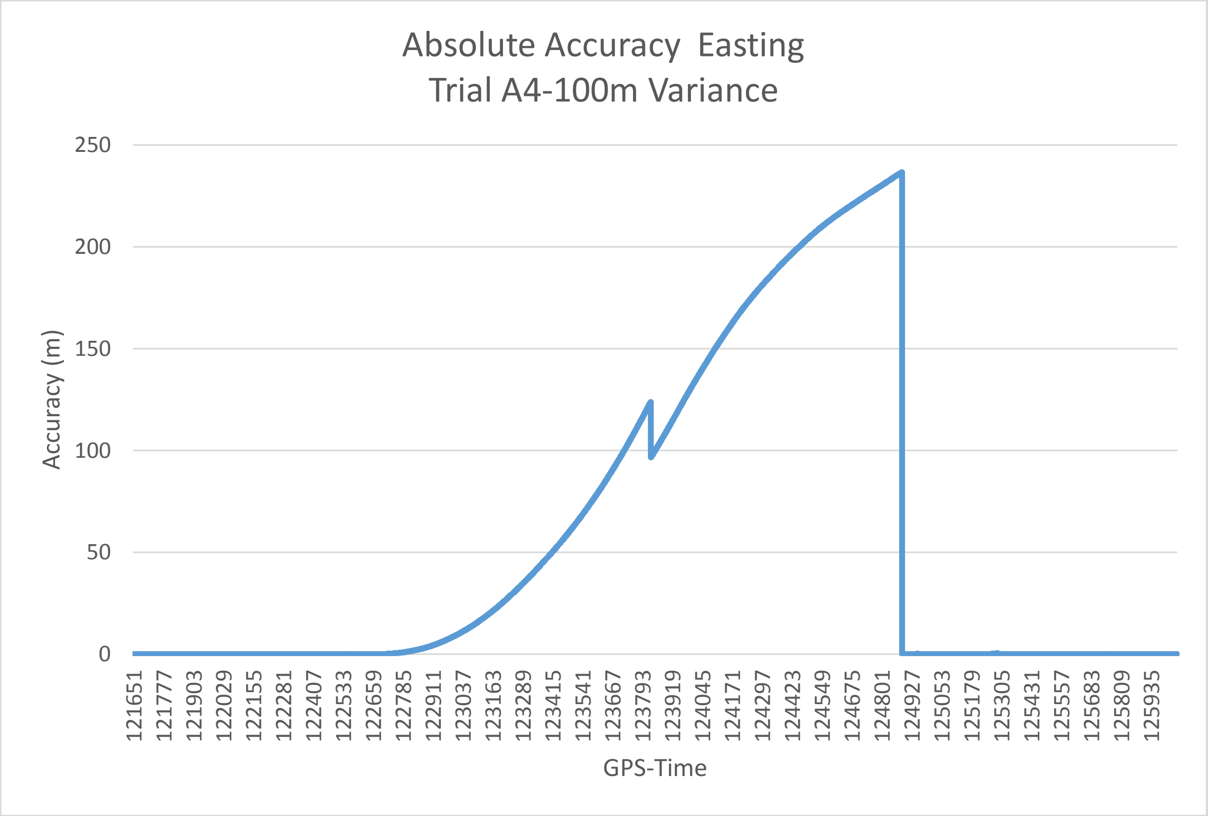
125683
125809
125935
GPS-Time


Absolute Accuracy Easting
Trial A4-100m Variance
Accuracy (m)
0
50
100
150
200
250
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
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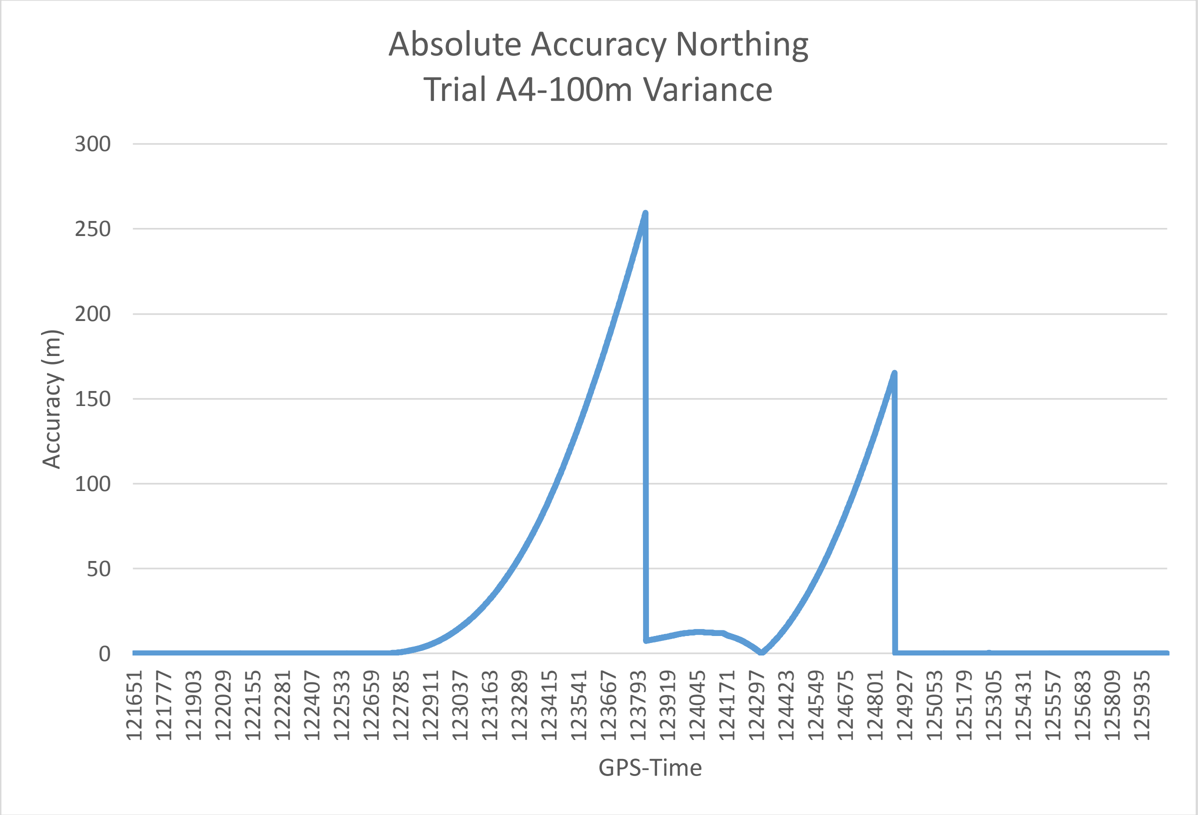
125683
125809
125935
GPS-Time


Absolute Accuracy Northing
Trial A4-100m Variance
Accuracy (m)
0
50
100
150
200
250
300
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
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124045
124171
124297
124423
124549
124675
124801
124927
125053
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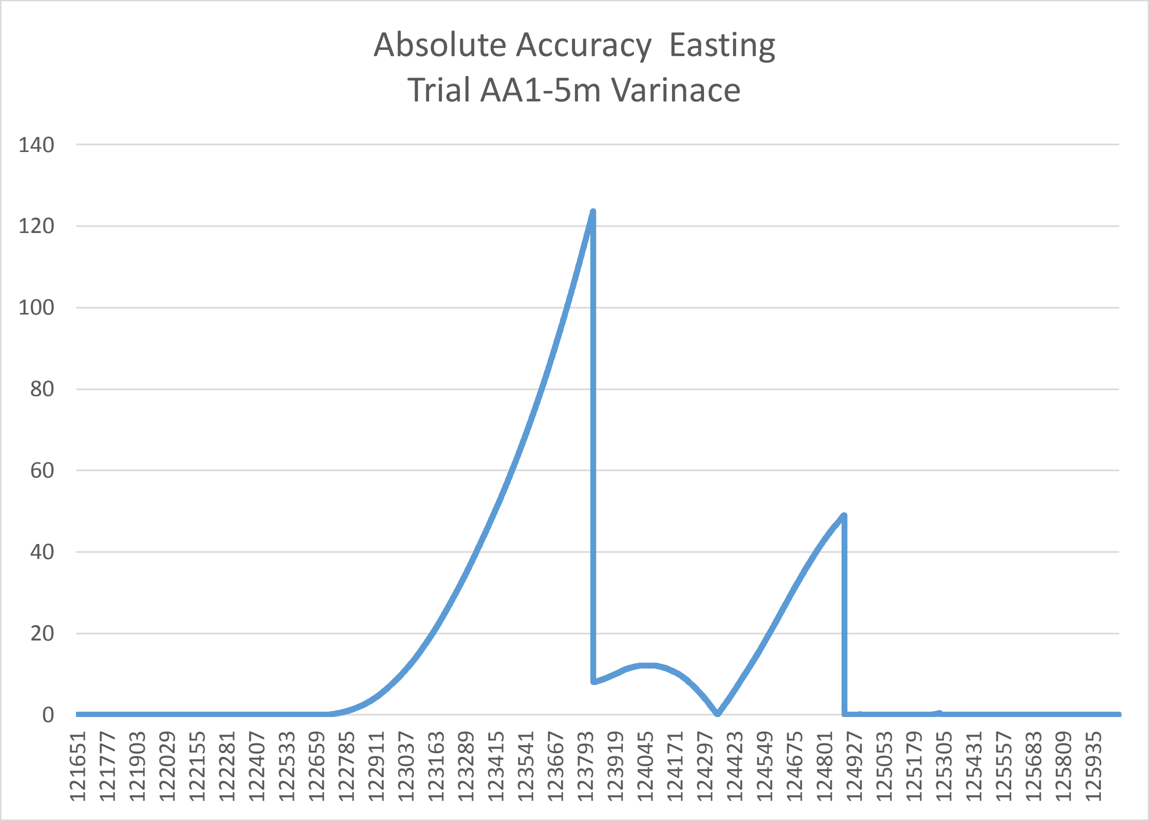
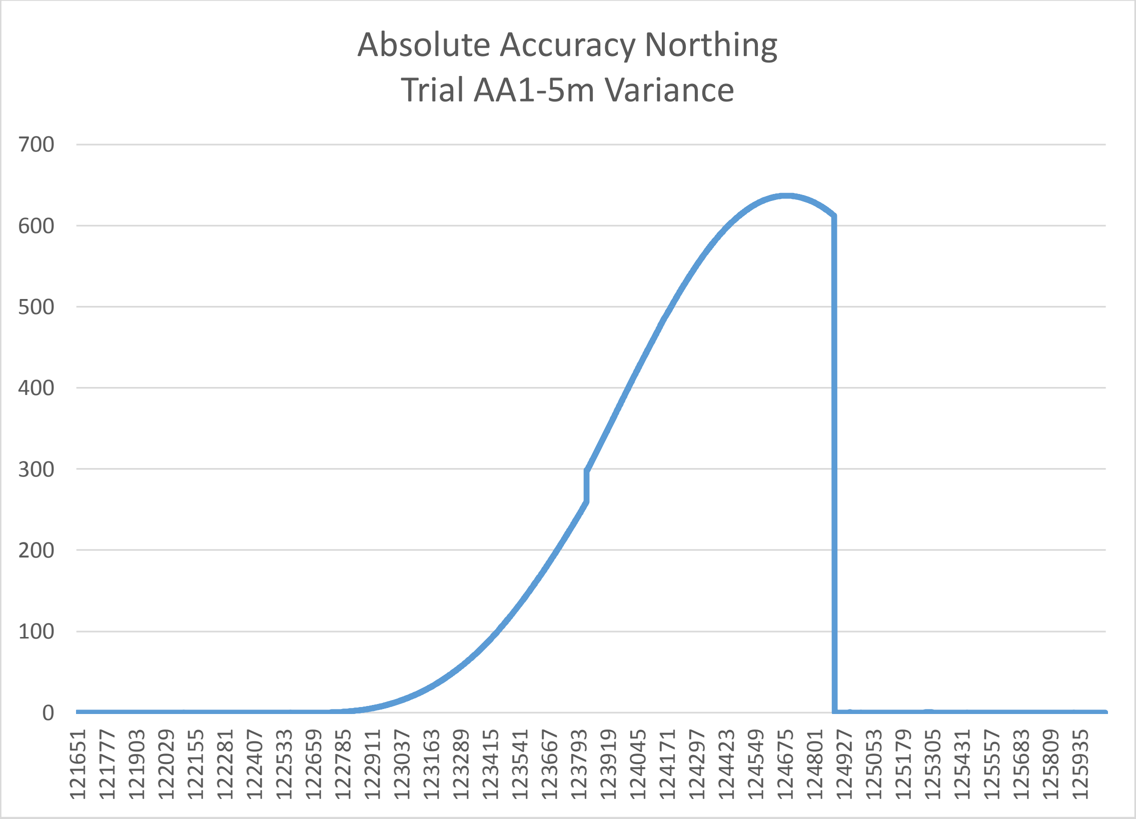
125683
125809
125935
GPS-Time

C5: Trial AA1-5m Vaiance

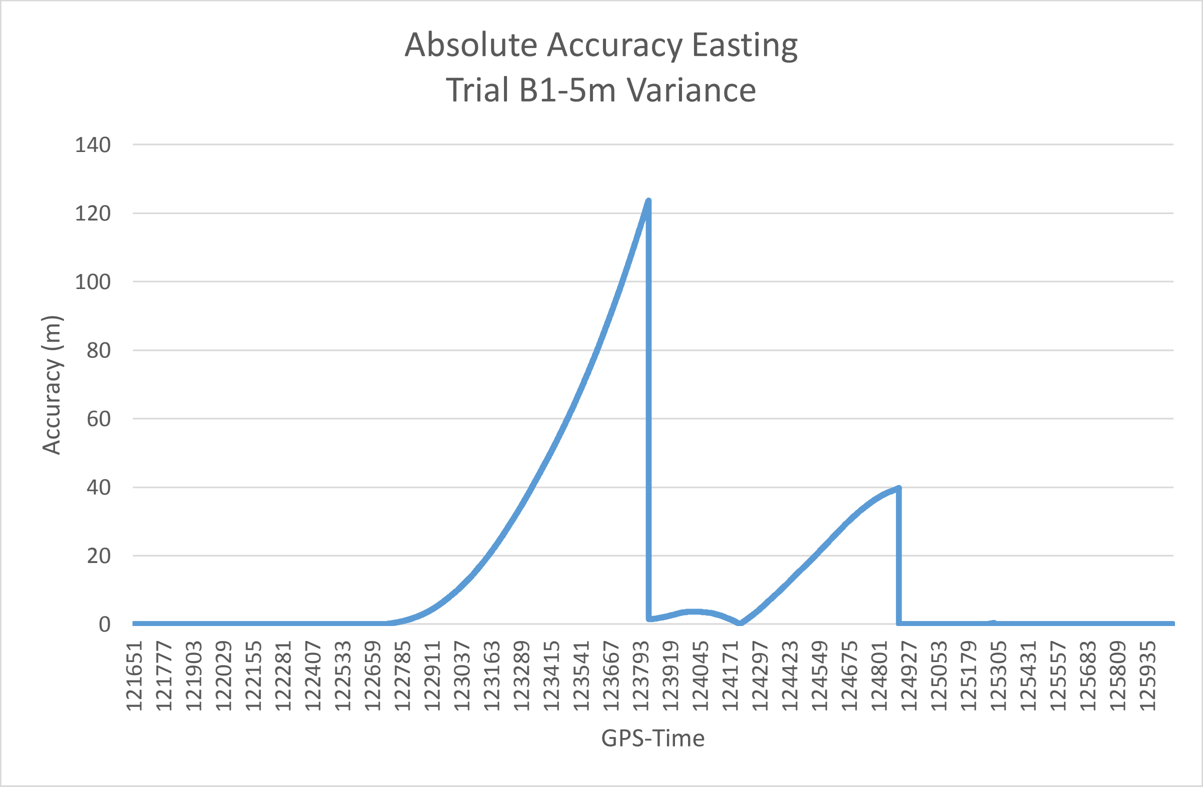
Manipulated Error: 300m North

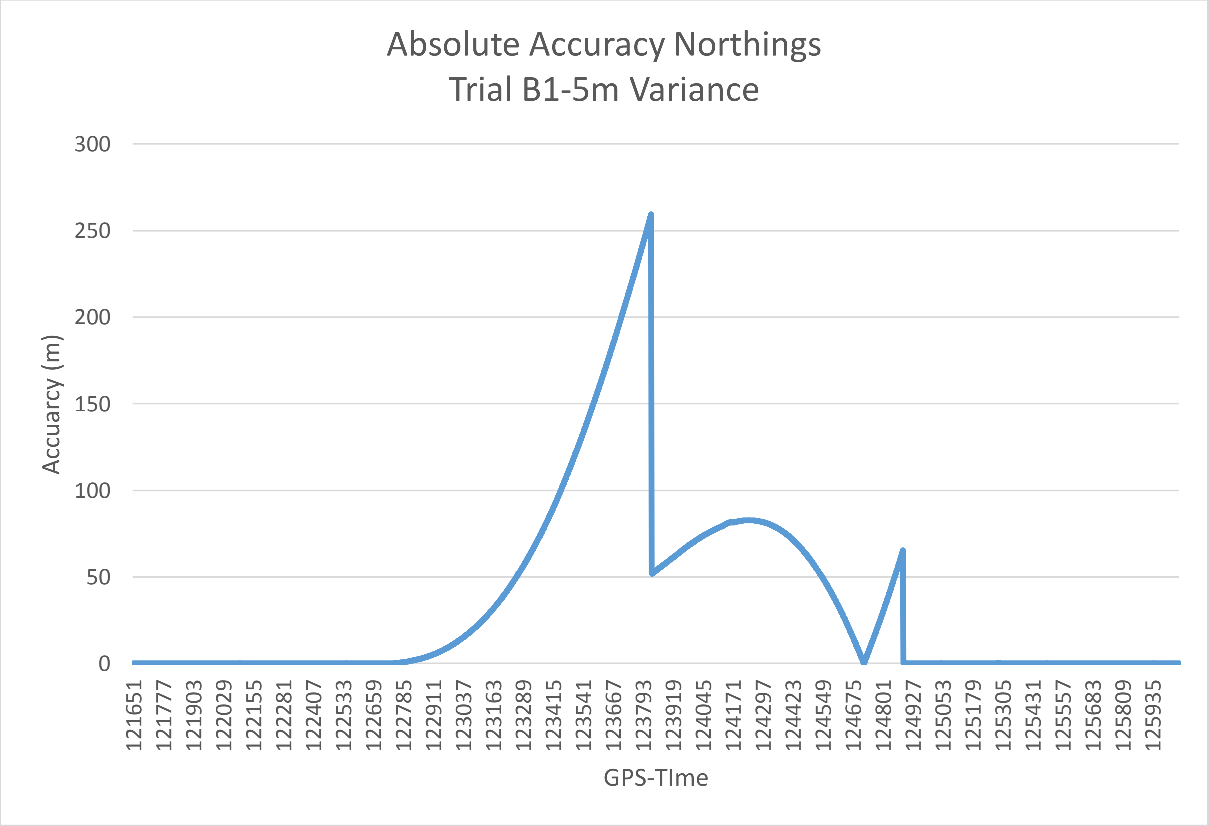


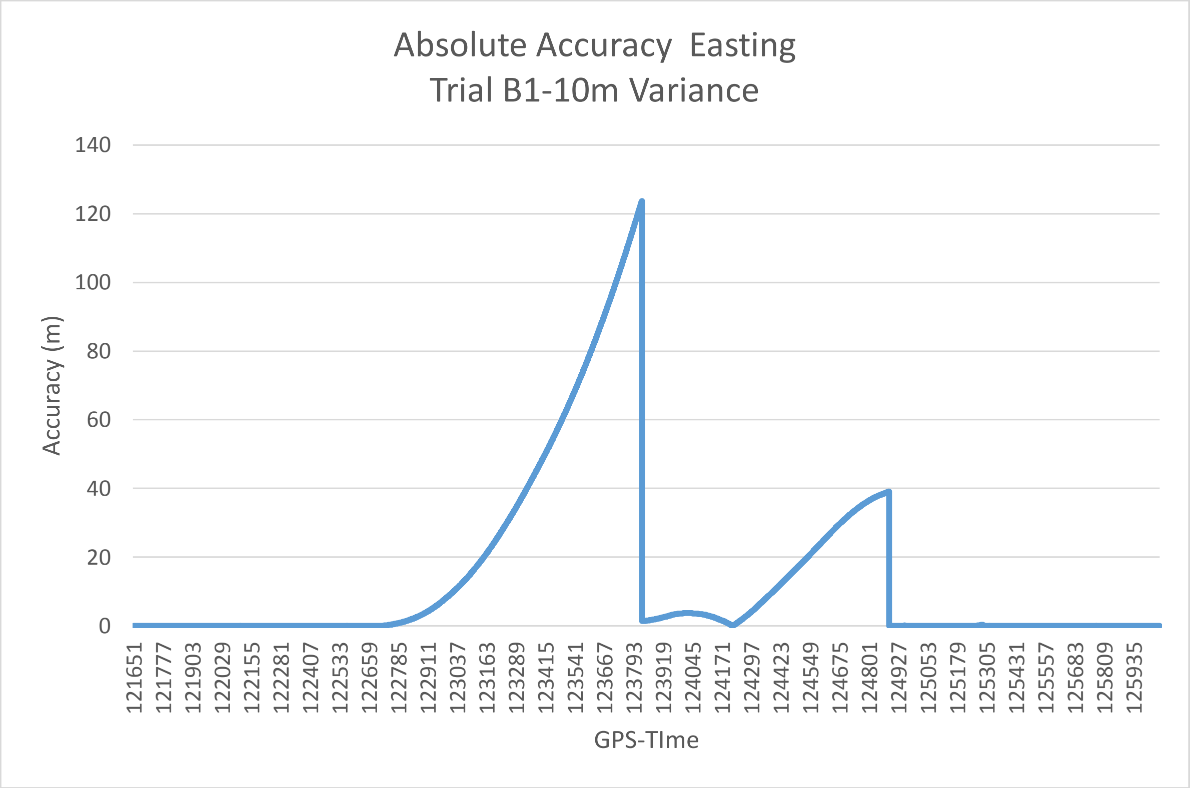


C6: Trial B1

Manipulated Error: 50m North






Absolute Accuracy Easting
Trial B1-10m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
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124297
124423
124549
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125053
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125305
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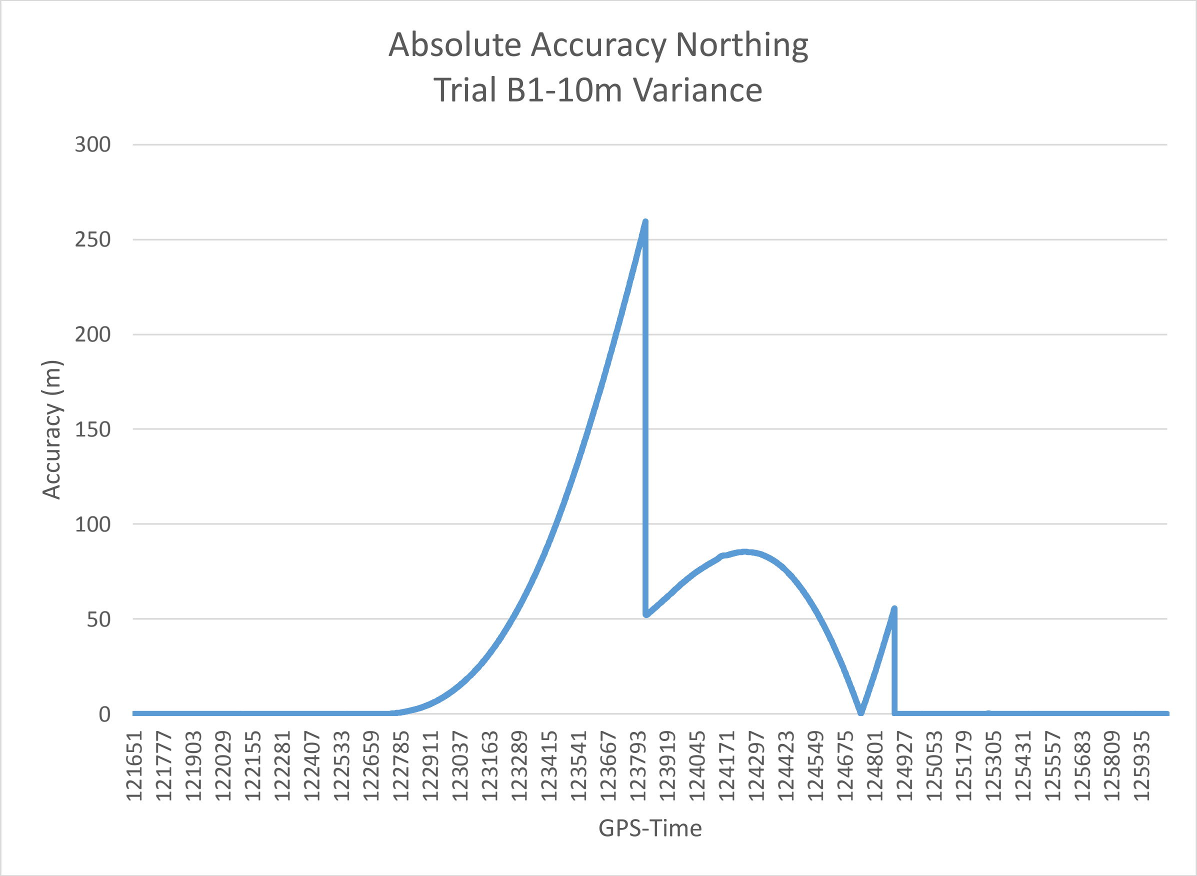
125557
125683
125809
125935
GPS-TIme


Absolute Accuracy Northing
Trial B1-10m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
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123289
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124297
124423
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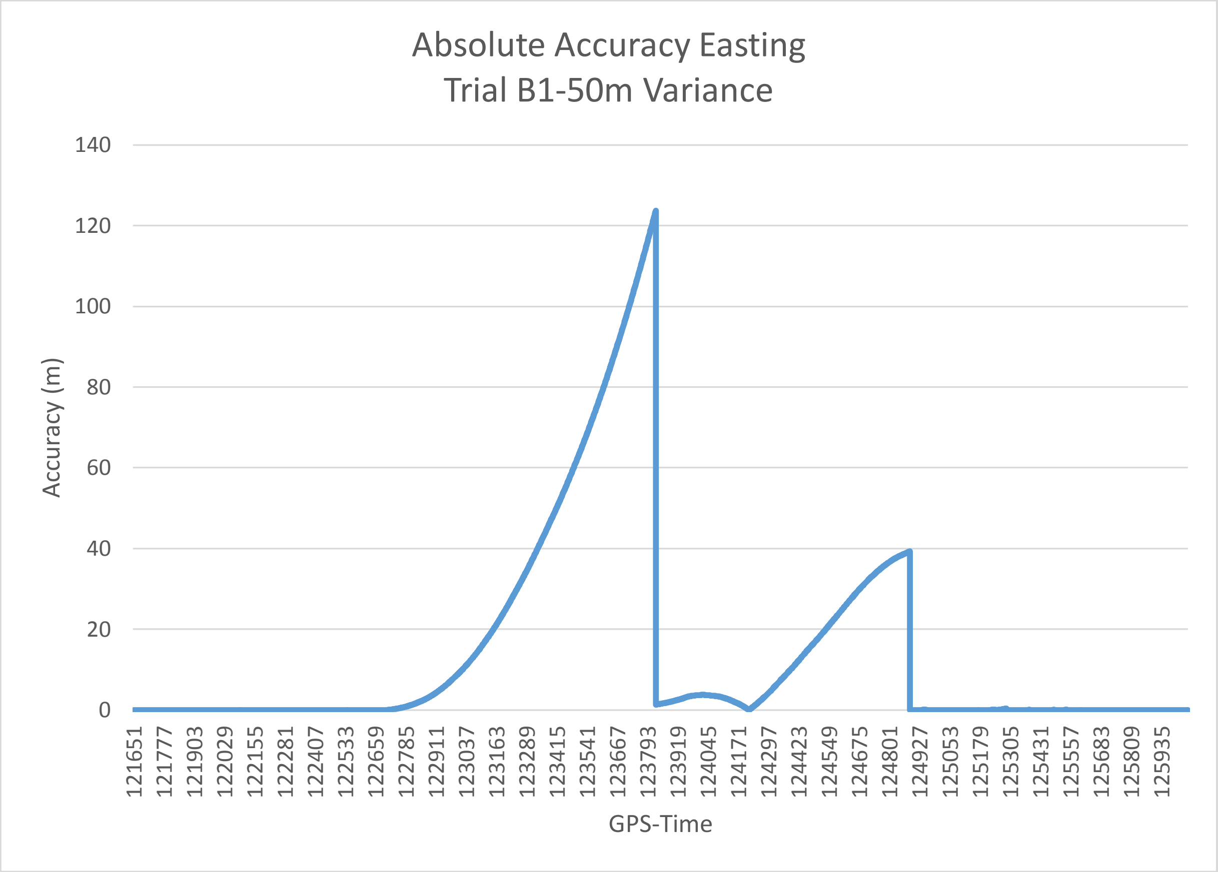
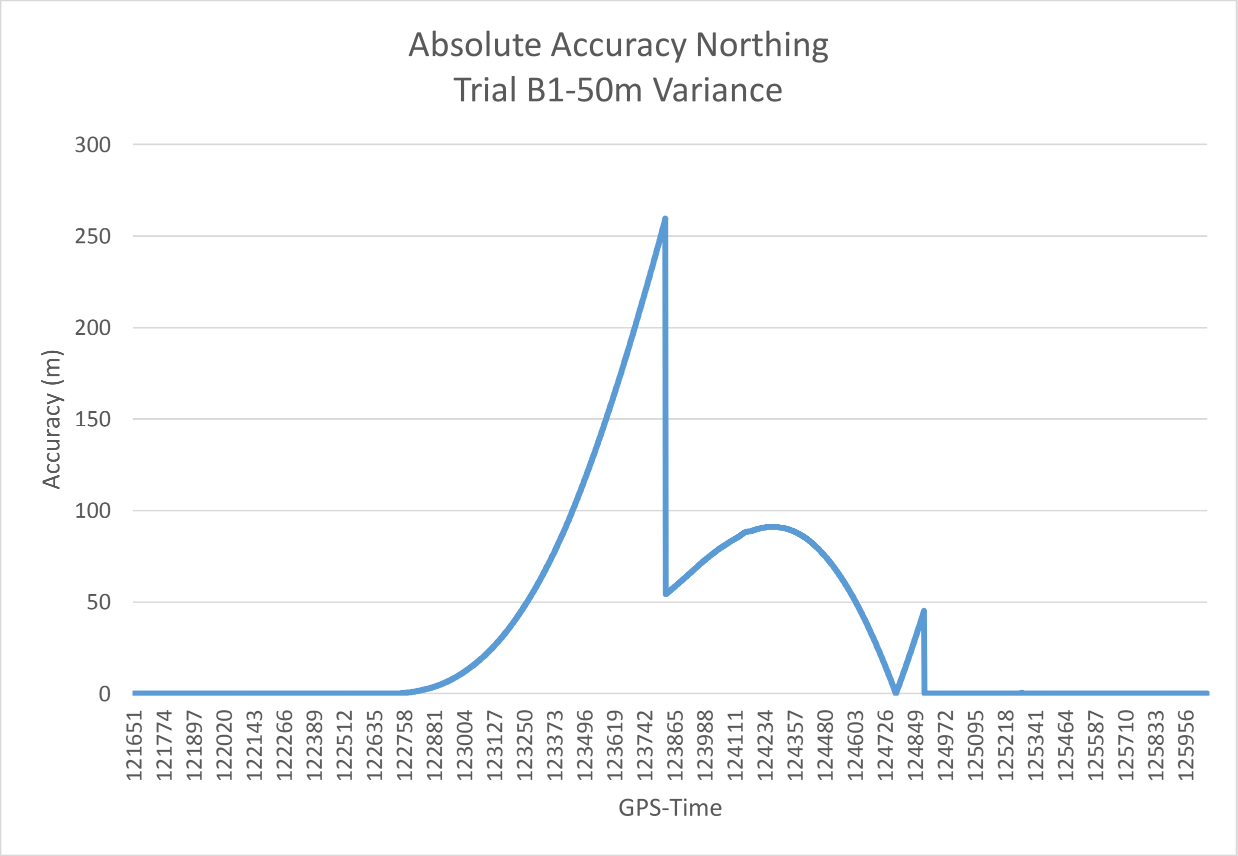
125683
125809
125935
GPS-Time


Absolute Accuracy Easting
Trial B1-50m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
124549
124675
124801
124927
125053
125179
125305
125431
125557
125683
125809
125935
GPS-Time


Absolute Accuracy Northing
Trial B1-50m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121774
121897
122020
122143
122266
122389
122512
122635
122758
122881
123004
123127
123250
123373
123496
123619
123742
123865
123988
124111
124234
124357
124480
124603
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125095
125218
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125587
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125833
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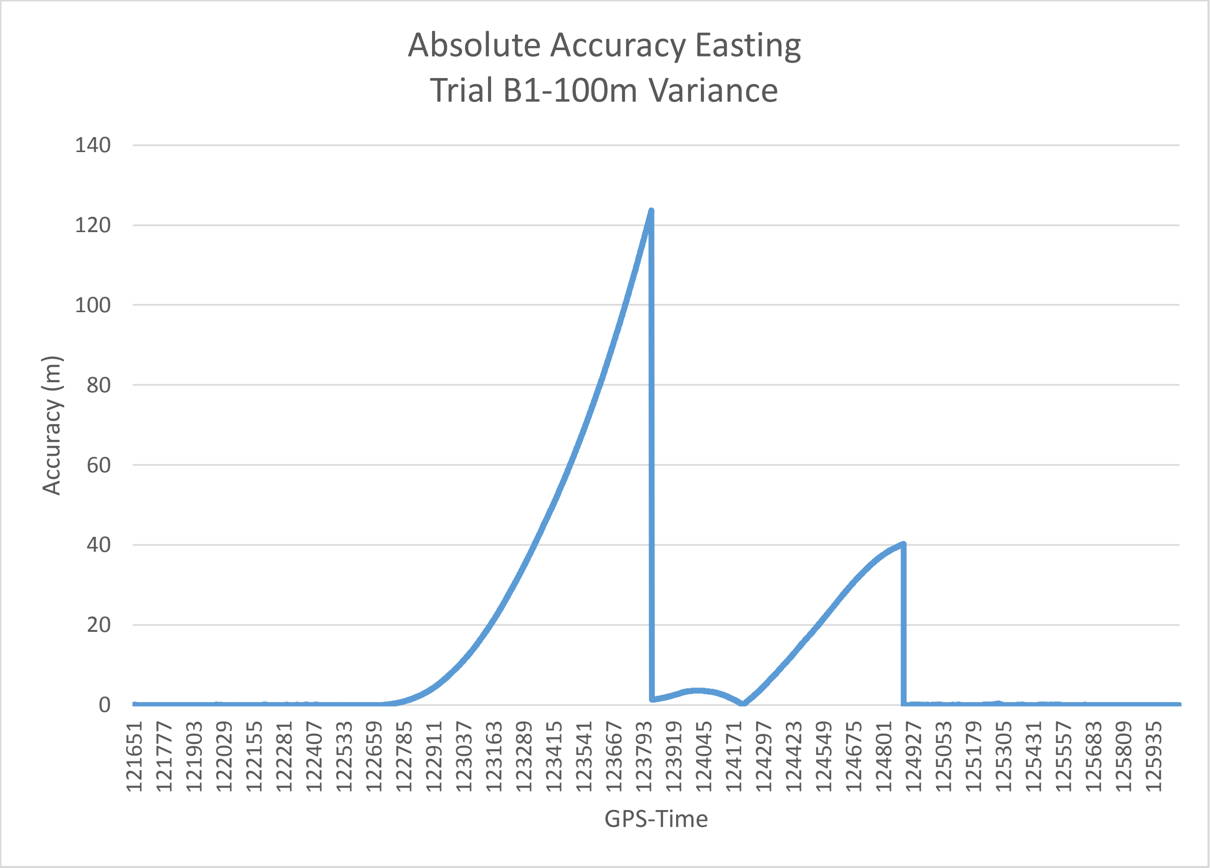
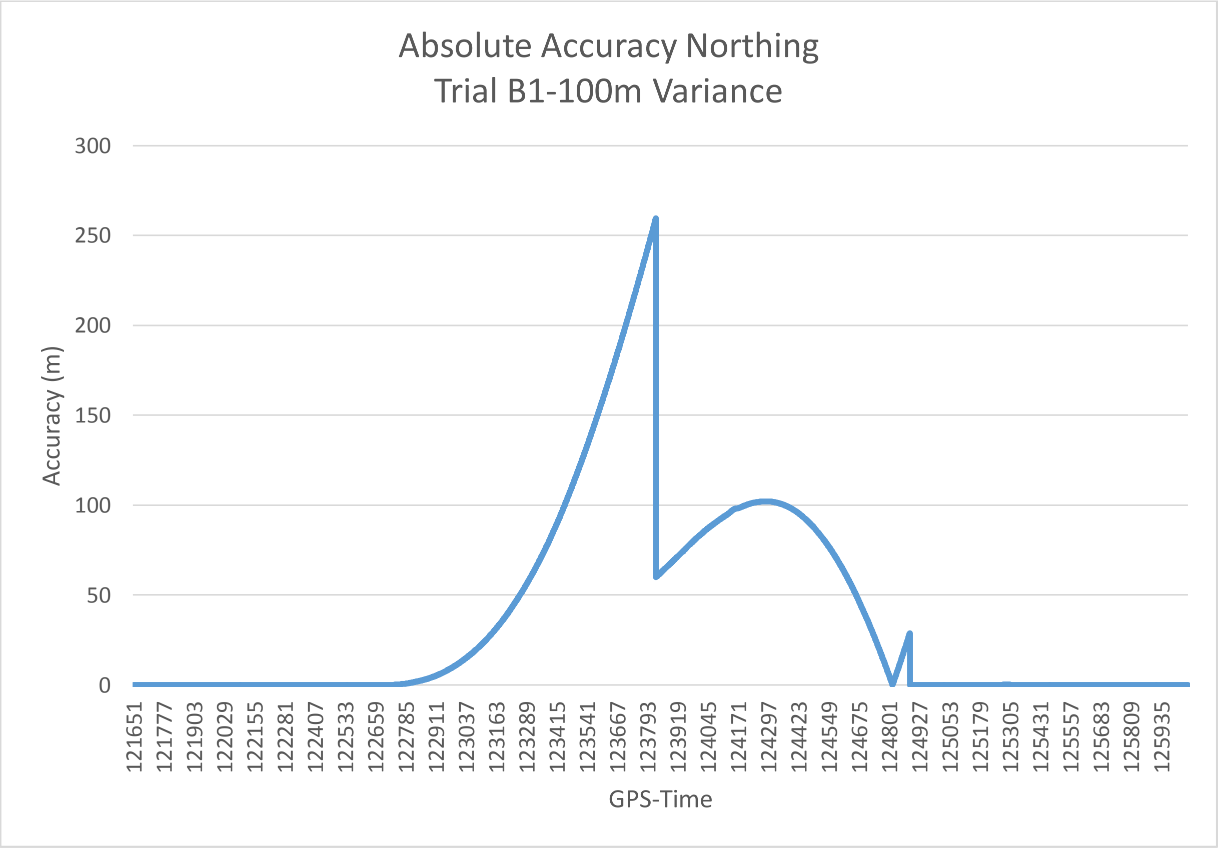
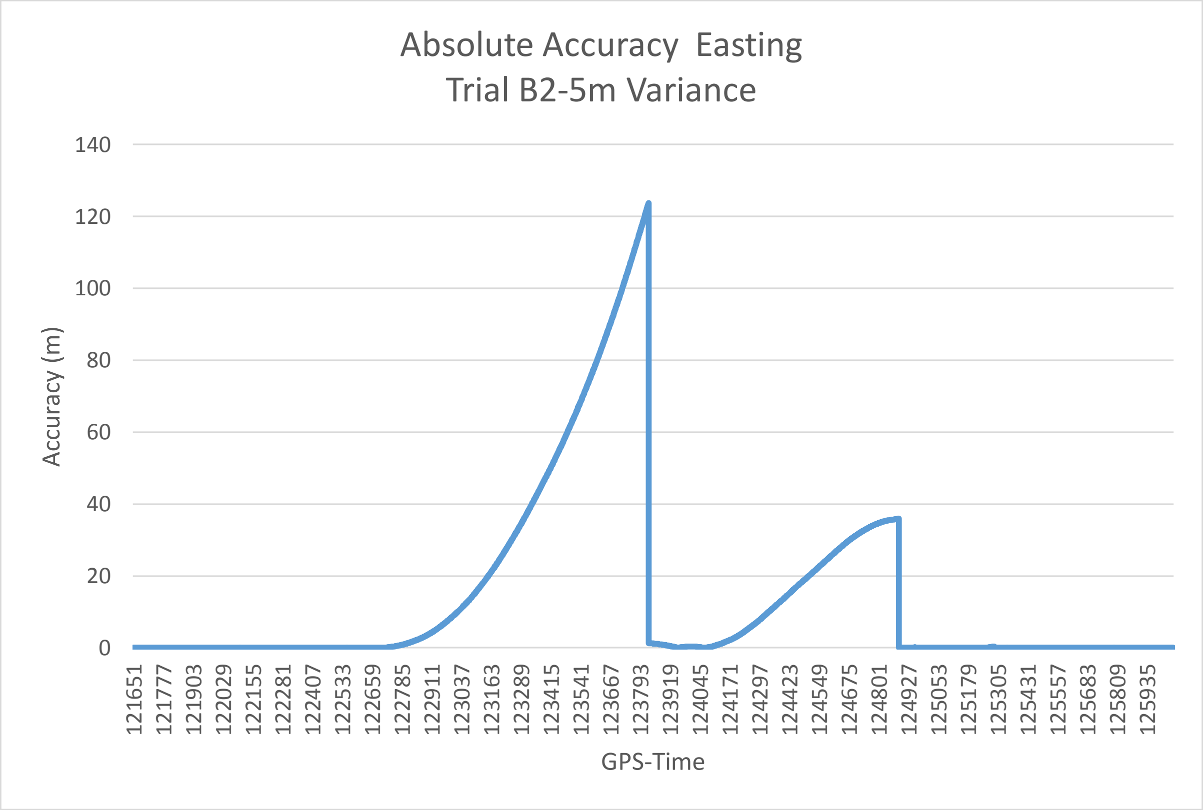
GPS-Time

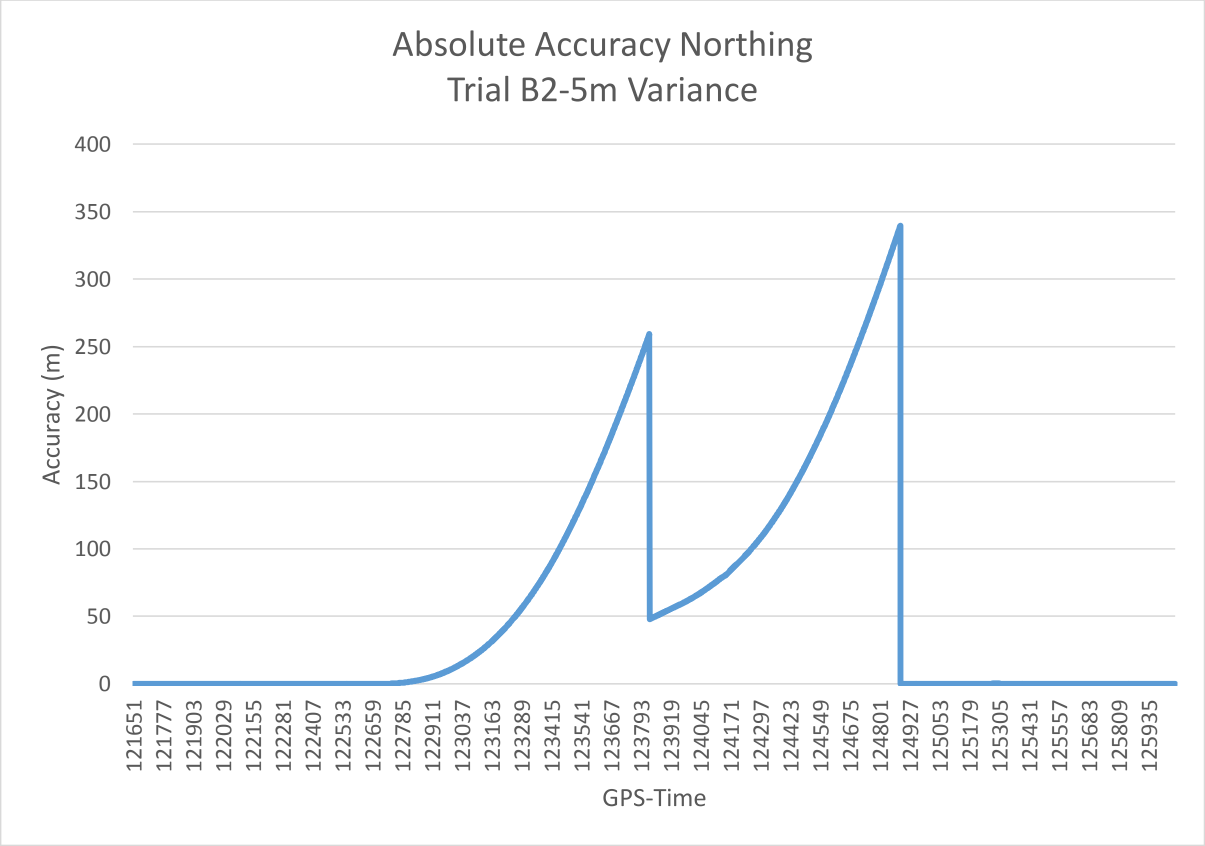

Absolute Accuracy Easting
Trial B1-100m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
124549
124675
124801
124927
125053
125179
125305
125431
125557
125683
125809
125935
GPS-Time

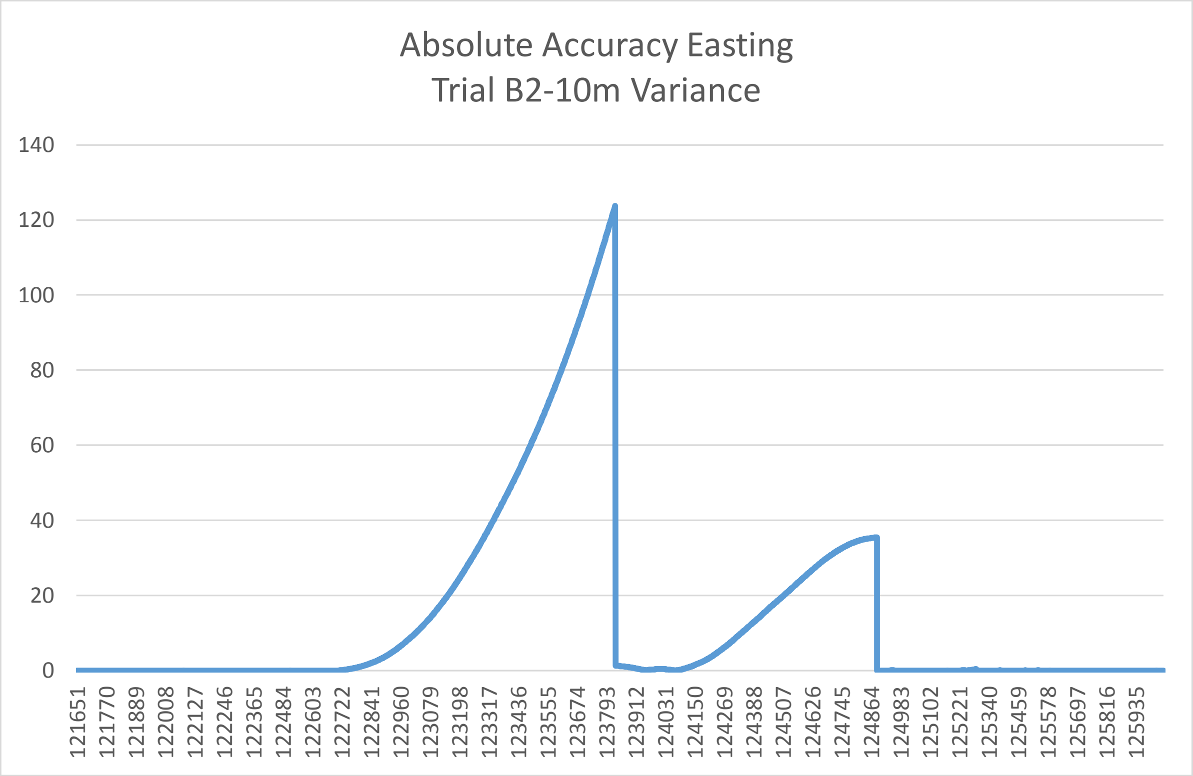

Absolute Accuracy Northing
Trial B1-100m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
124549
124675
124801
124927
125053
125179
125305
125431
125557
125683
125809
125935
GPS-Time

C7: Trial B2

Manipulated Error: 50m South






Absolute Accuracy Easting
Trial B2-10m Variance
140
120
100
80
60
40
20
0
121651
121770
121889
122008
122127
122246
122365
122484
122603
122722
122841
122960
123079
123198
123317
123436
123555
123674
123793
123912
124031
124150
124269
124388
124507
124626
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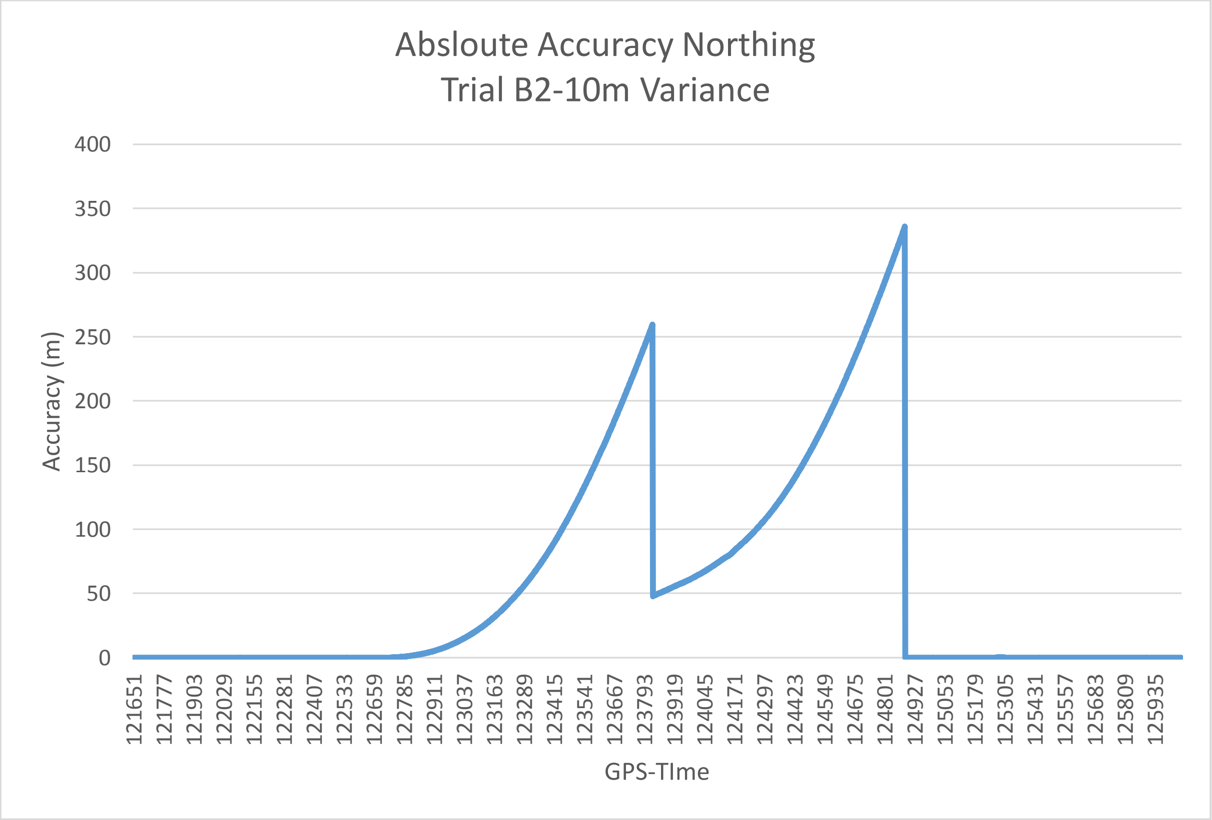
125697
125816
125935


Absloute Accuracy Northing
Trial B2-10m Variance
Accuracy (m)
400
350
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
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124045
124171
124297
124423
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125053
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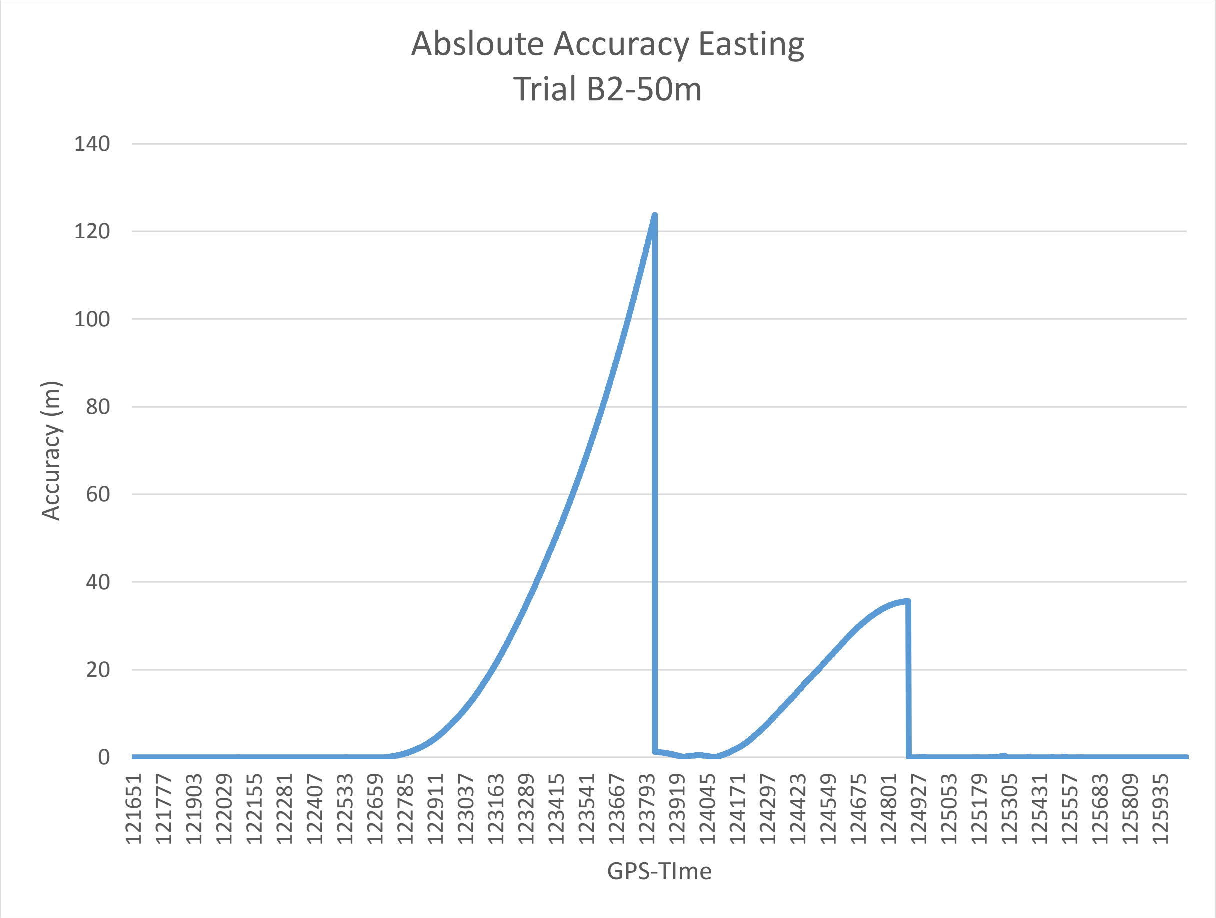
125683
125809
125935
GPS-TIme


Absloute Accuracy Easting
Trial B2-50m
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
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124675
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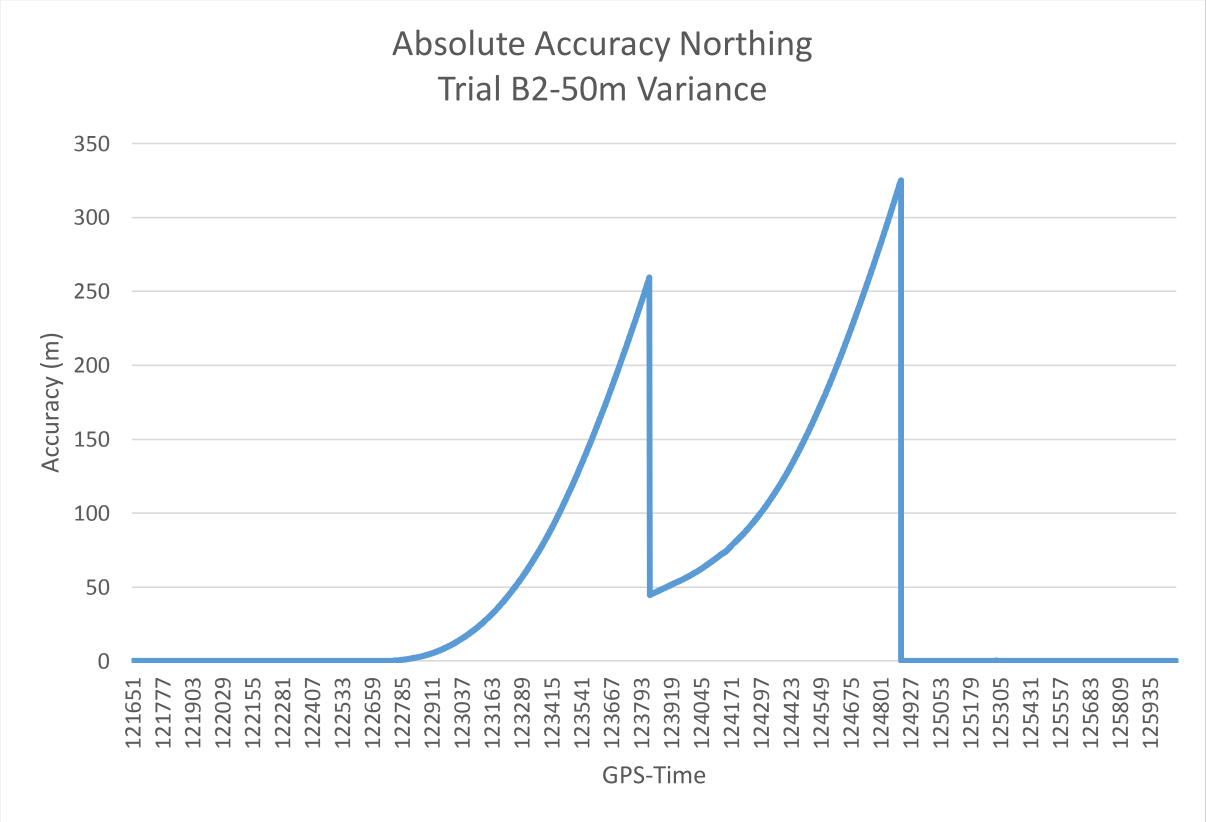
124927
125053
125179
125305
125431
125557
125683
125809
125935
GPS-TIme


Absolute Accuracy Northing
Trial B2-50m Variance
Accuracy (m)
350
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
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124675
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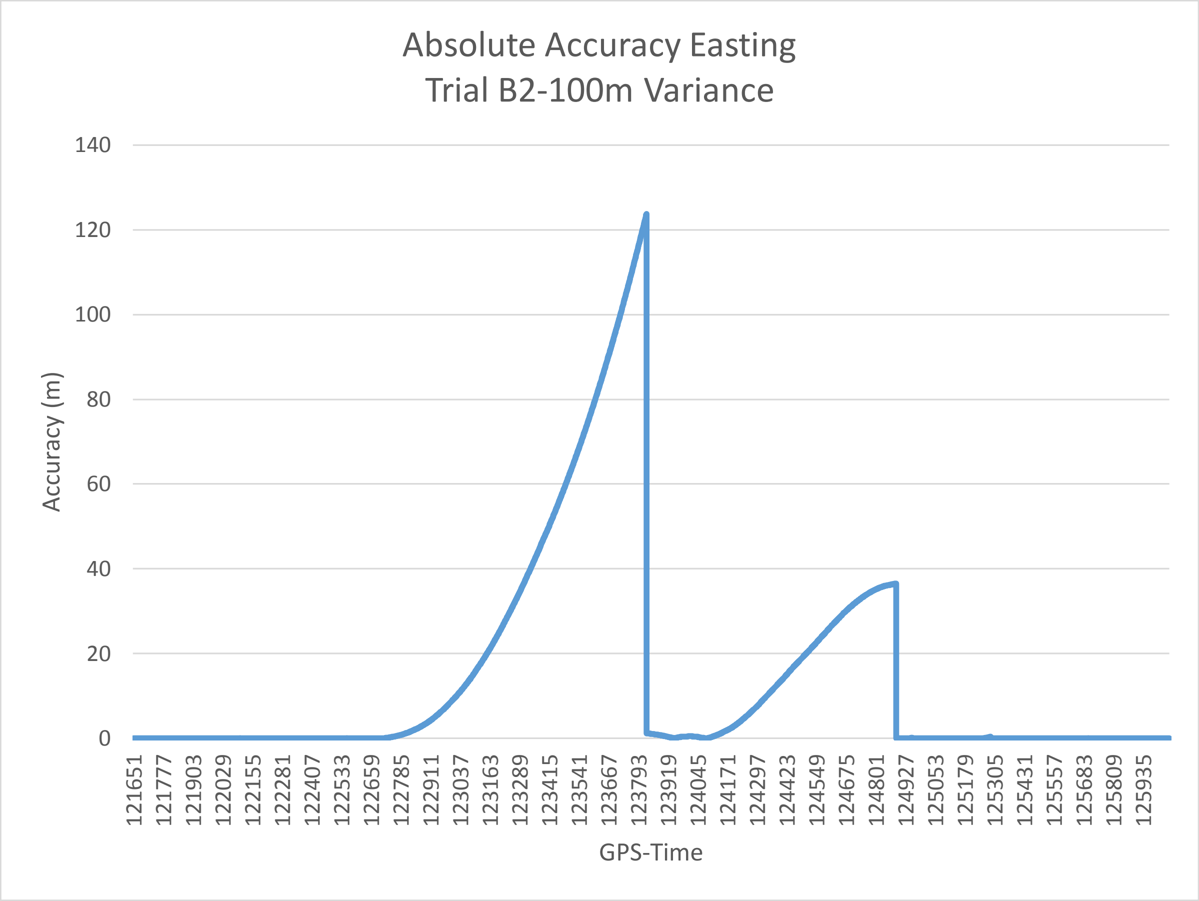
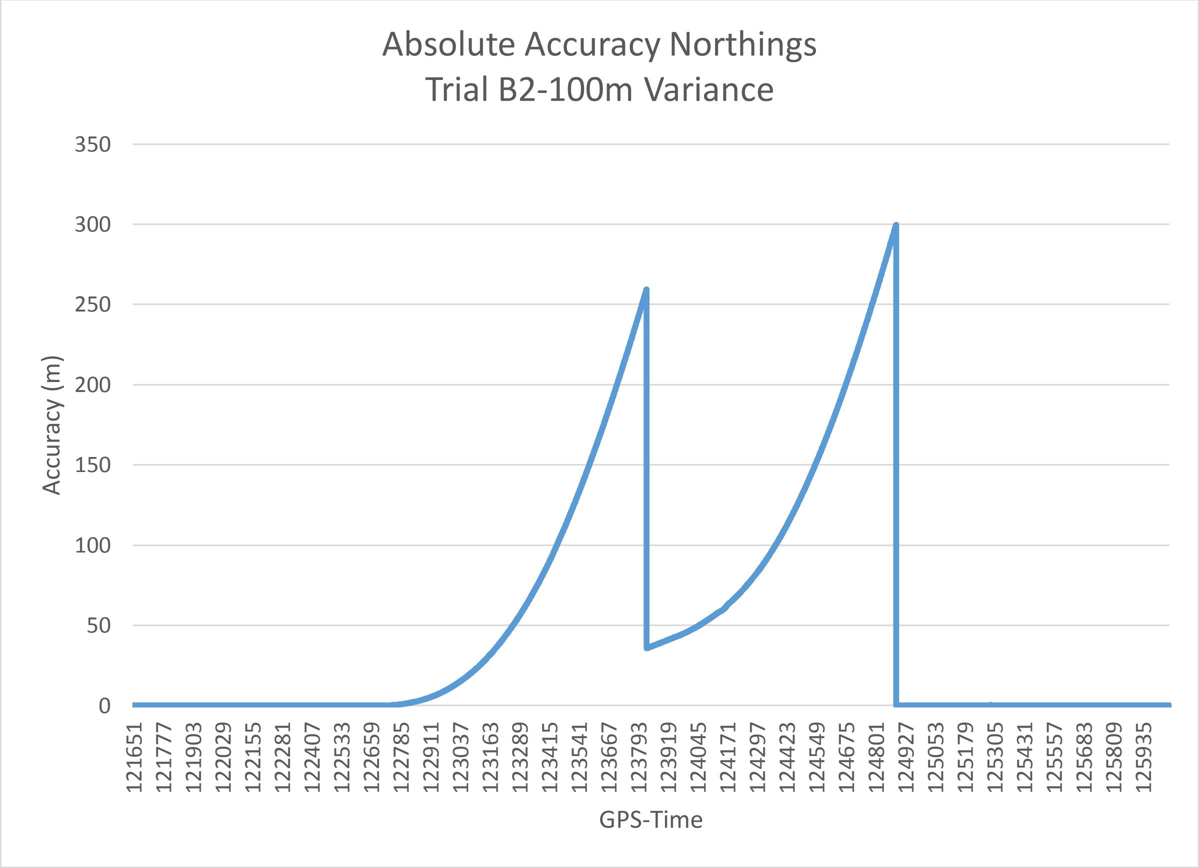
124927
125053
125179
125305
125431
125557
125683
125809
125935
GPS-Time


Absolute Accuracy Easting
Trial B2-100m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
124549
124675
124801
124927
125053
125179
125305
125431
125557
125683
125809
125935
GPS-Time


Absolute Accuracy Northings
Trial B2-100m Variance
Accuracy (m)
350
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
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124171
124297
124423
124549
124675
124801
124927
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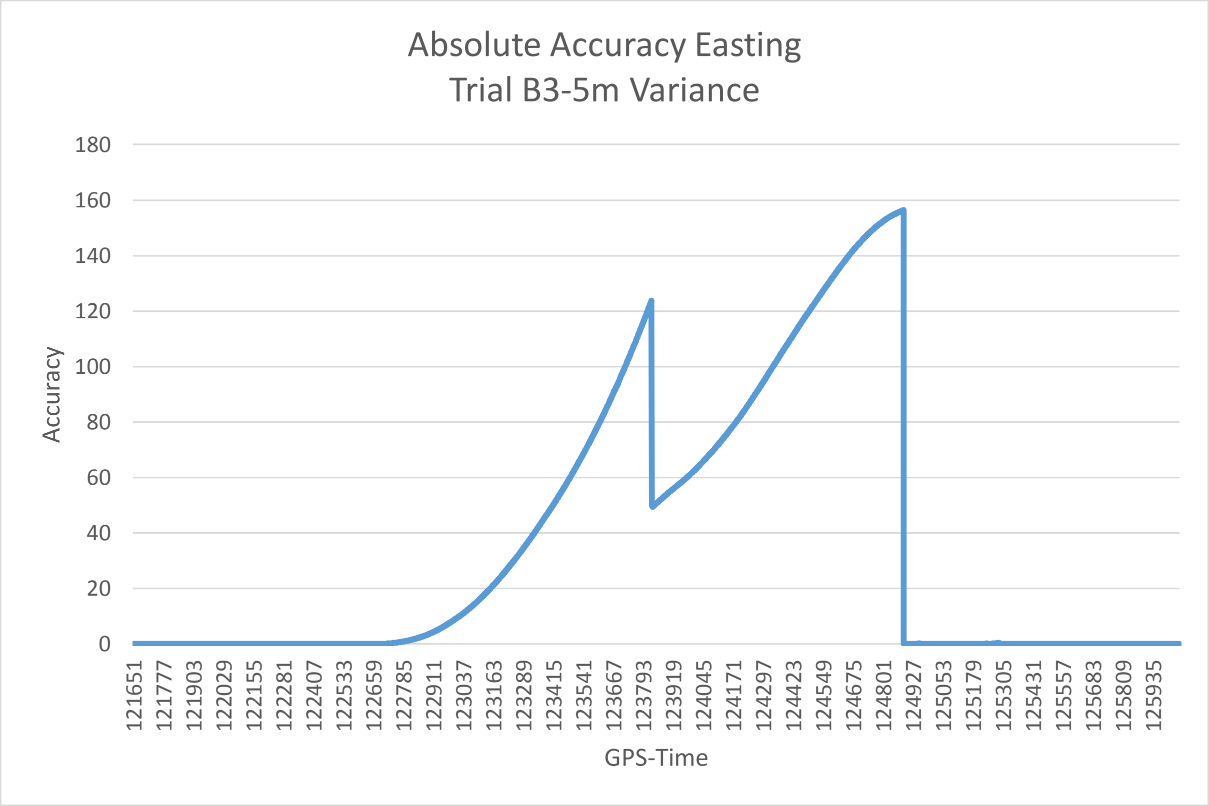
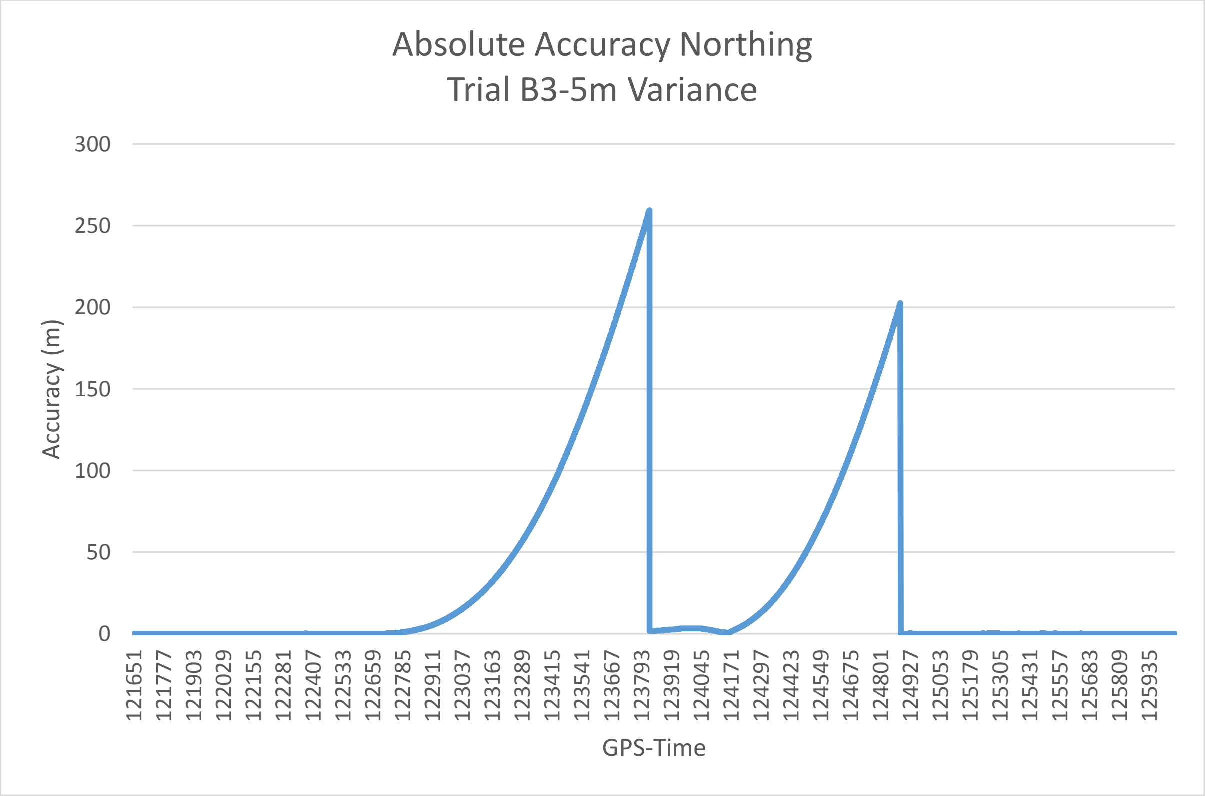
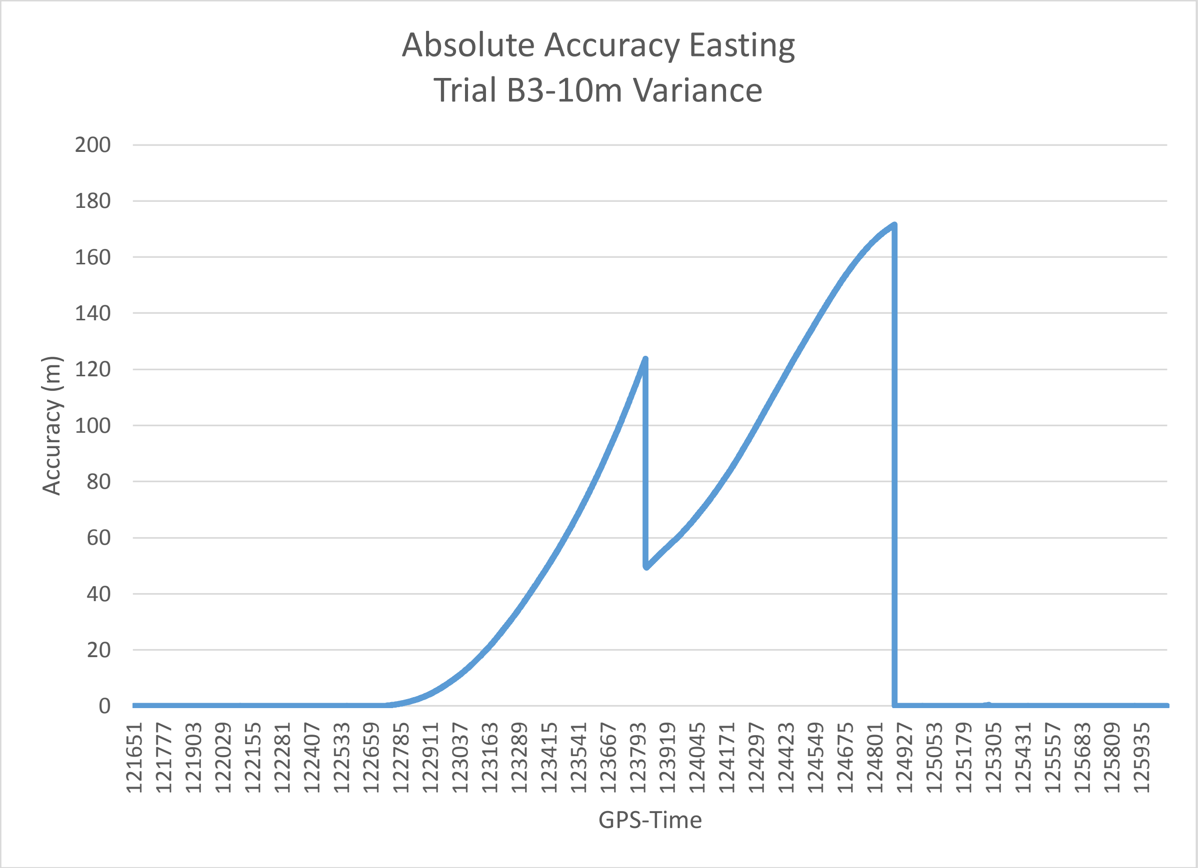
125431
125557
125683
125809
125935
GPS-Time

C8: Trial B3

Manipulated Error: 50m East






Absolute Accuracy Easting
Trial B3-10m Variance
Accuracy (m)
200
180
160
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
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124171
124297
124423
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124927
125053
125179
125305
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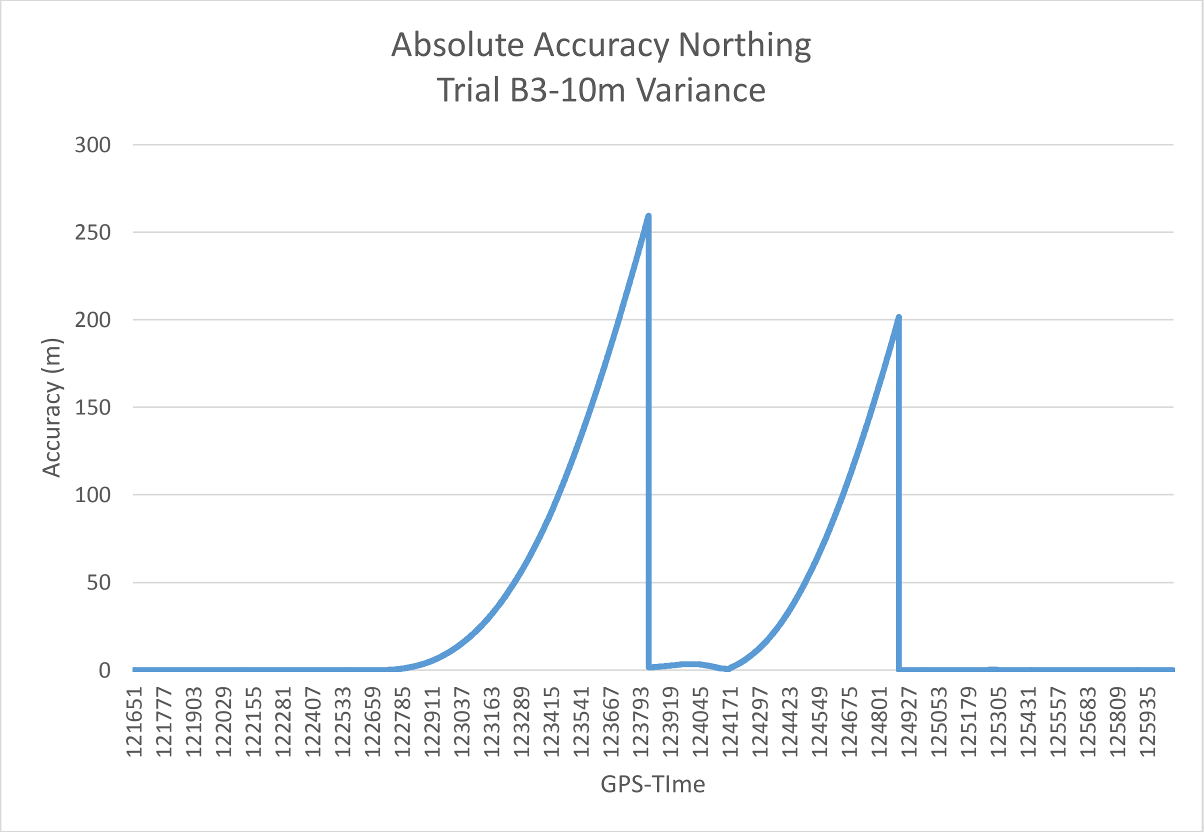
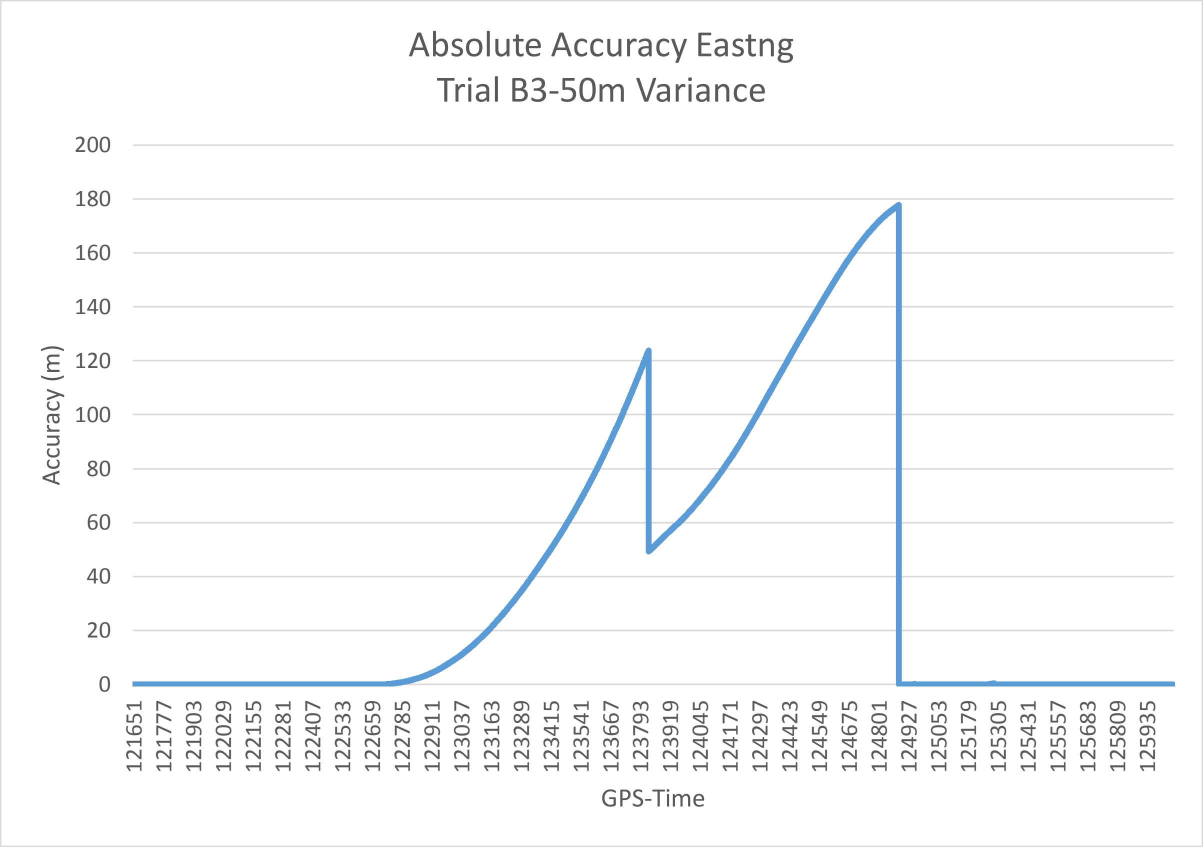
125557
125683
125809
125935
GPS-Time


Absolute Accuracy Northing
Trial B3-10m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
124549
124675
124801
124927
125053
125179
125305
125431
125557
125683
125809
125935
GPS-TIme


Absolute Accuracy Eastng
Trial B3-50m Variance
Accuracy (m)
200
180
160
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
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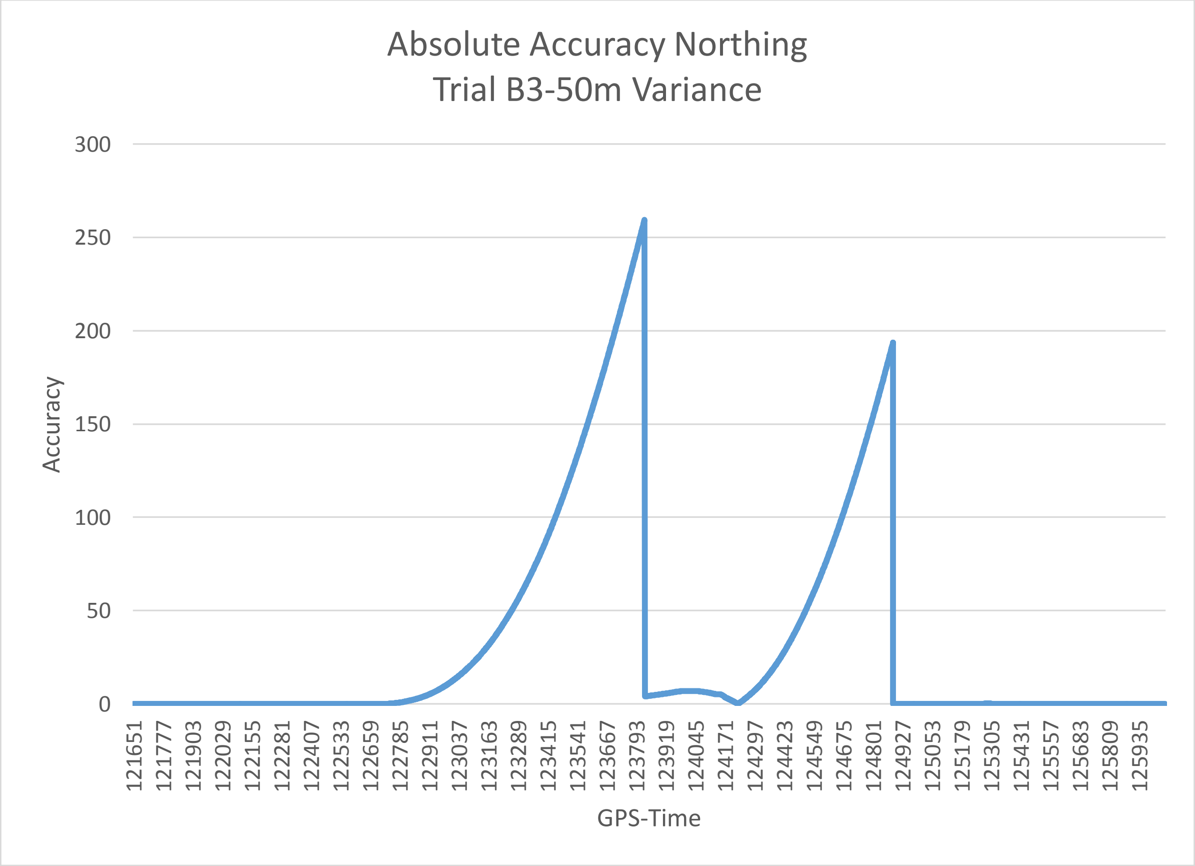
GPS-Time


Absolute Accuracy Northing
Trial B3-50m Variance
Accuracy
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
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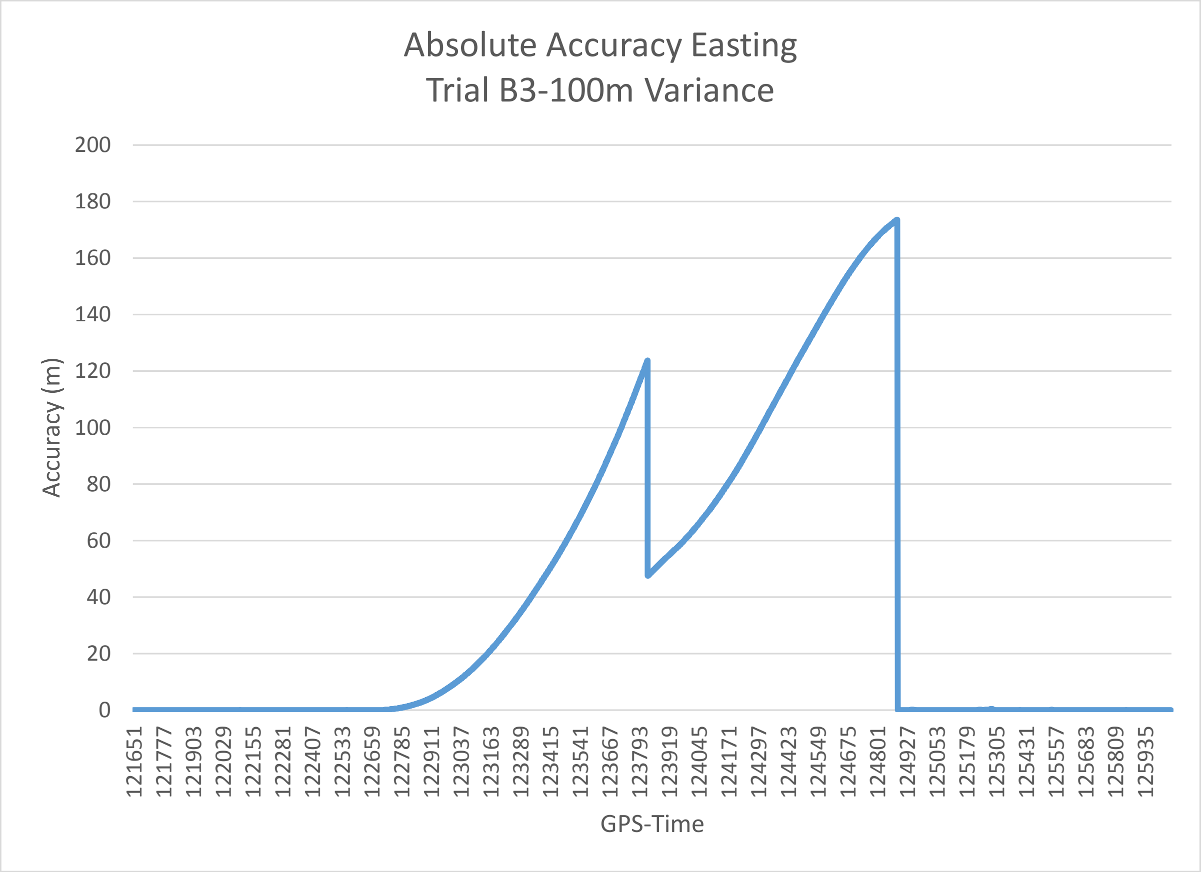
125809
125935
GPS-Time


Absolute Accuracy Easting
Trial B3-100m Variance
Accuracy (m)
200
180
160
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
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124171
124297
124423
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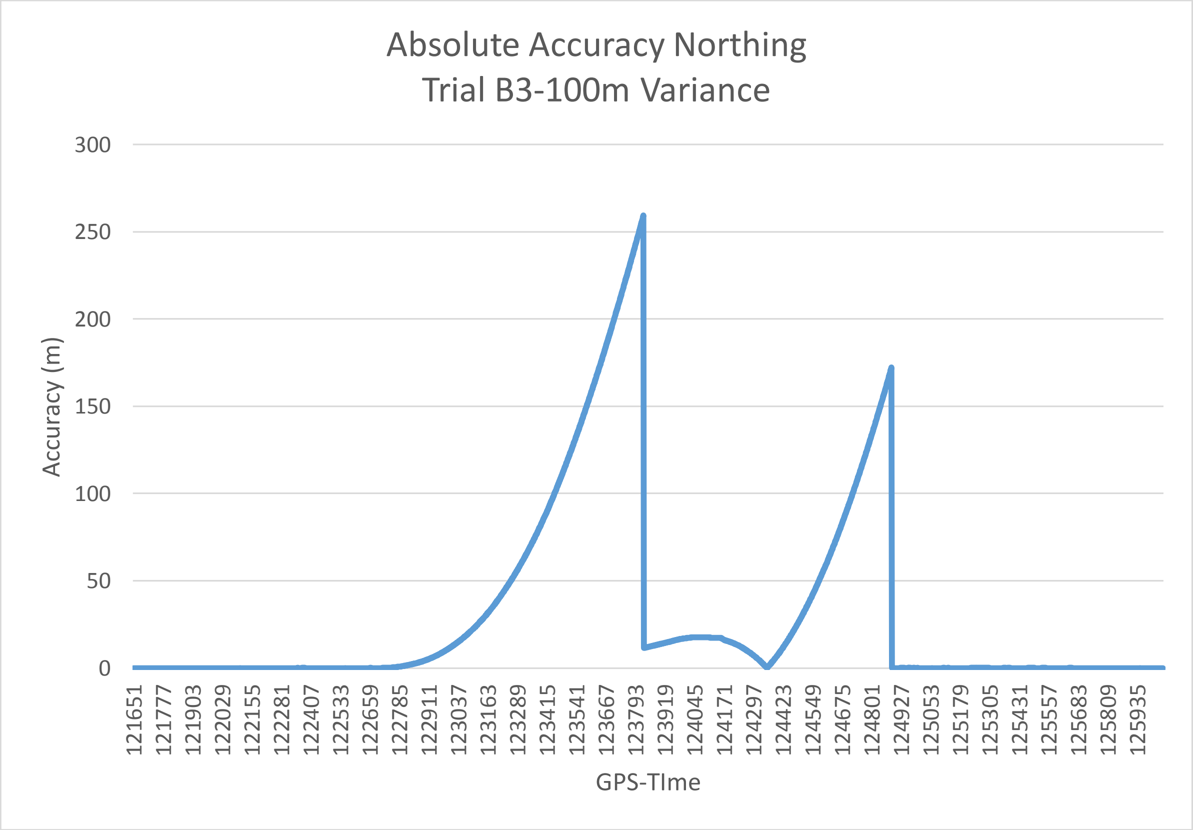
125683
125809
125935
GPS-Time


Absolute Accuracy Northing
Trial B3-100m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
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124171
124297
124423
124549
124675
124801
124927
125053
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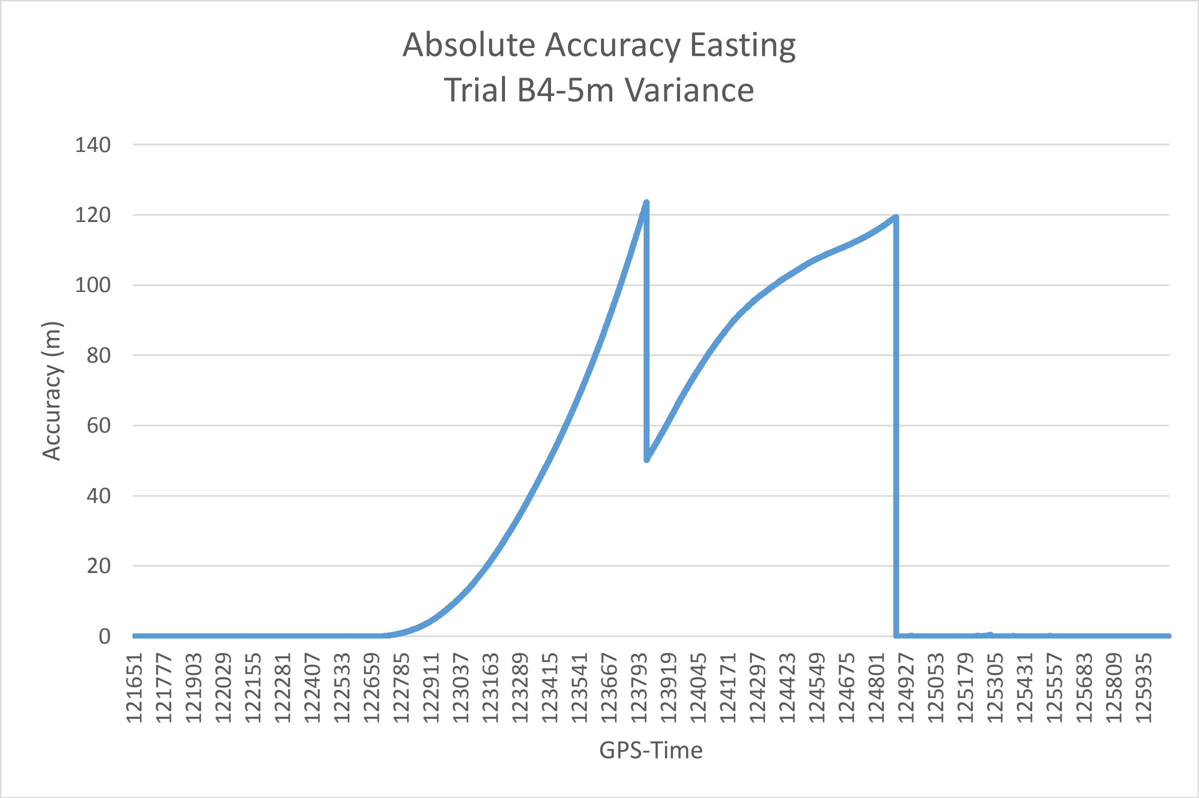
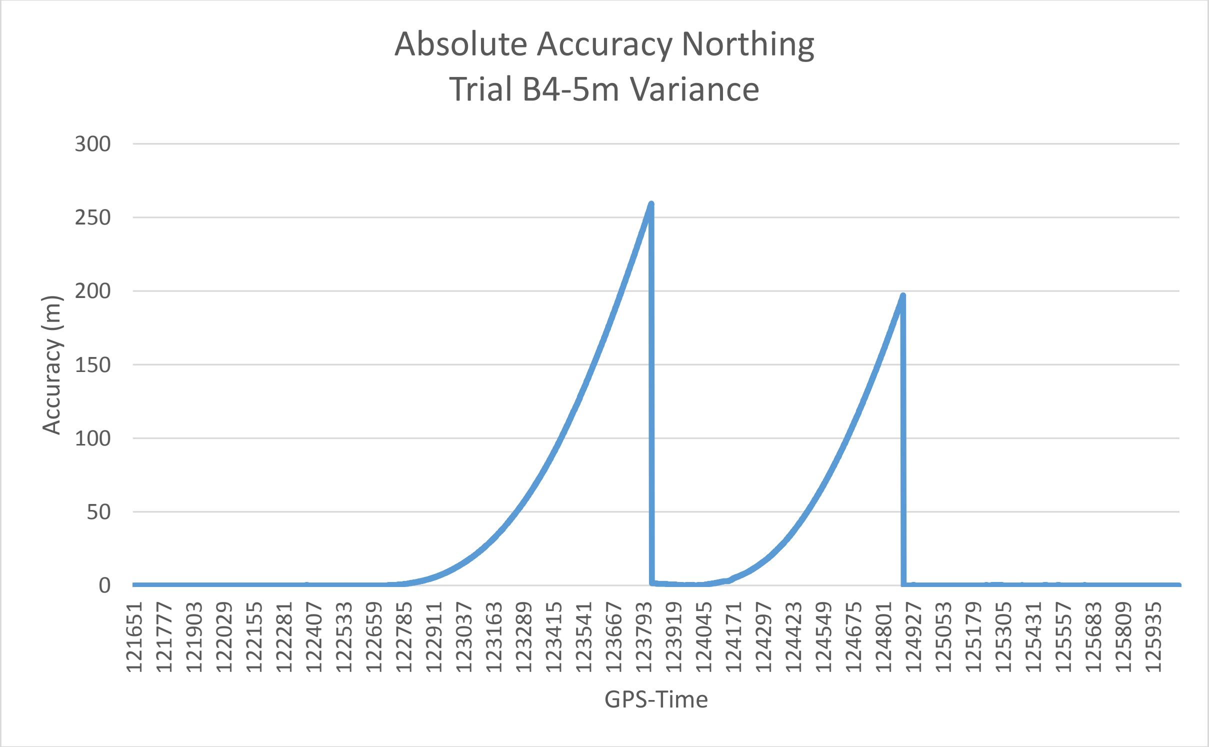
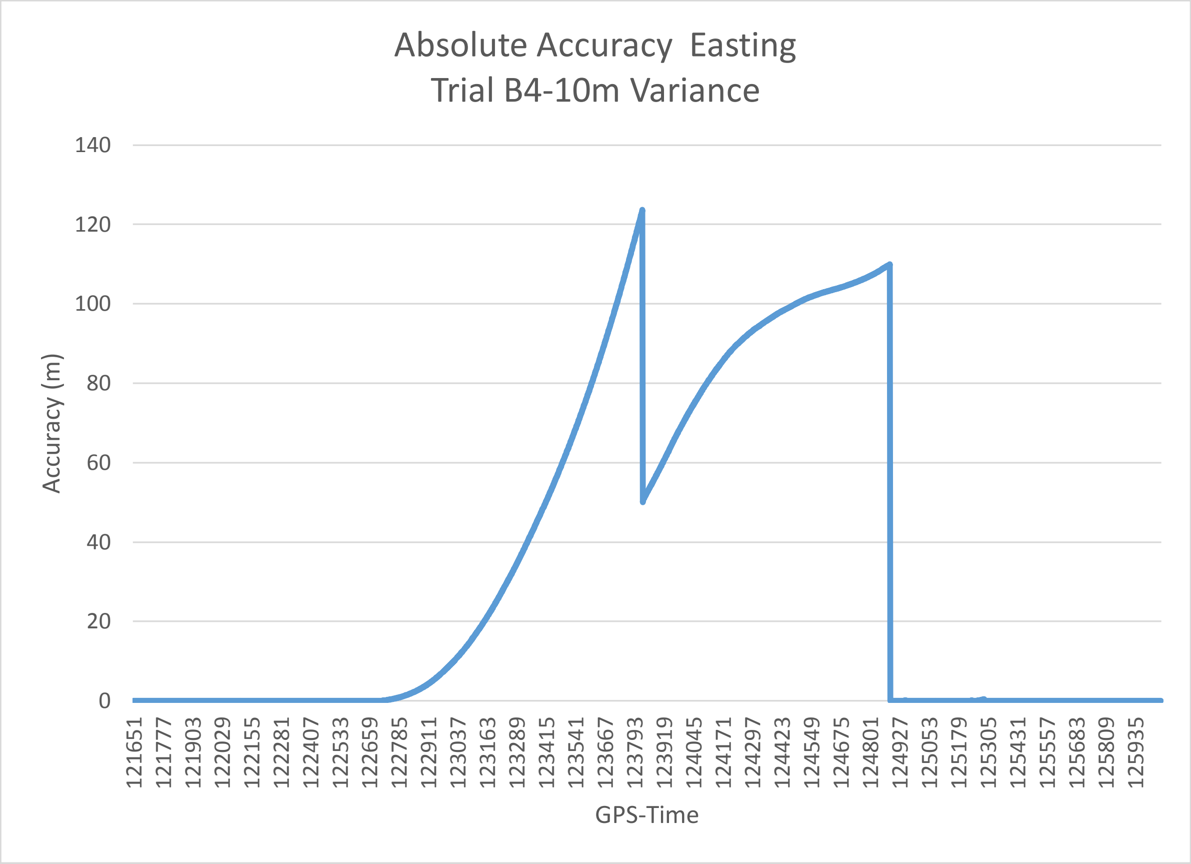
125431
125557
125683
125809
125935
GPS-TIme

C9: Trial B4

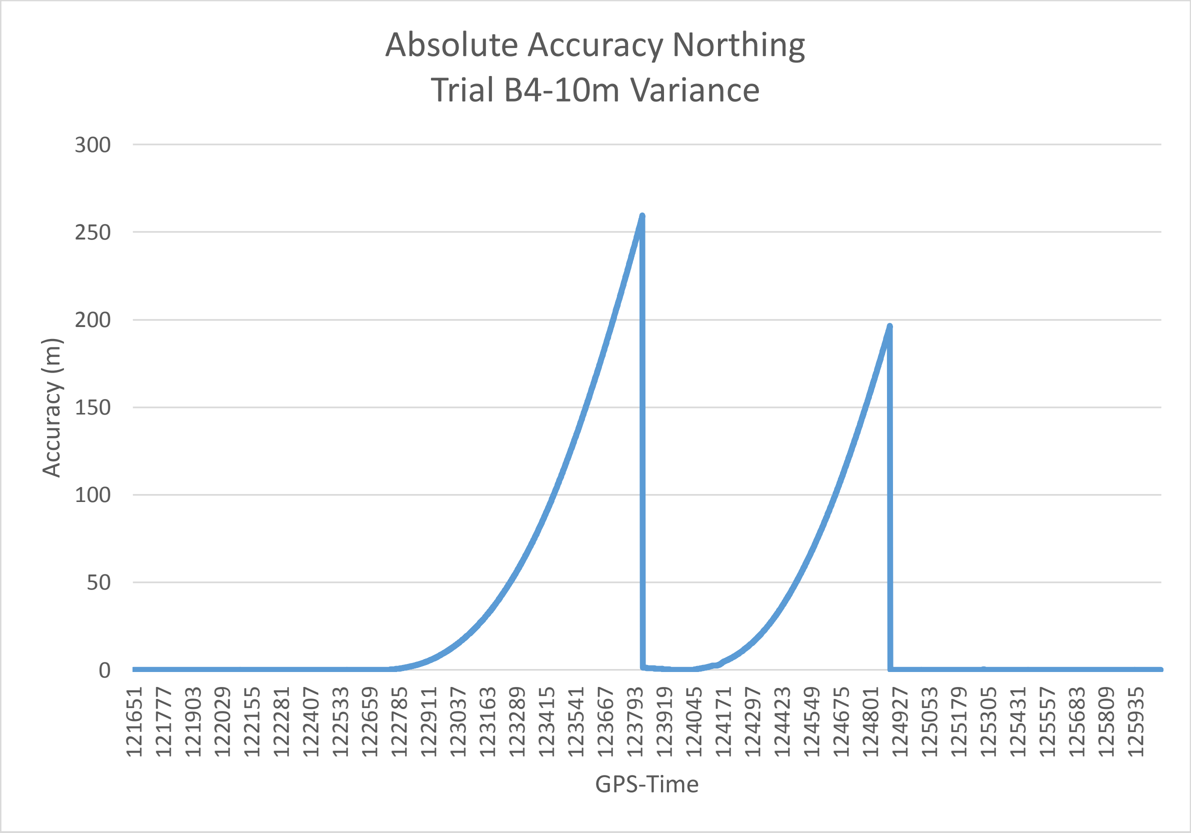
Manipulated Error: 100m West



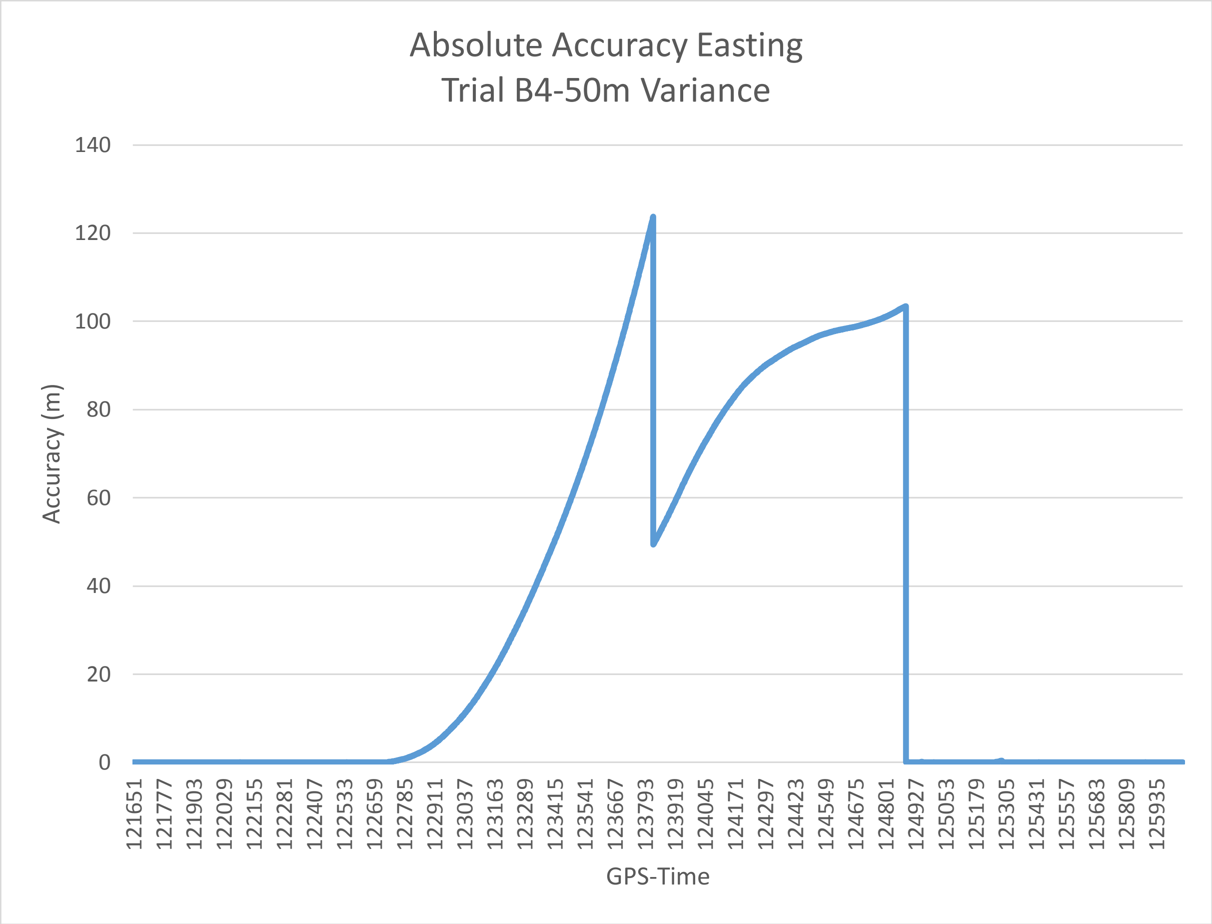




Absolute Accuracy Easting
Trial B4-10m Variance



Absolute Accuracy Northing
Trial B4-10m Variance


Absolute Accuracy Easting
Trial B4-50m Variance
140
120
100
80
60
40
20
0
Accuracy (m)
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
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124171
124297
124423
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125179
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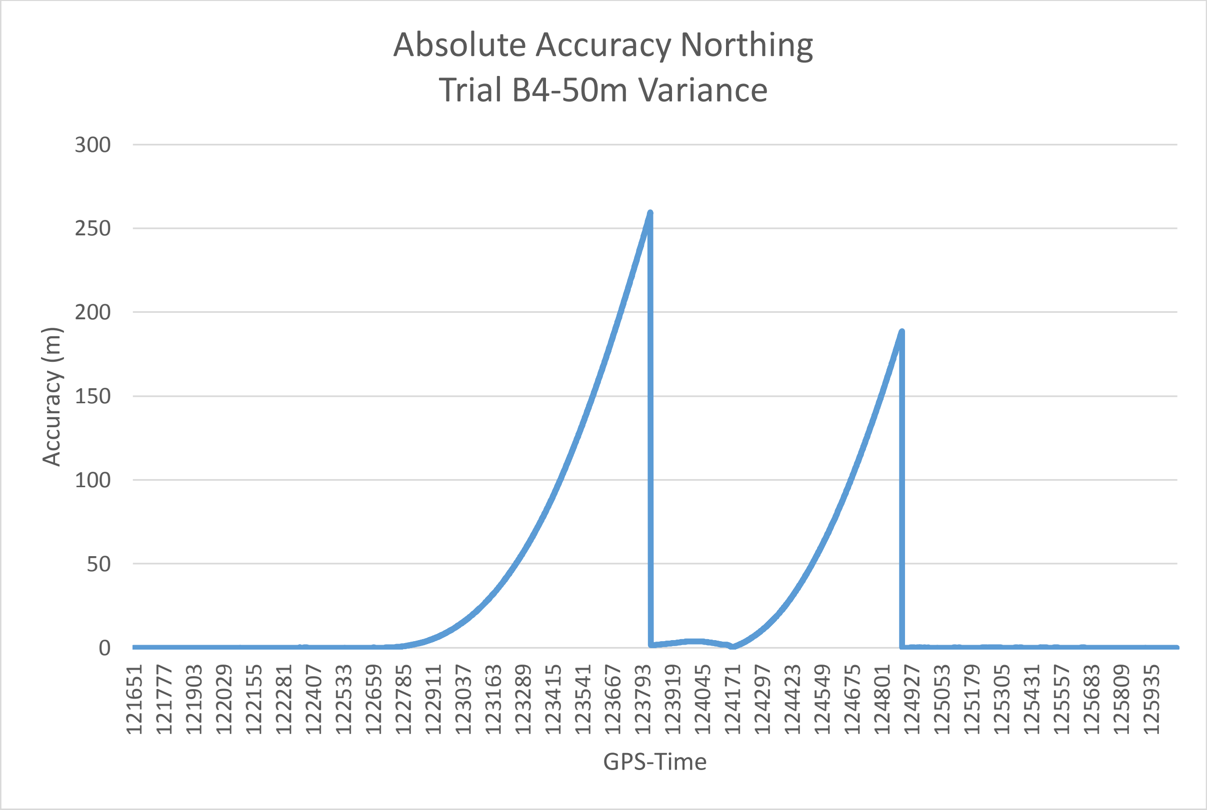
125557
125683
125809
125935
GPS-Time


Absolute Accuracy Northing
Trial B4-50m Variance
300
250
200
150
100
50
0
Accuracy (m)
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
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125053
125179
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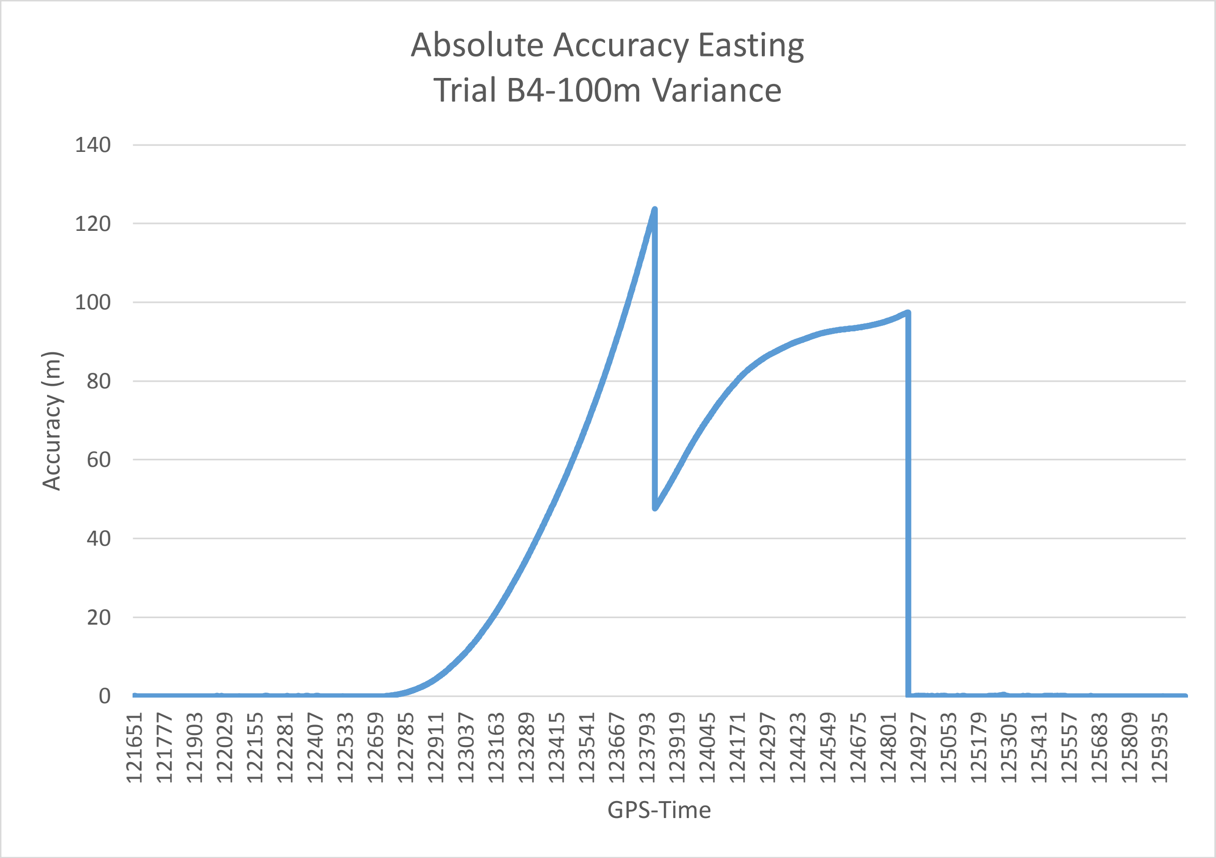
125683
125809
125935
GPS-Time


Absolute Accuracy Easting
Trial B4-100m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
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124045
124171
124297
124423
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124927
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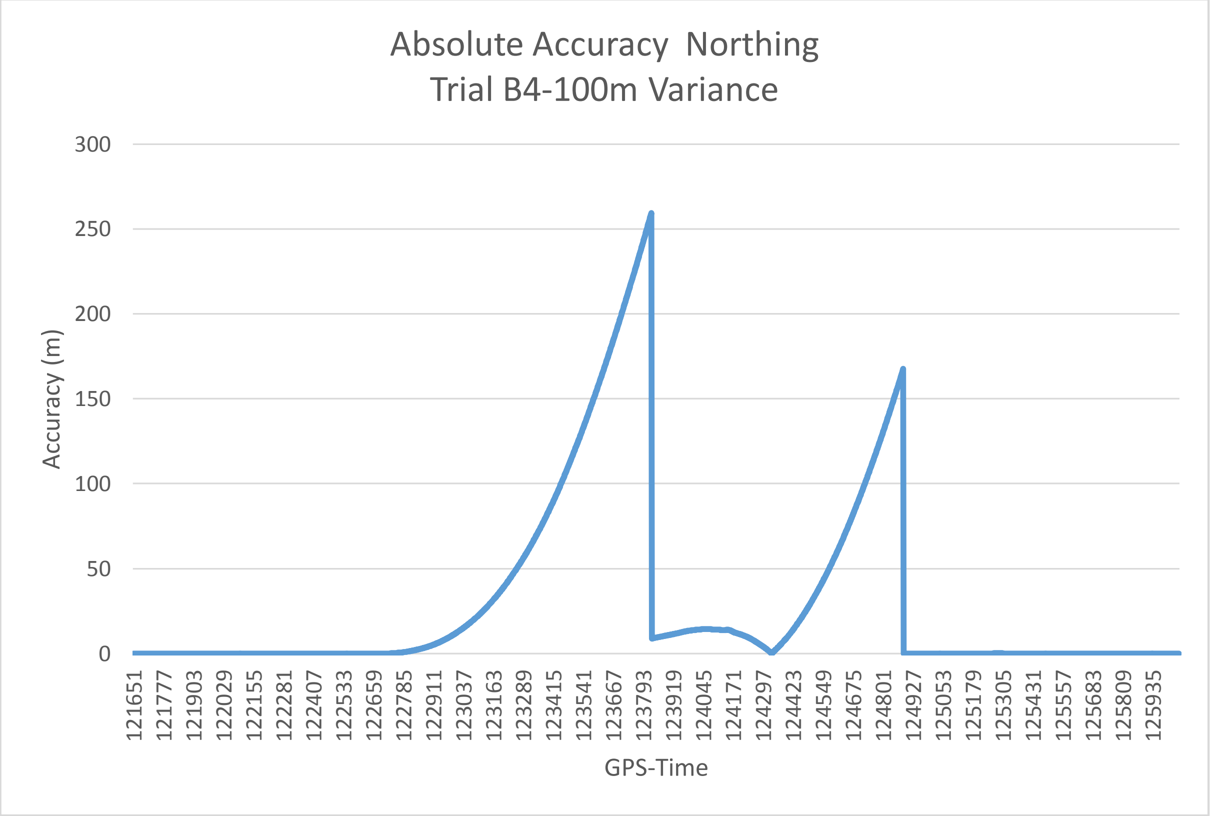
125557
125683
125809
125935
GPS-Time


Absolute Accuracy Northing
Trial B4-100m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
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123667
123793
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124171
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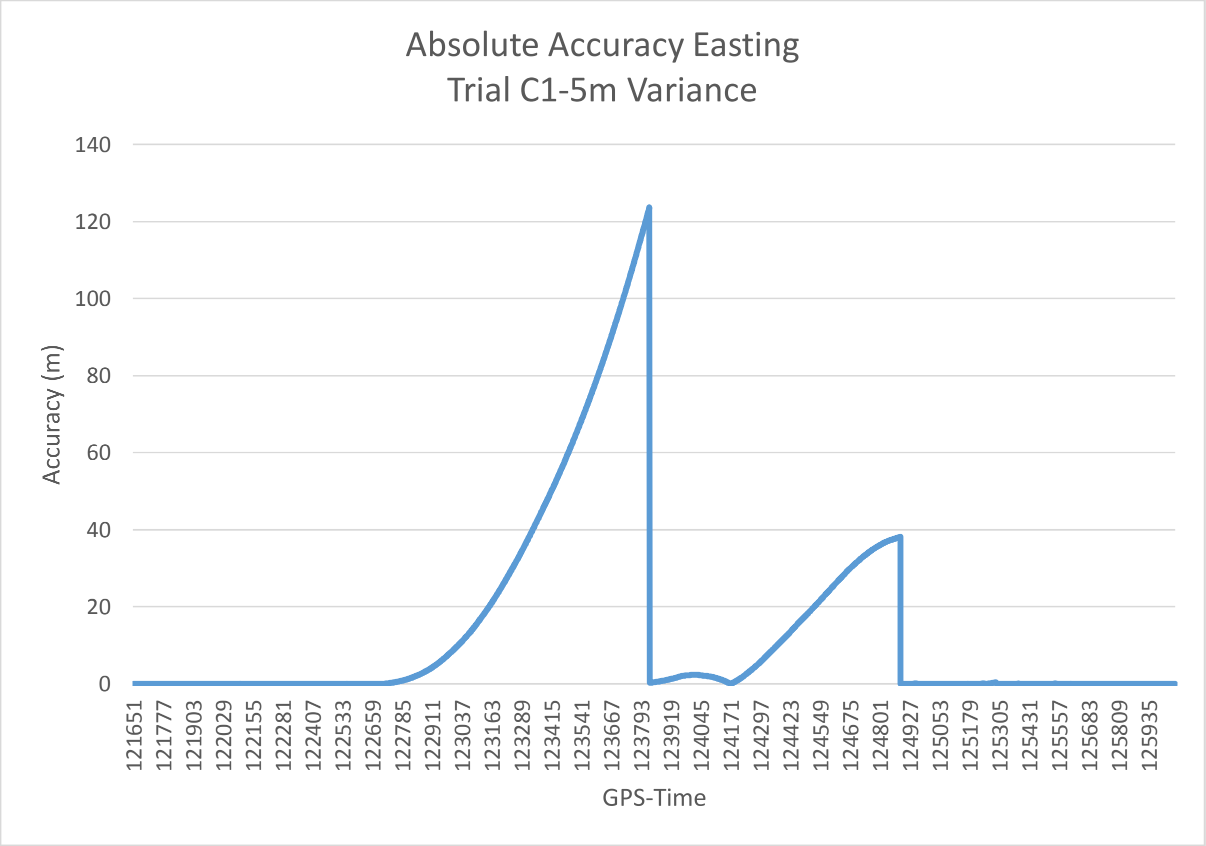
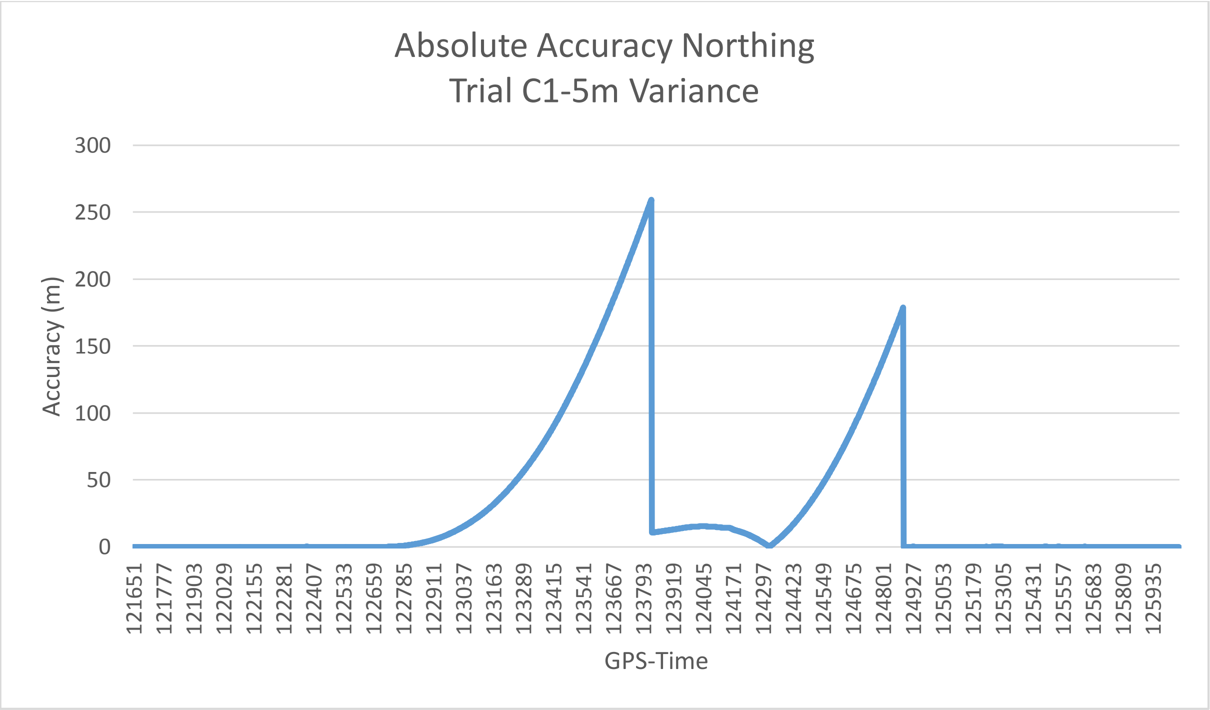
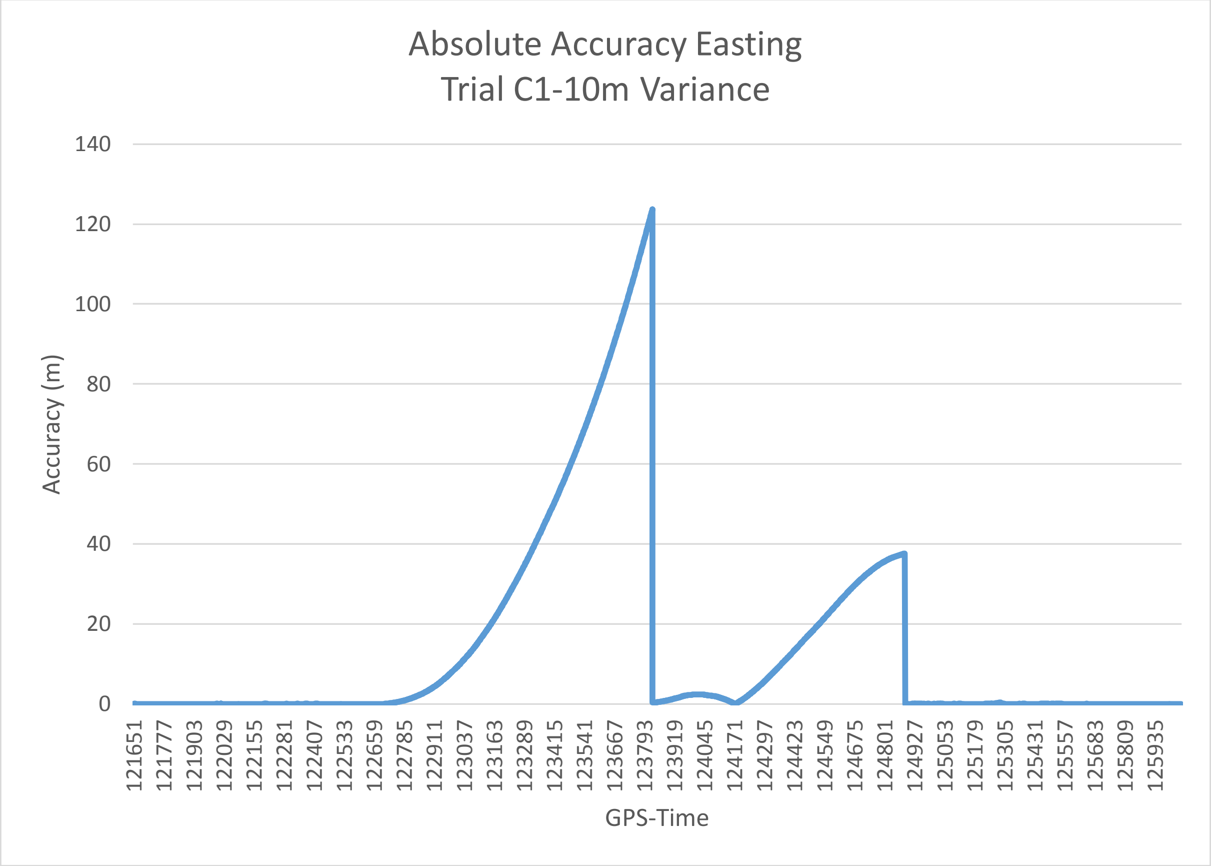
125935
GPS-Time

C10: Trial C1

Manipulated Error: 10 m North






Absolute Accuracy Easting
Trial C1-10m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
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124423
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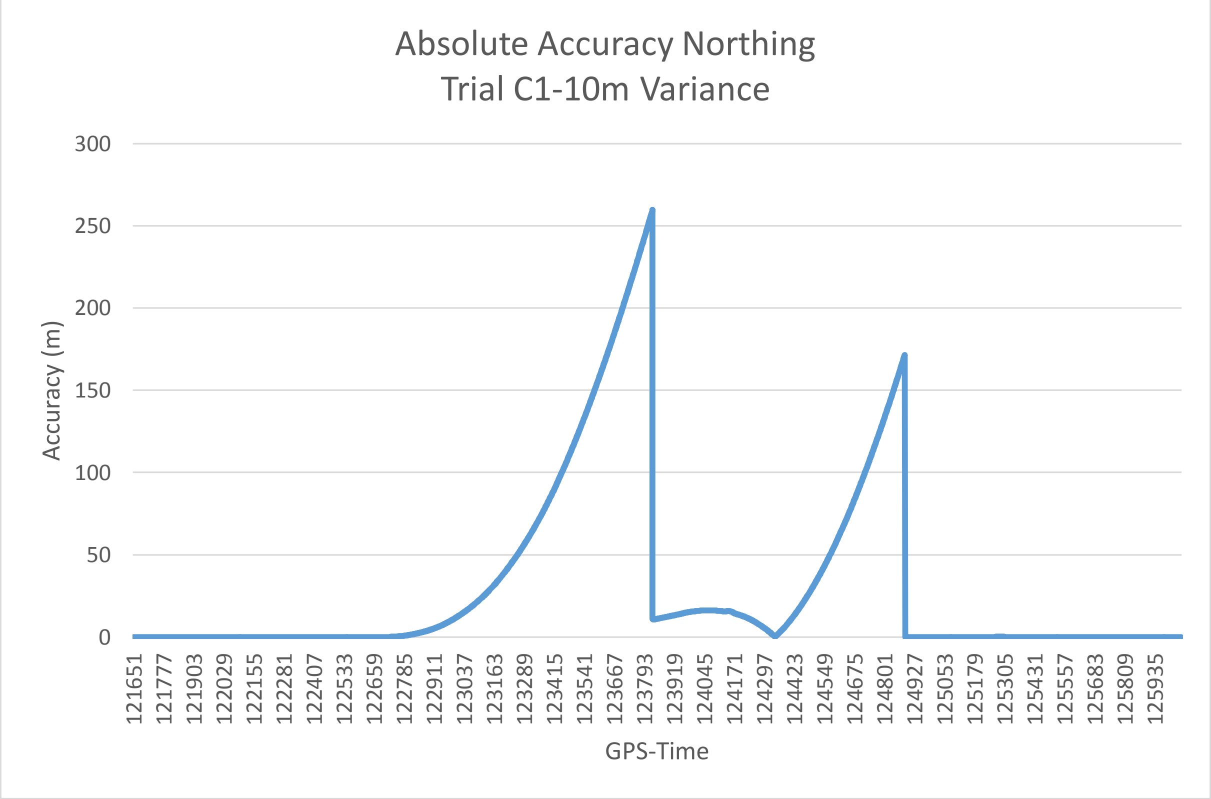
125935
GPS-Time


Absolute Accuracy Northing
Trial C1-10m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
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124423
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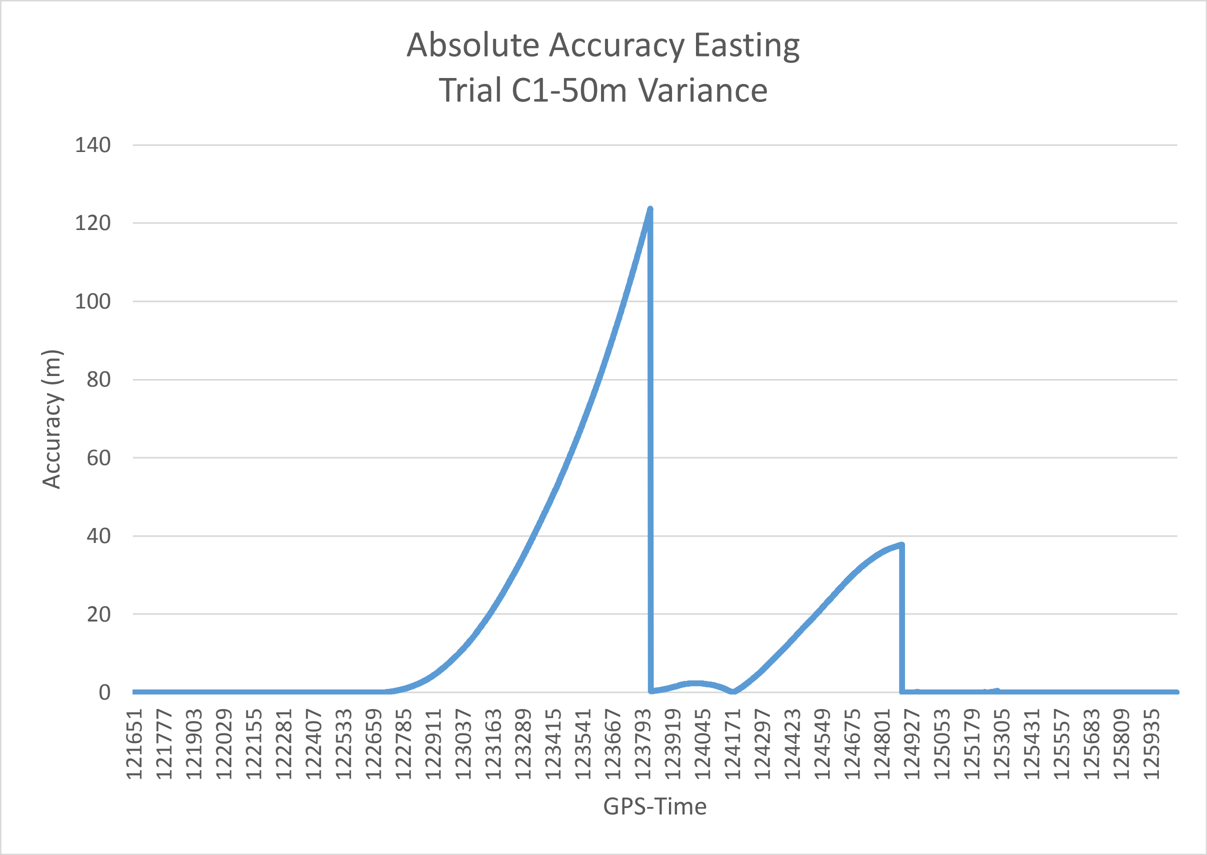
GPS-Time


Absolute Accuracy Easting
Trial C1-50m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
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123163
123289
123415
123541
123667
123793
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124171
124297
124423
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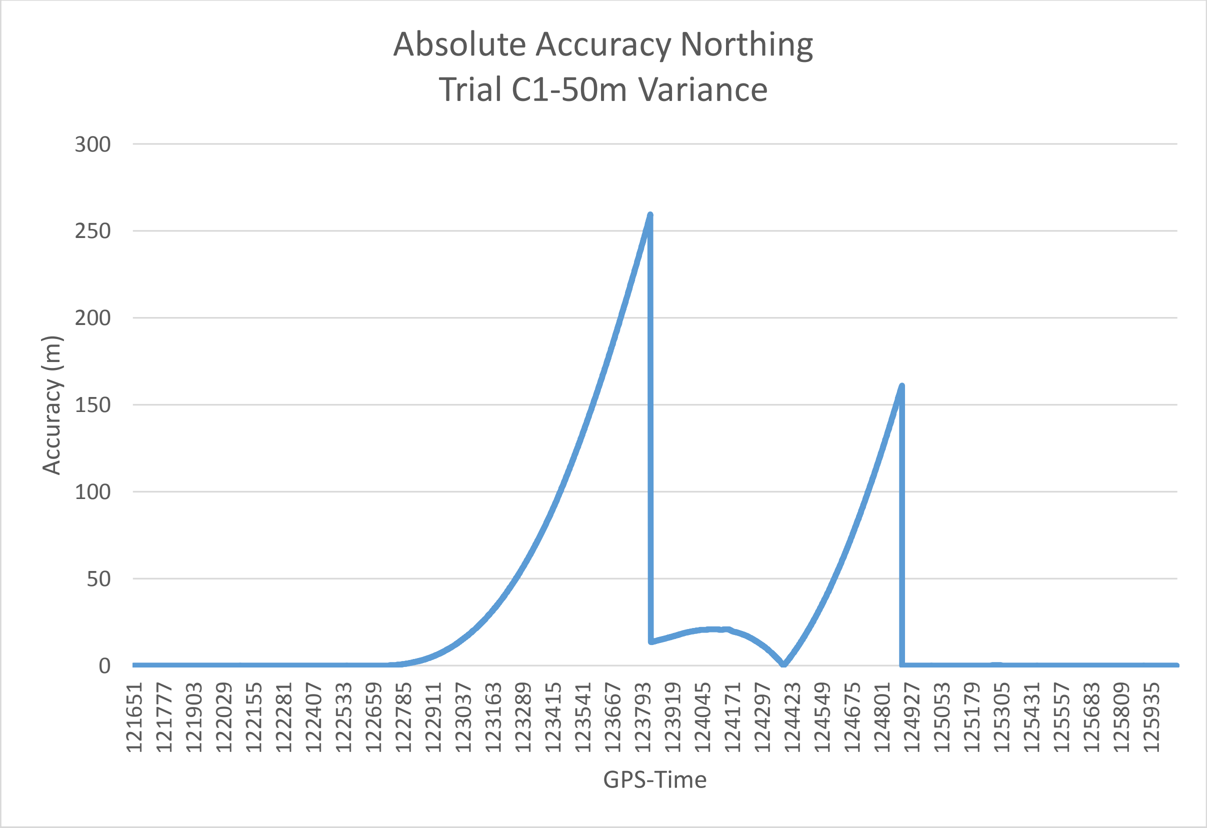
125935
GPS-Time


Absolute Accuracy Northing
Trial C1-50m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
124423
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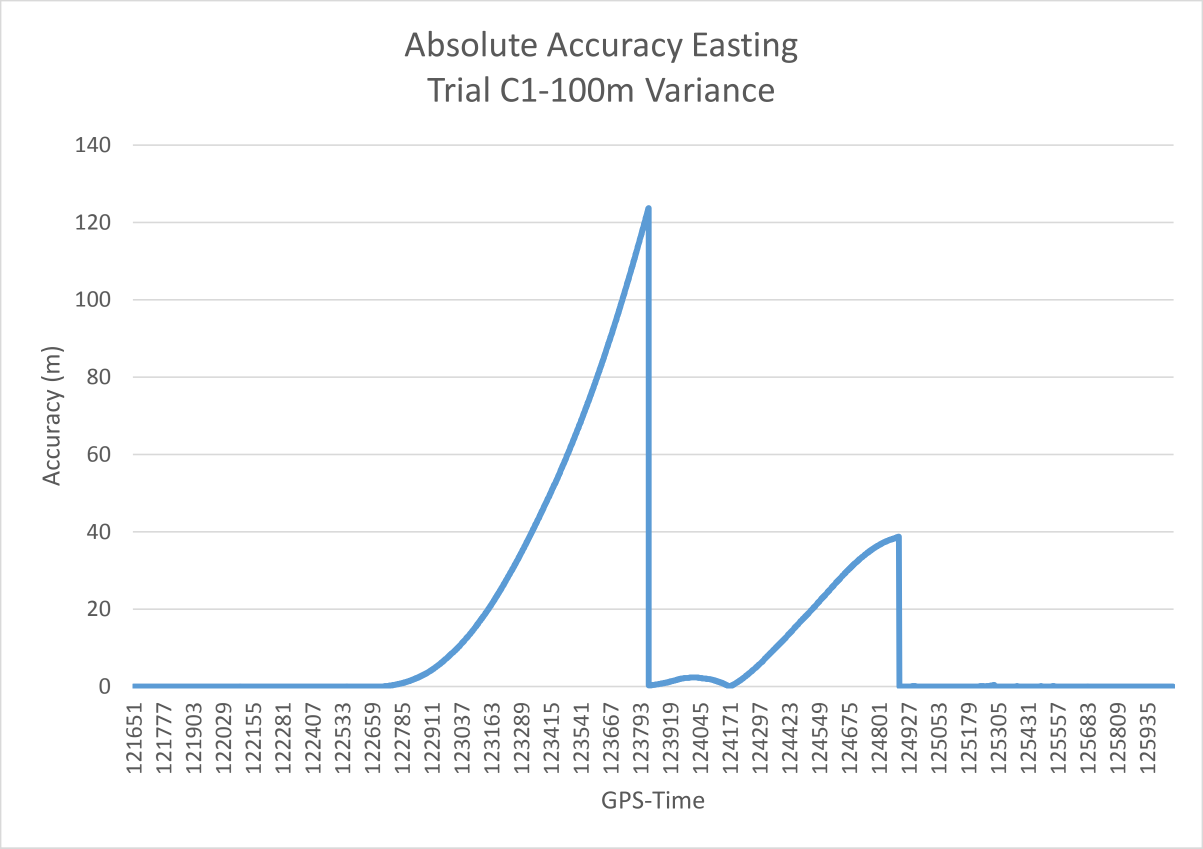
125557
125683
125809
125935
GPS-Time


Absolute Accuracy Easting
Trial C1-100m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
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124171
124297
124423
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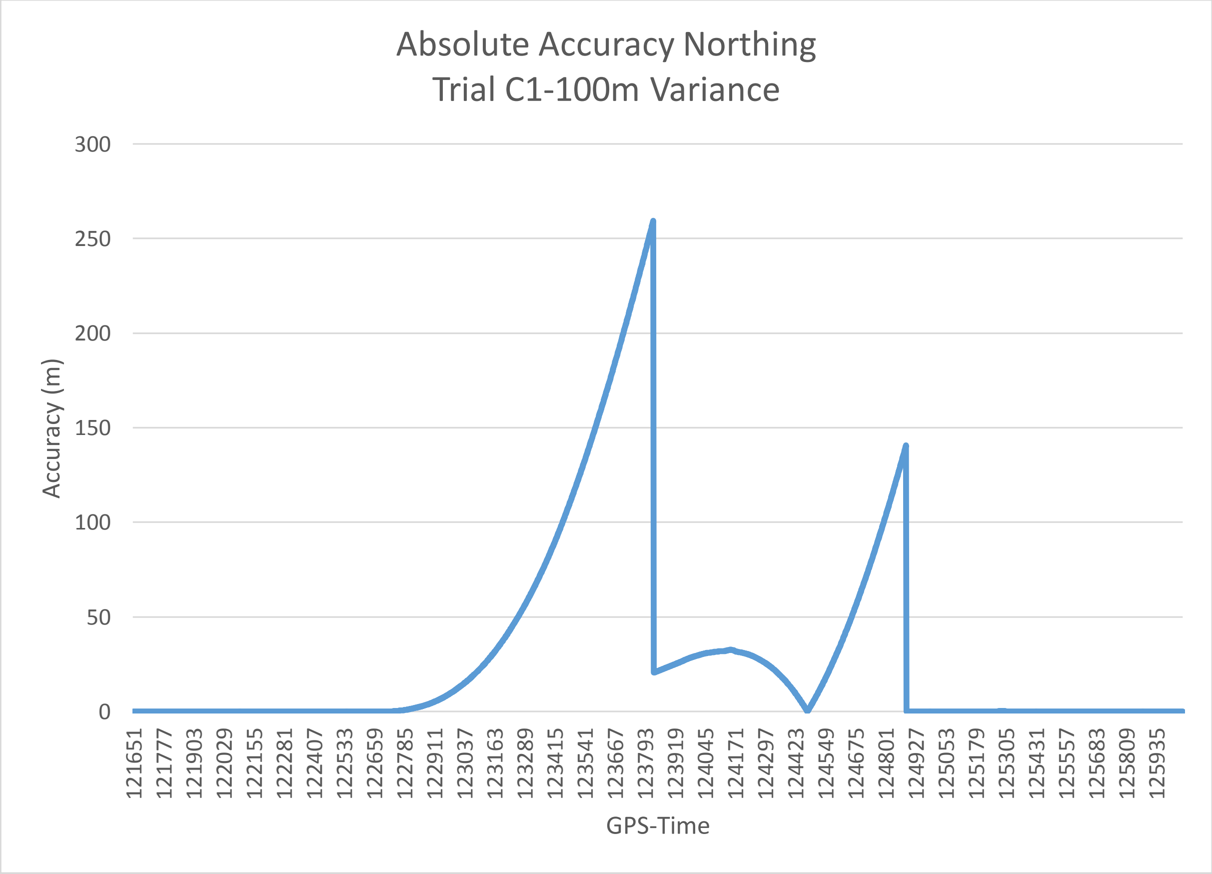
125935
GPS-Time


Absolute Accuracy Northing
Trial C1-100m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
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123415
123541
123667
123793
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124549
124675
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124927
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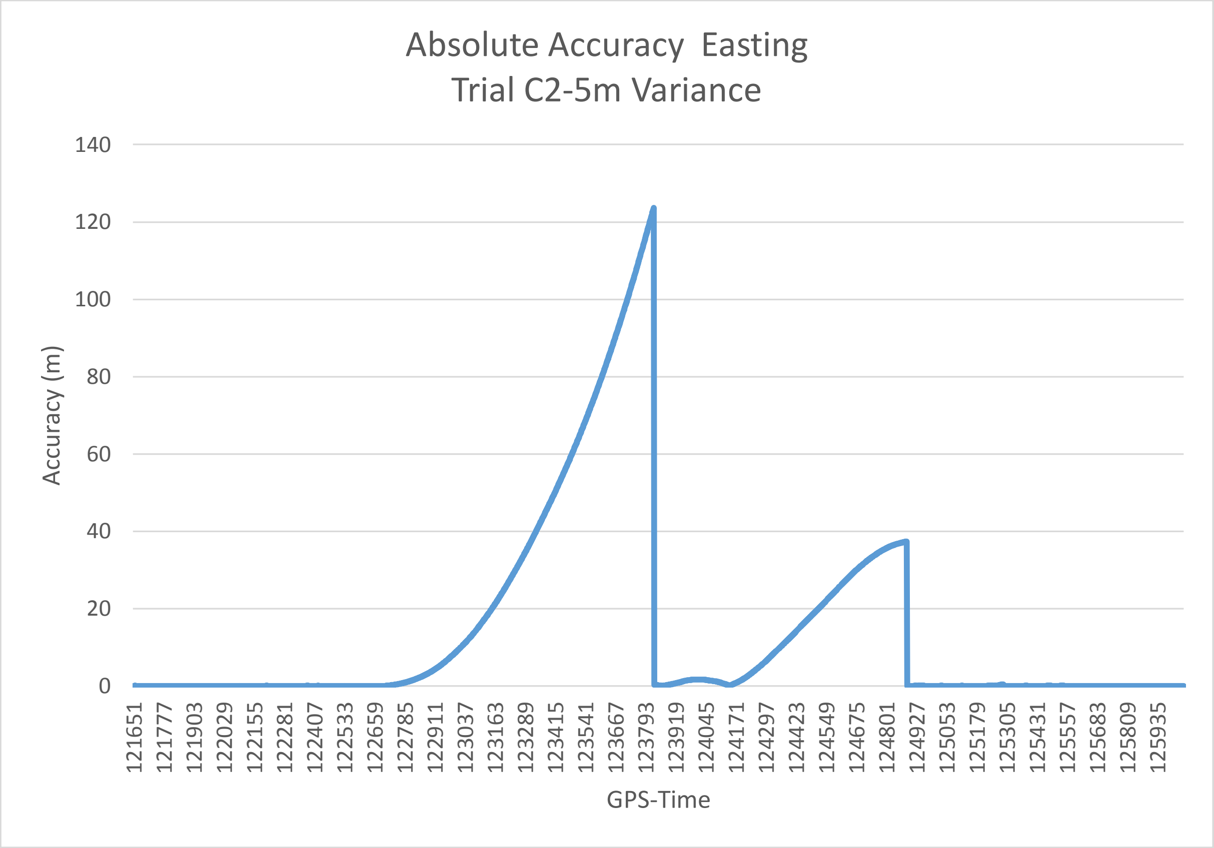
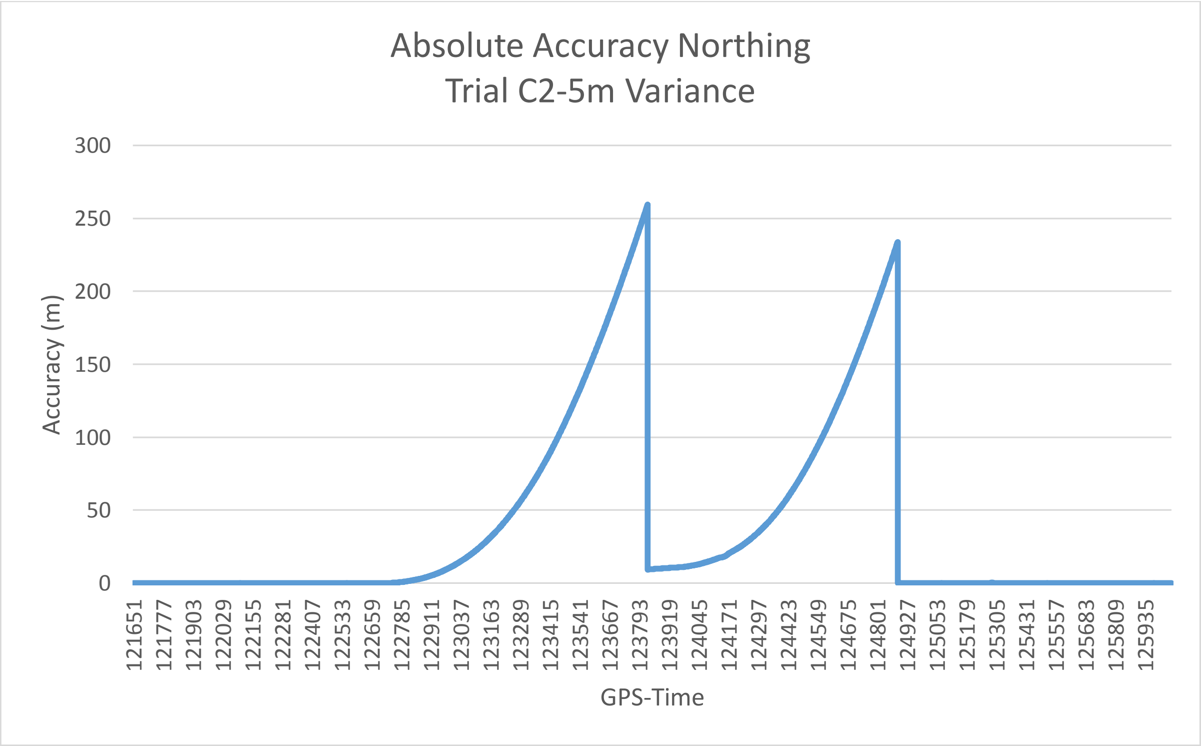
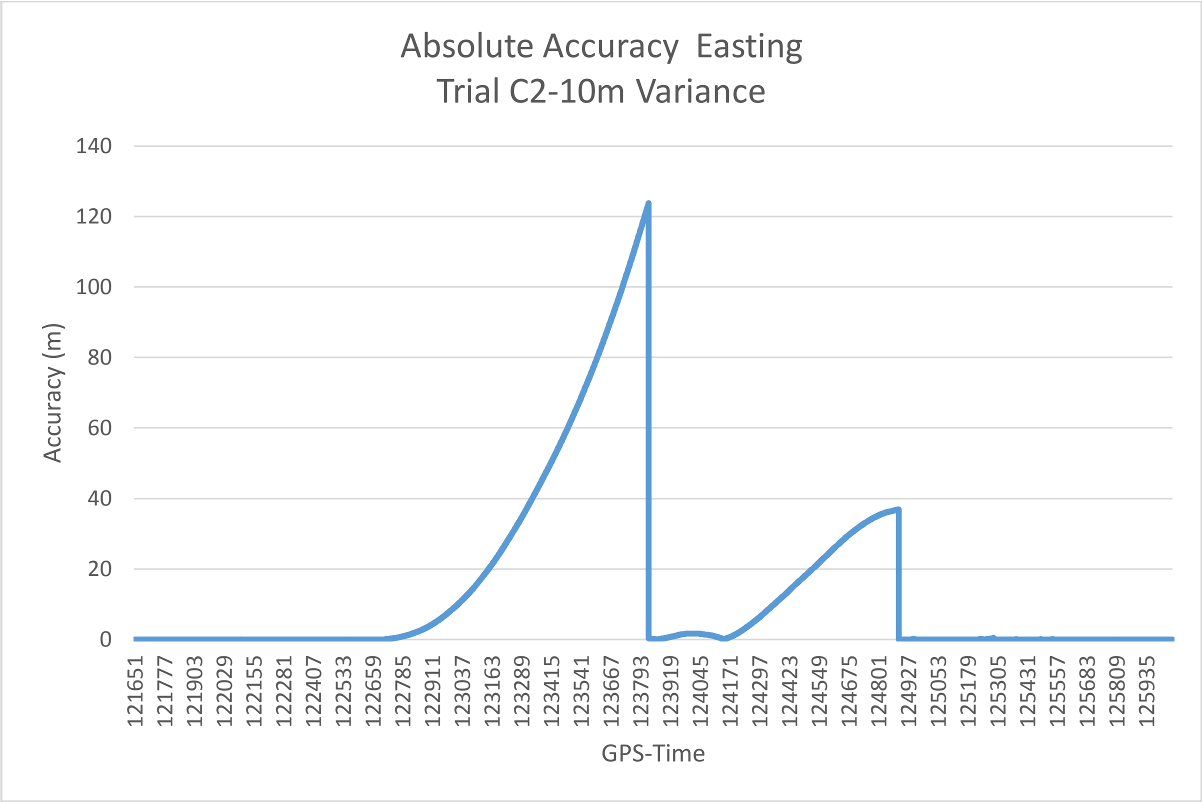
125557
125683
125809
125935
GPS-Time

C11: Trial C2

Manipulated Error: 10 m South






Absolute Accuracy Easting
Trial C2-10m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
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123541
123667
123793
123919
124045
124171
124297
124423
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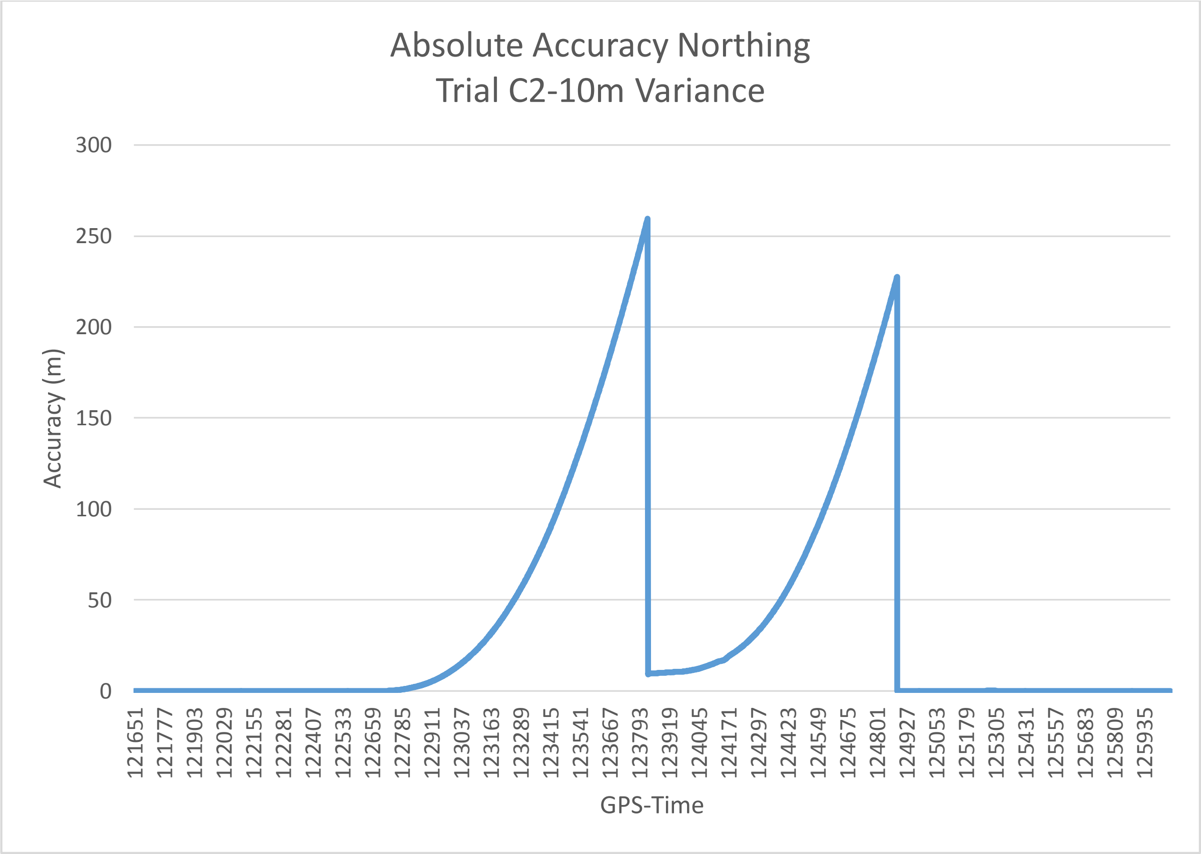
125683
125809
125935
GPS-Time


Absolute Accuracy Northing
Trial C2-10m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
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124171
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124423
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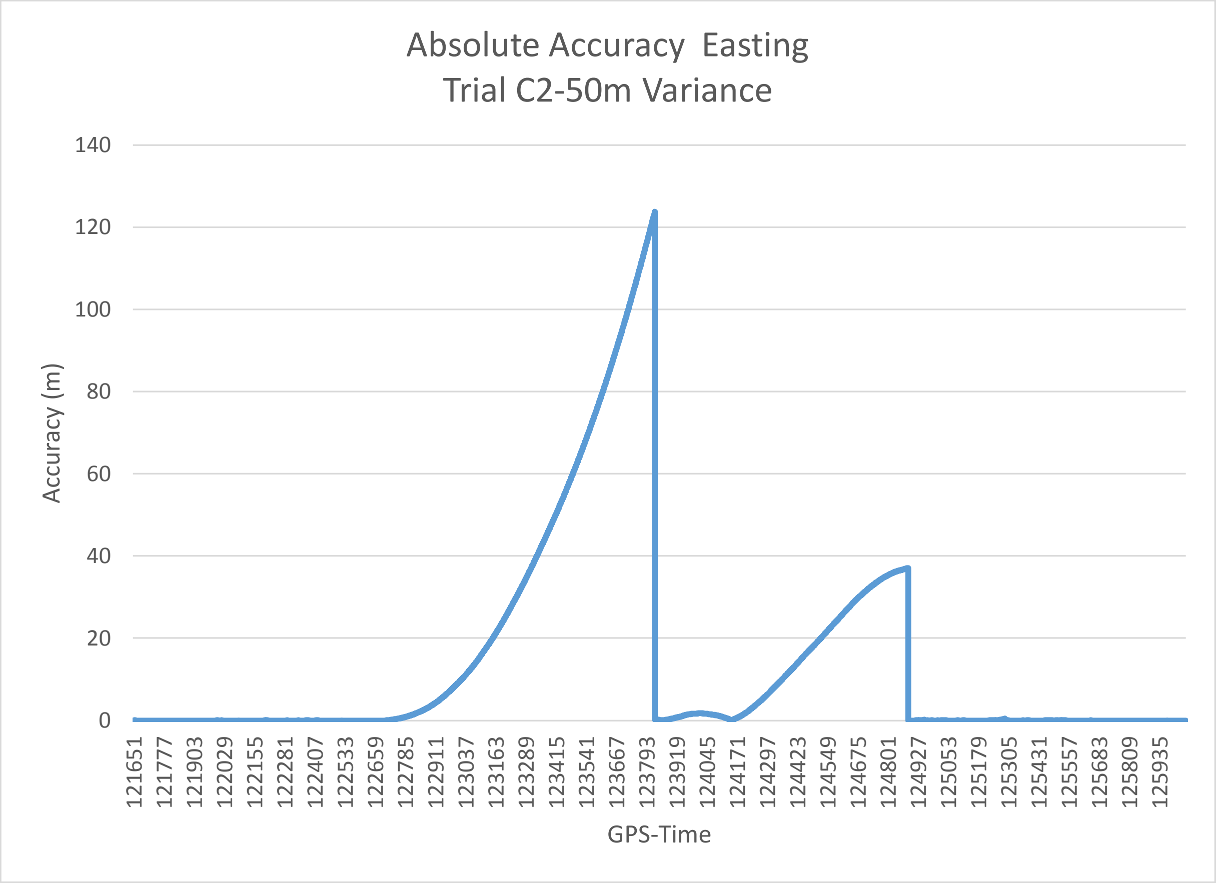
GPS-Time


Absolute Accuracy Easting
Trial C2-50m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
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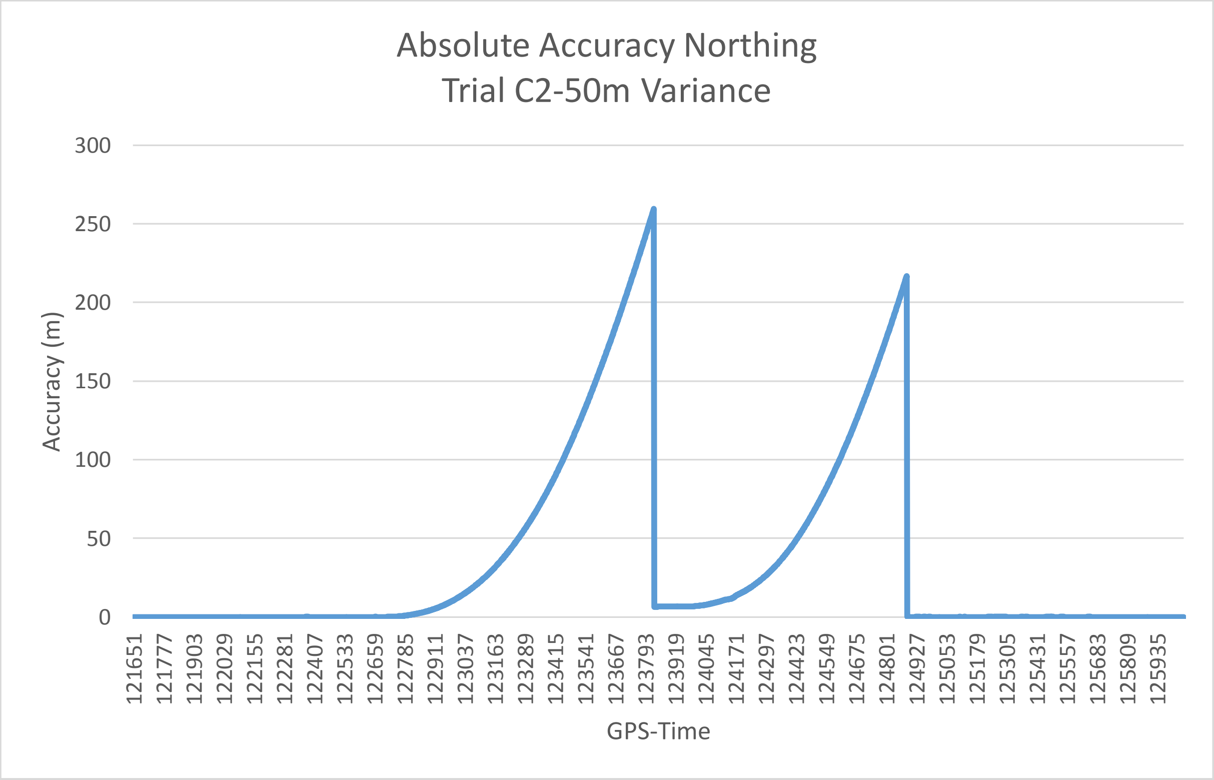
125935
GPS-Time


Absolute Accuracy Northing
Trial C2-50m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
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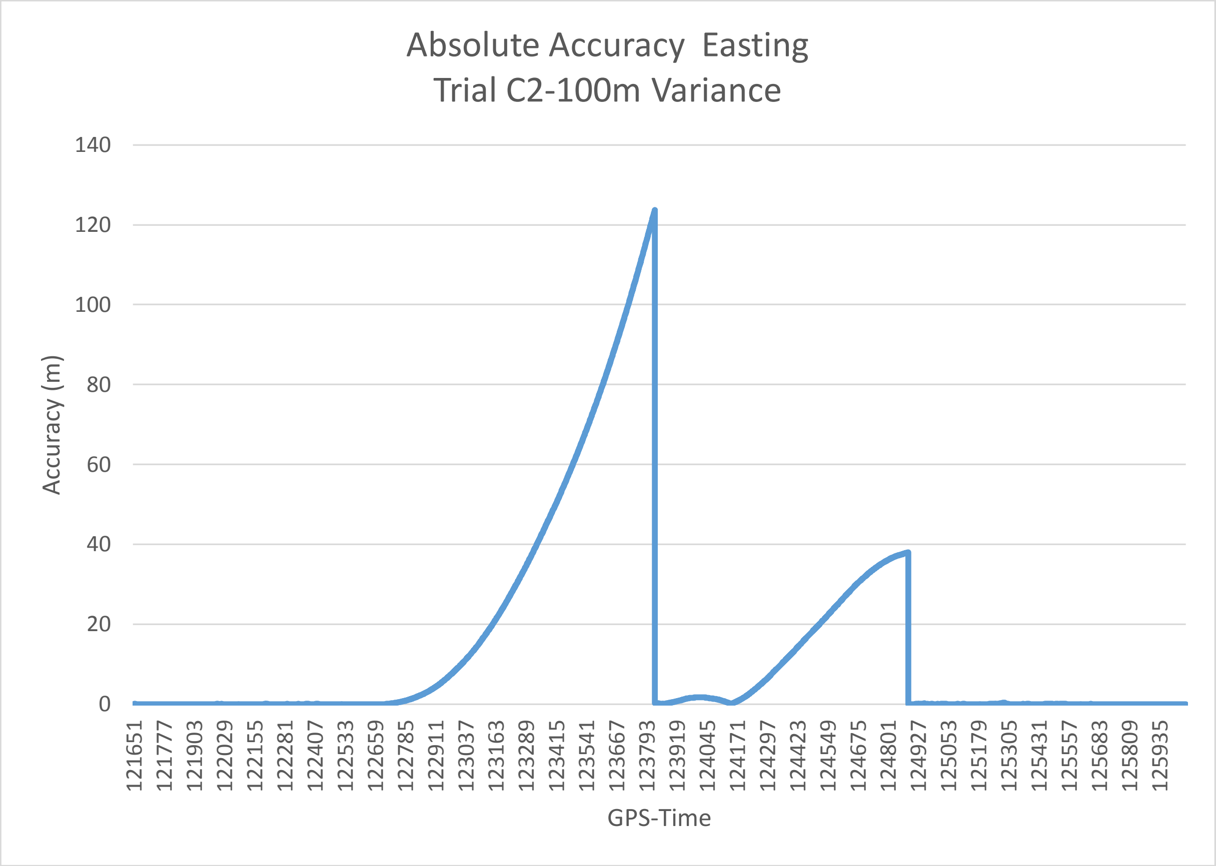
GPS-Time


Absolute Accuracy Easting
Trial C2-100m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
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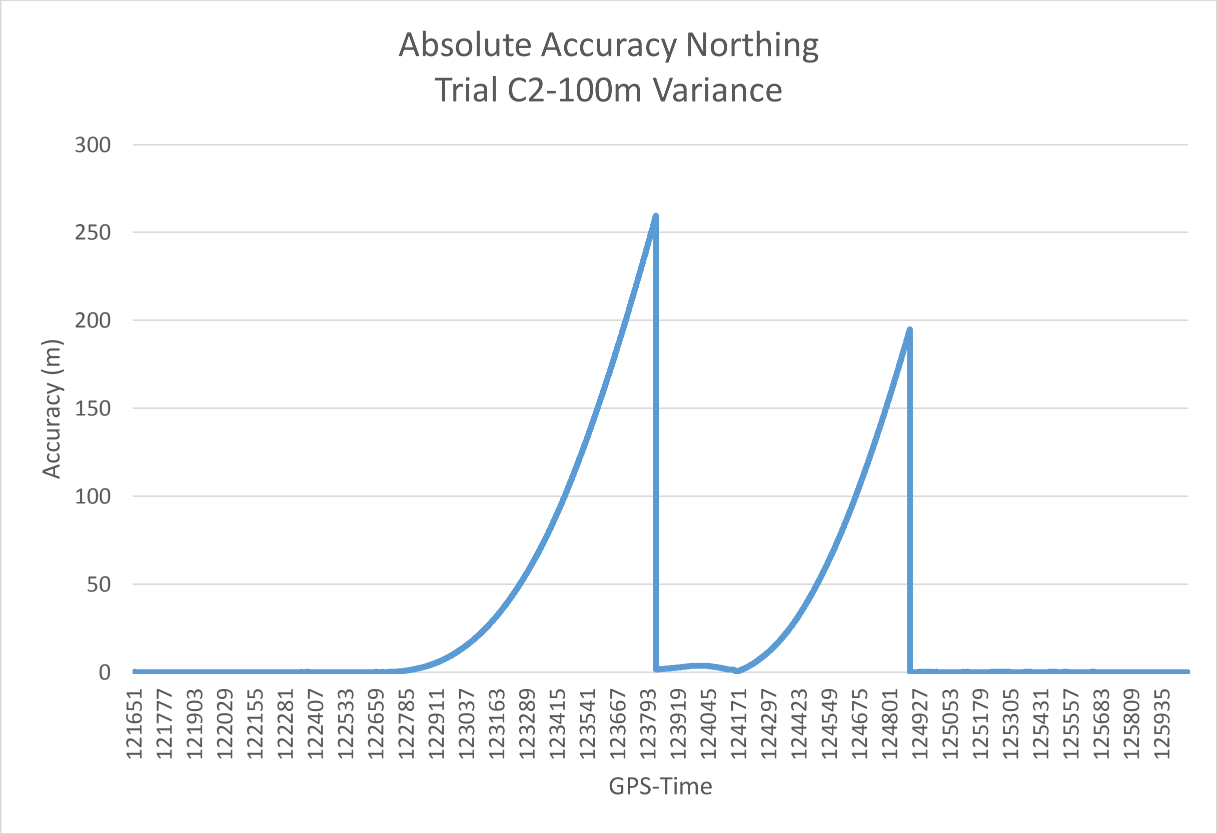
125935
GPS-Time


Absolute Accuracy Northing
Trial C2-100m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
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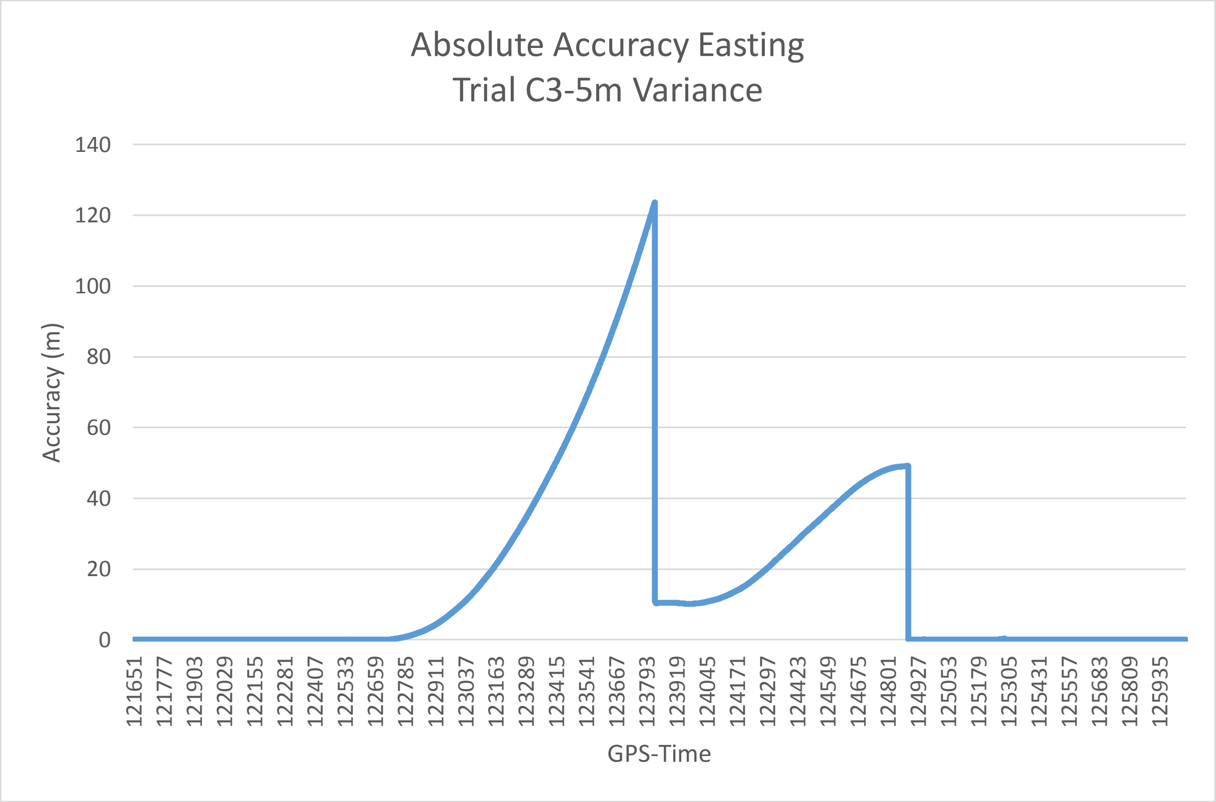
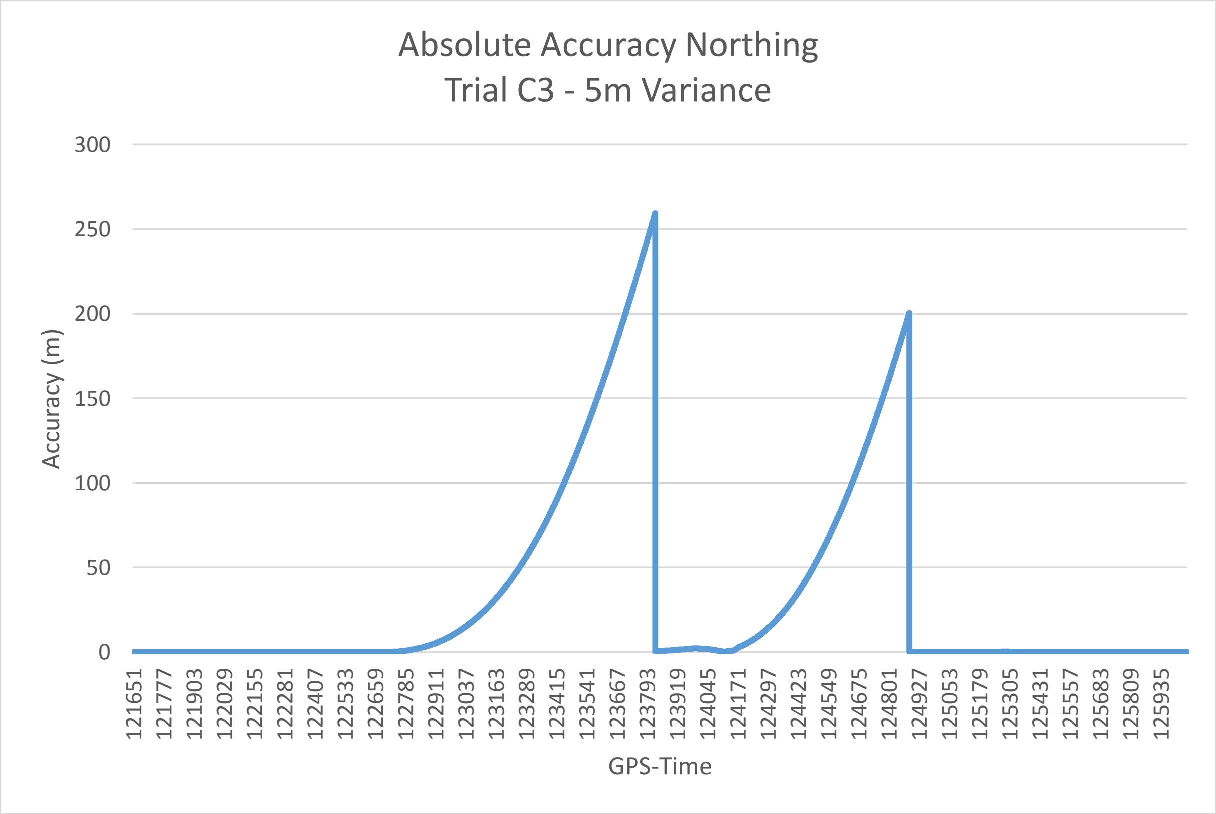
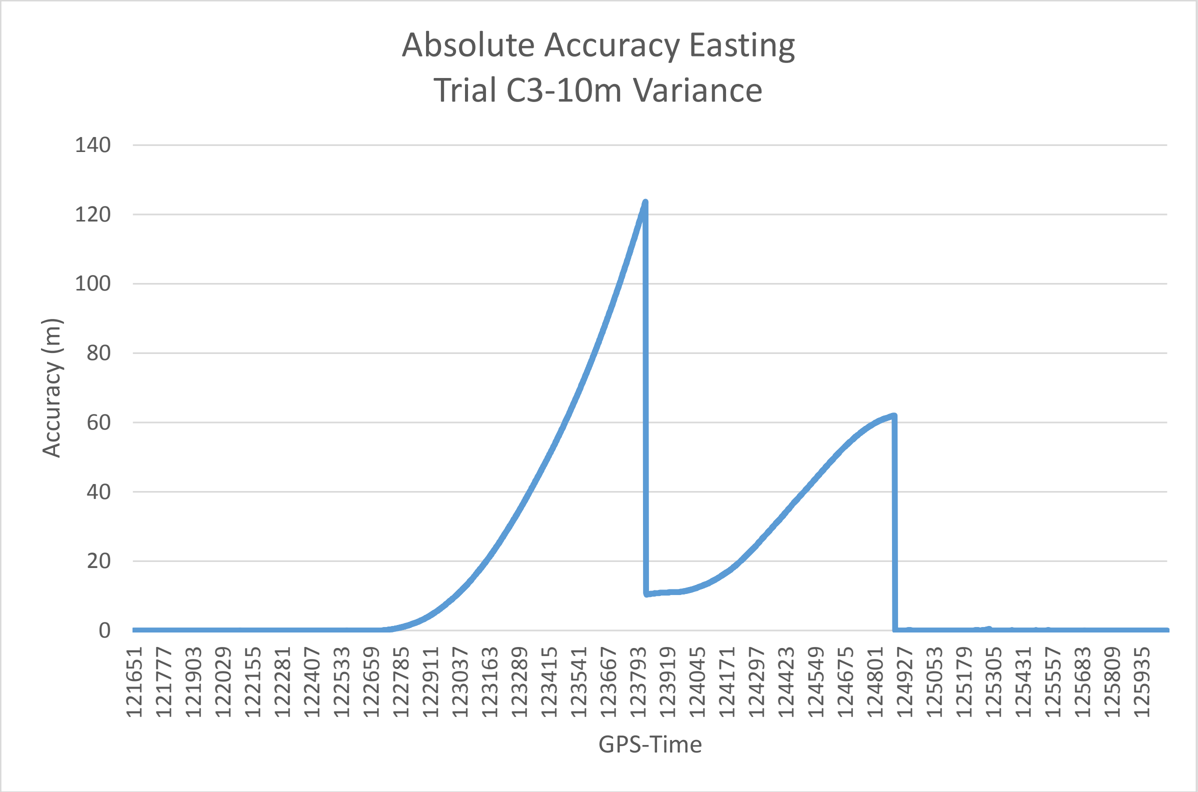
125683
125809
125935
GPS-Time

C12: Trial C3

Manipulated Error: 10 m East






Absolute Accuracy Easting
Trial C3-10m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
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123163
123289
123415
123541
123667
123793
123919
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124171
124297
124423
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124927
125053
125179
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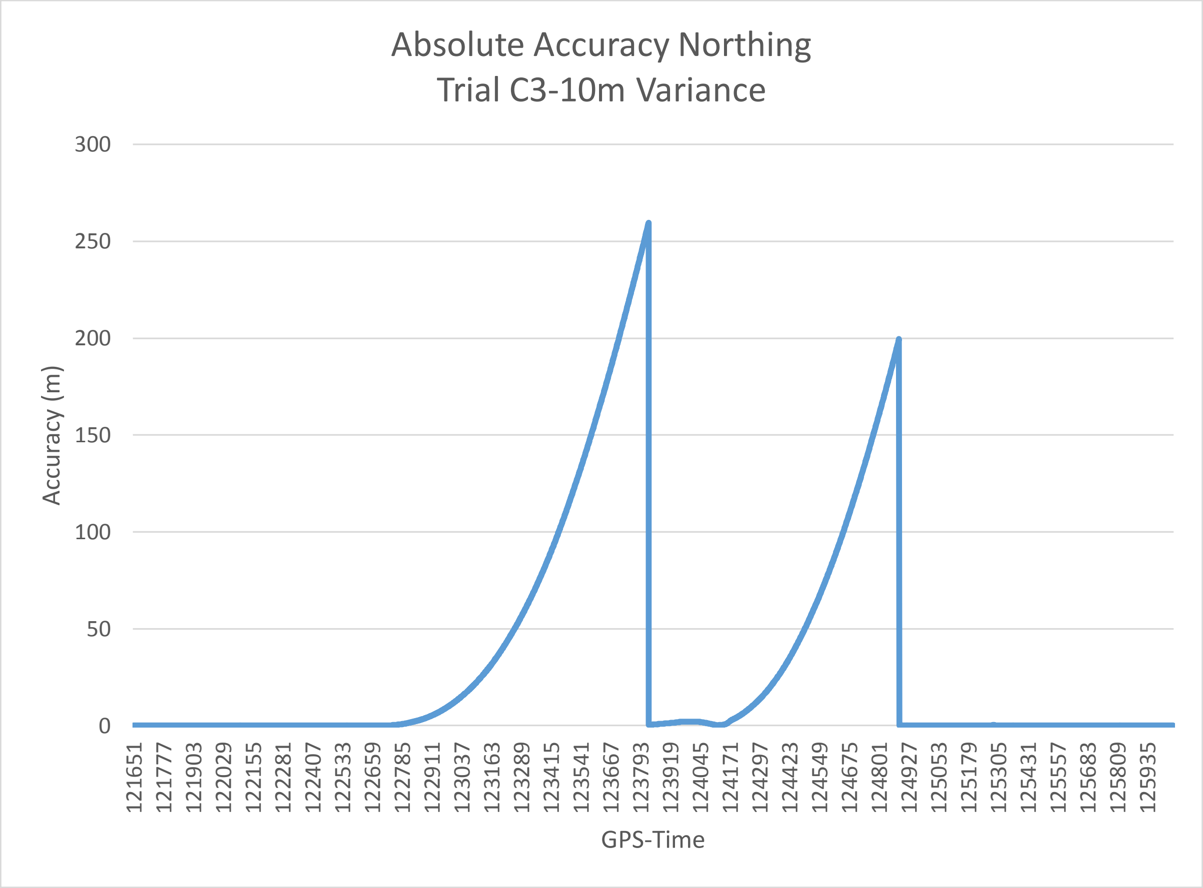
125557
125683
125809
125935
GPS-Time


Absolute Accuracy Northing
Trial C3-10m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
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122785
122911
123037
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123415
123541
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124423
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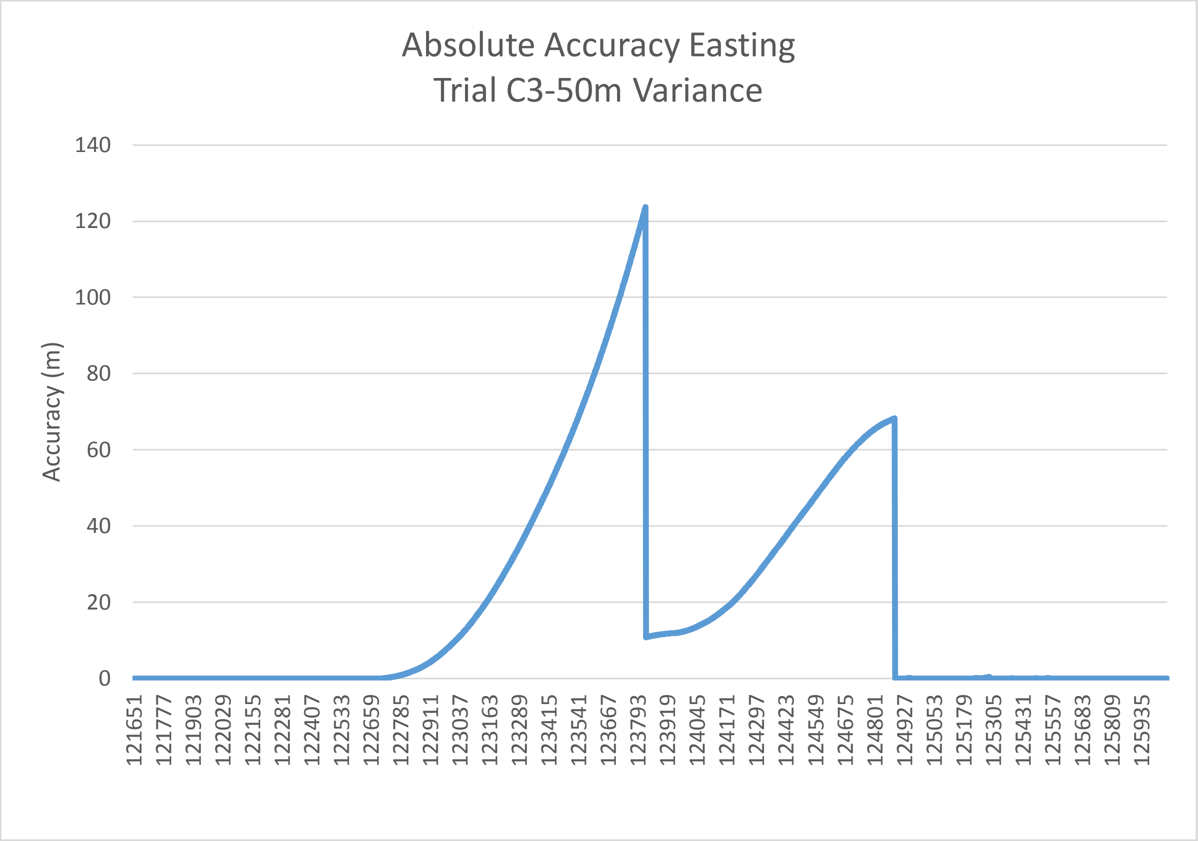
GPS-Time


Absolute Accuracy Easting
Trial C3-50m Variance
Accuracy (m)
0
20
40
60
80
100
120
140
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
124045
124171
124297
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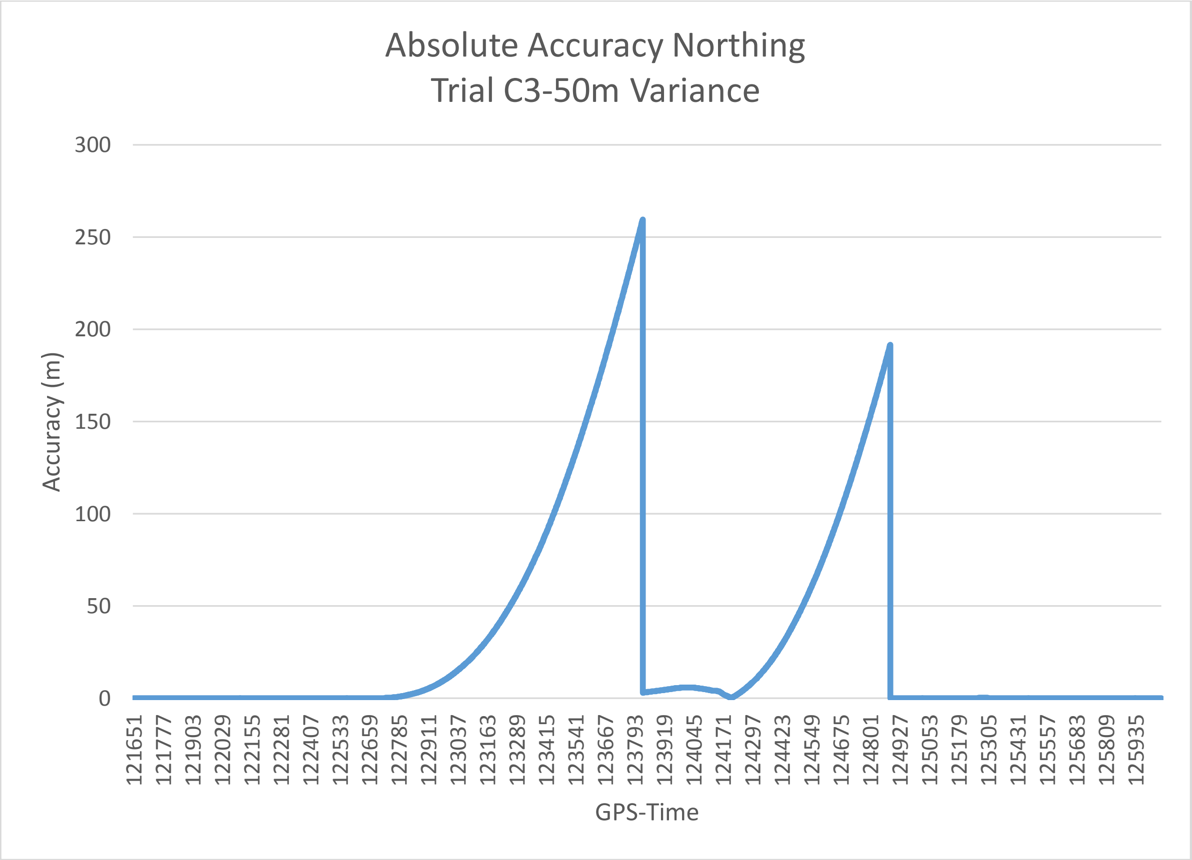
Absolute Accuracy Northing
Trial C3-50m Variance
Accuracy (m)
0
50
100
150
200
250
300
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
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123667
123793
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124297
124423
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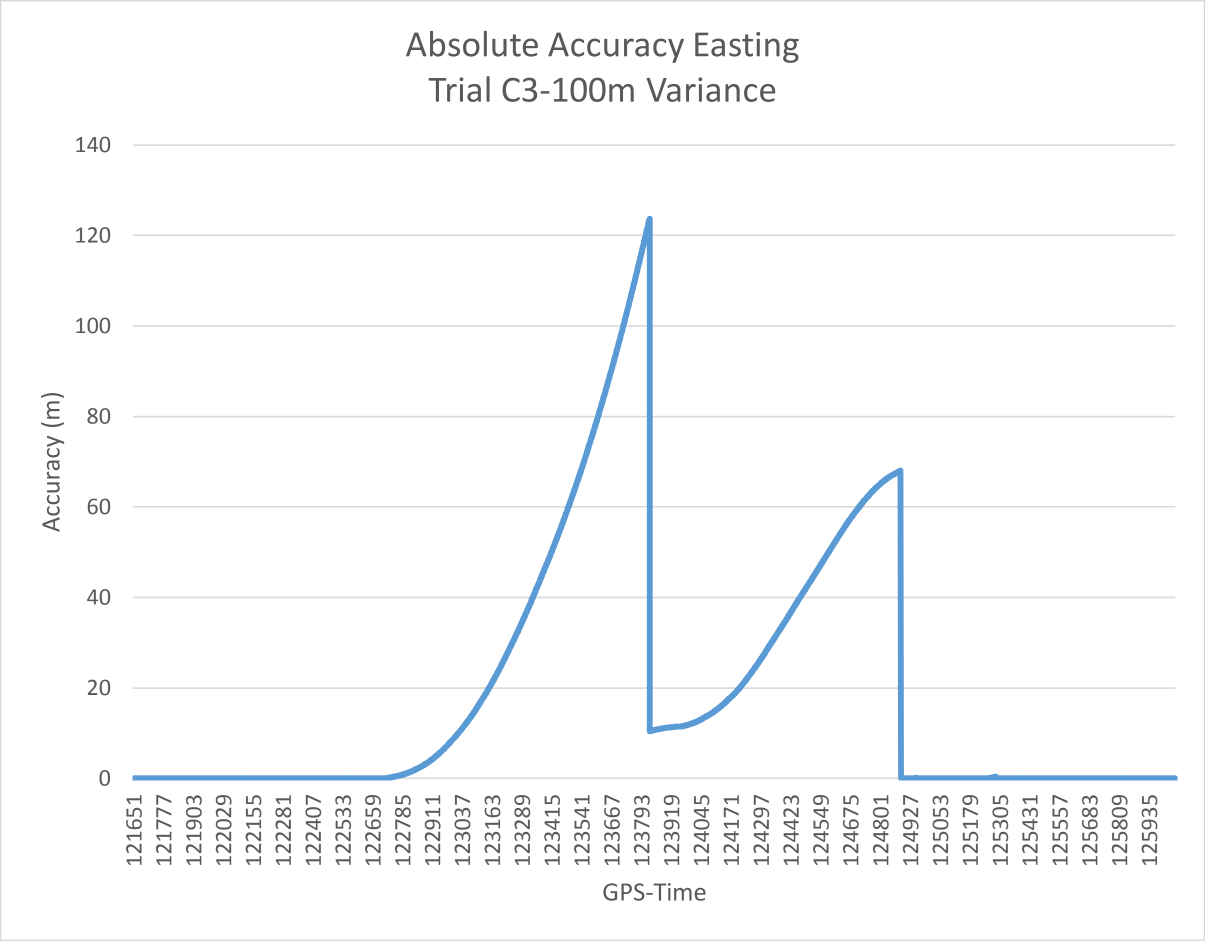
GPS-Time


Absolute Accuracy Easting
Trial C3-100m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
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123667
123793
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124171
124297
124423
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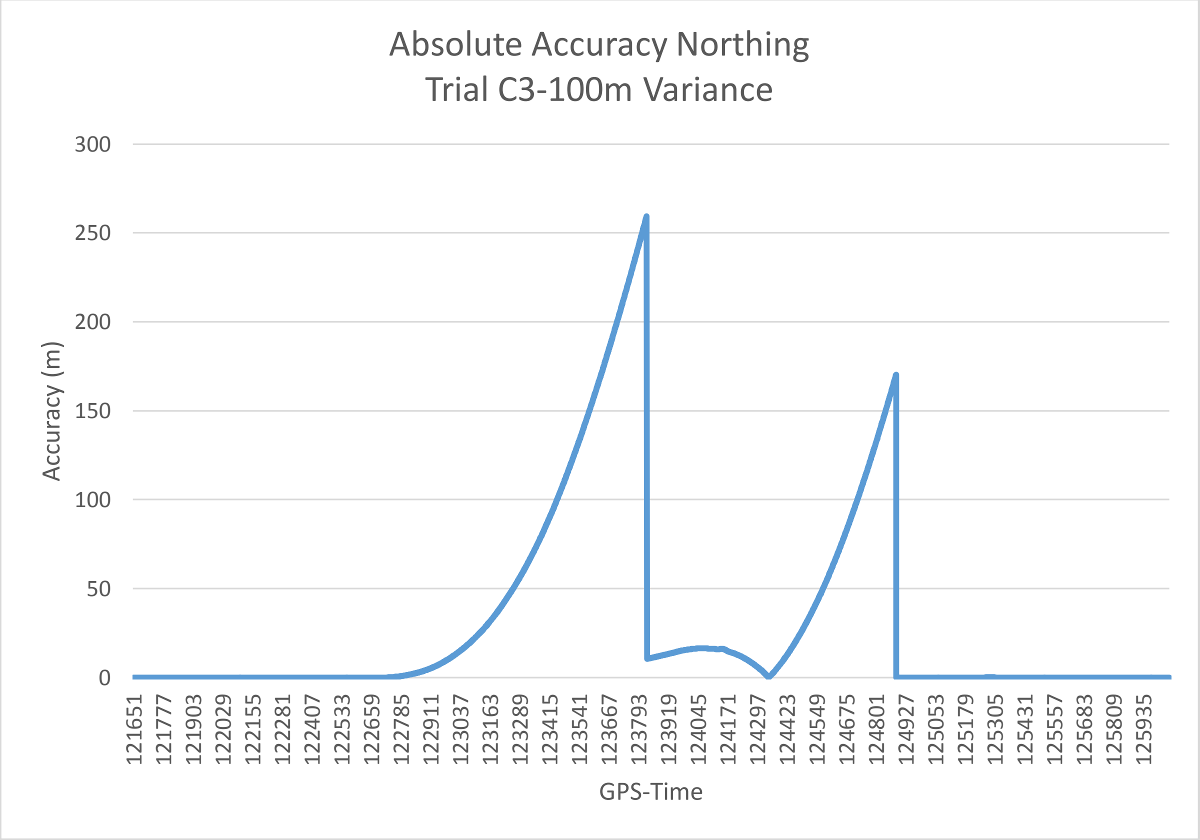
125683
125809
125935
GPS-Time


Absolute Accuracy Northing
Trial C3-100m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
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124297
124423
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124675
124801
124927
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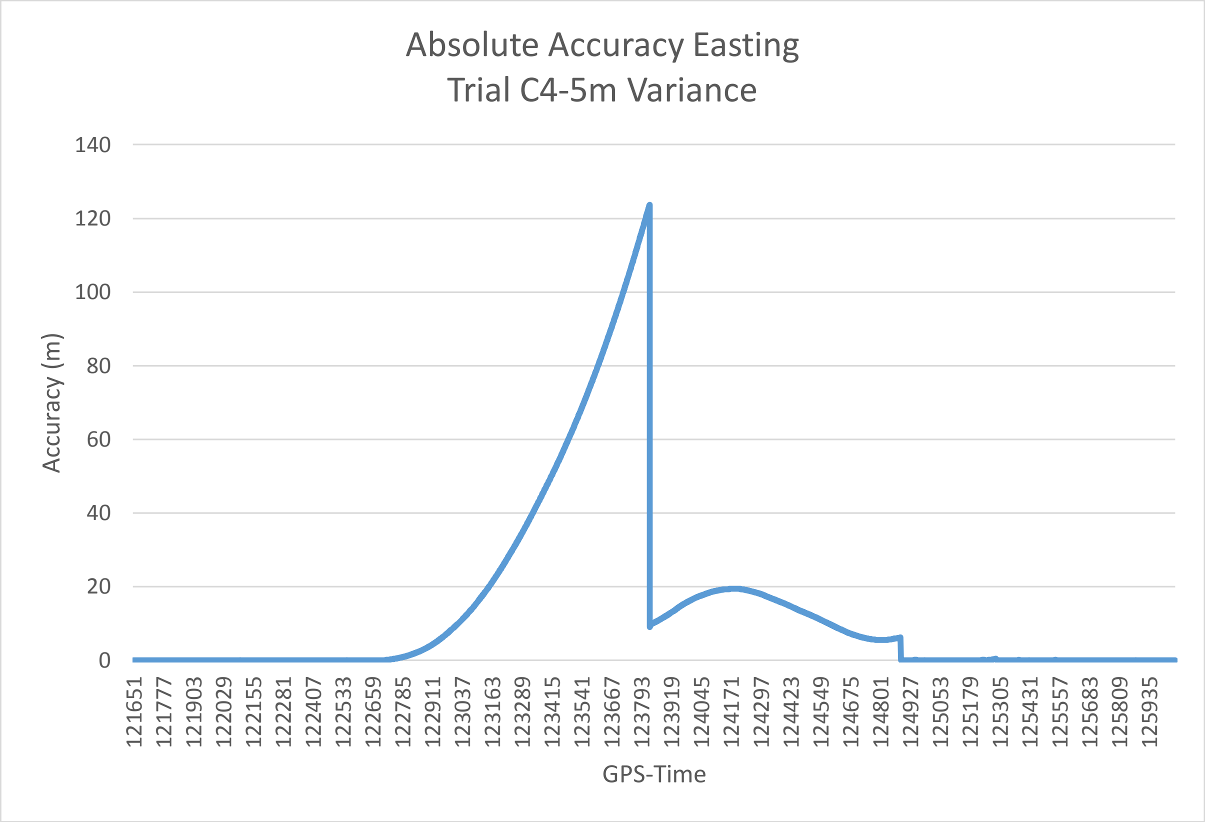
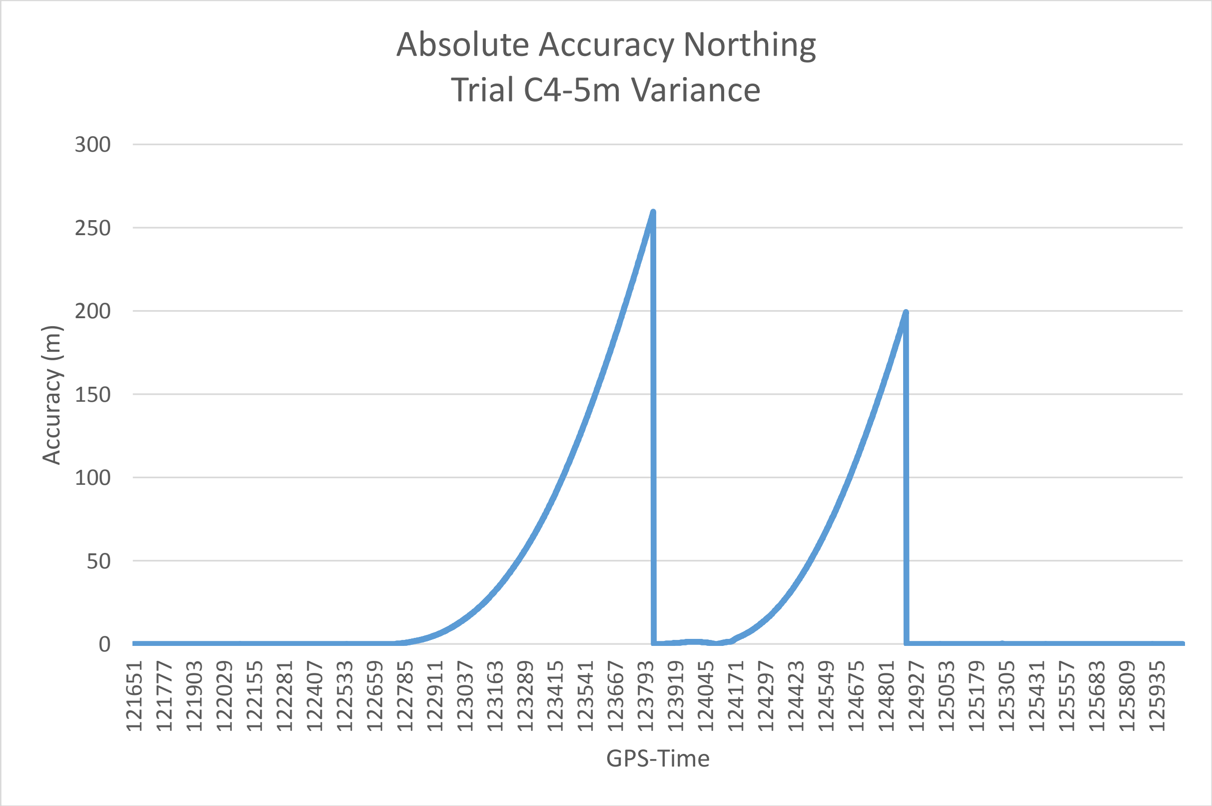
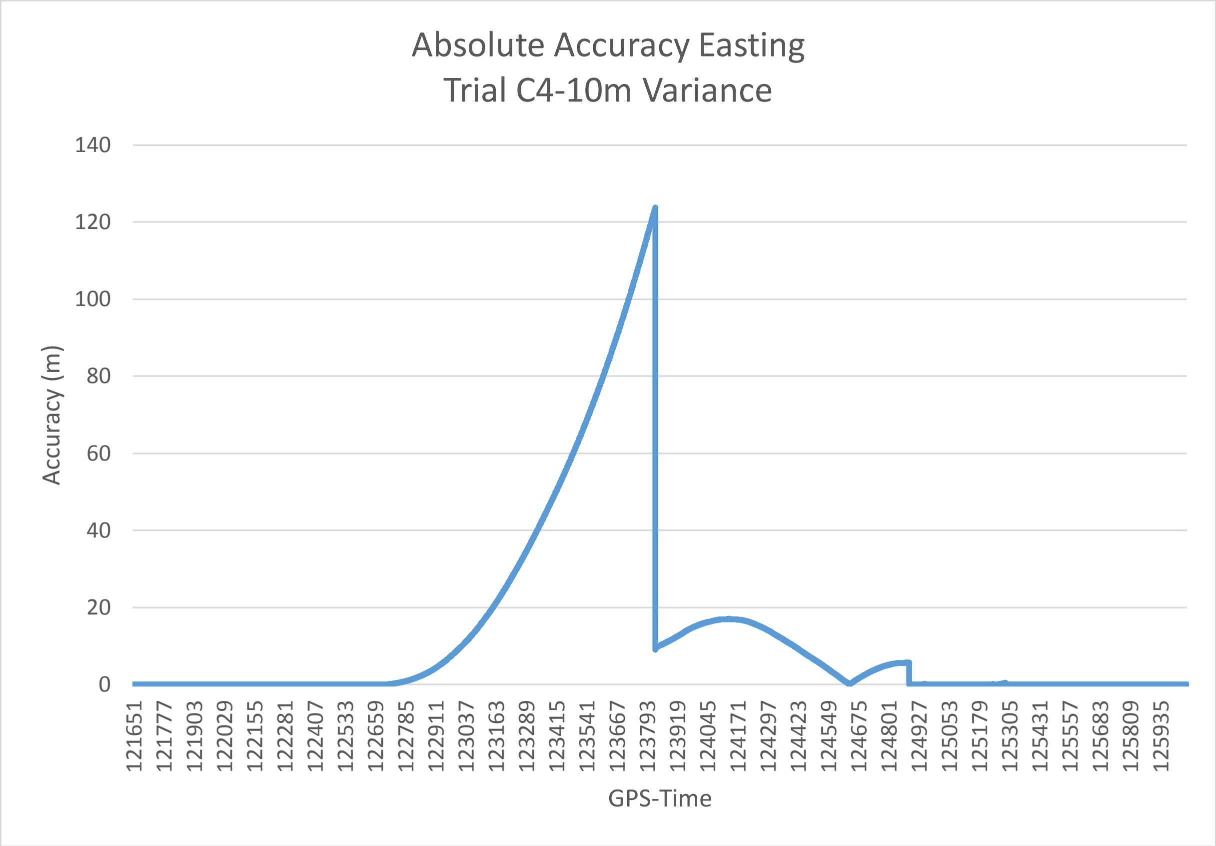
125683
125809
125935
GPS-Time

C13: Trial C4

Manipulated Error: 10 m West






Absolute Accuracy Easting
Trial C4-10m Variance
Accuracy (m)
0
20
40
60
80
100
120
140
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
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124171
124297
124423
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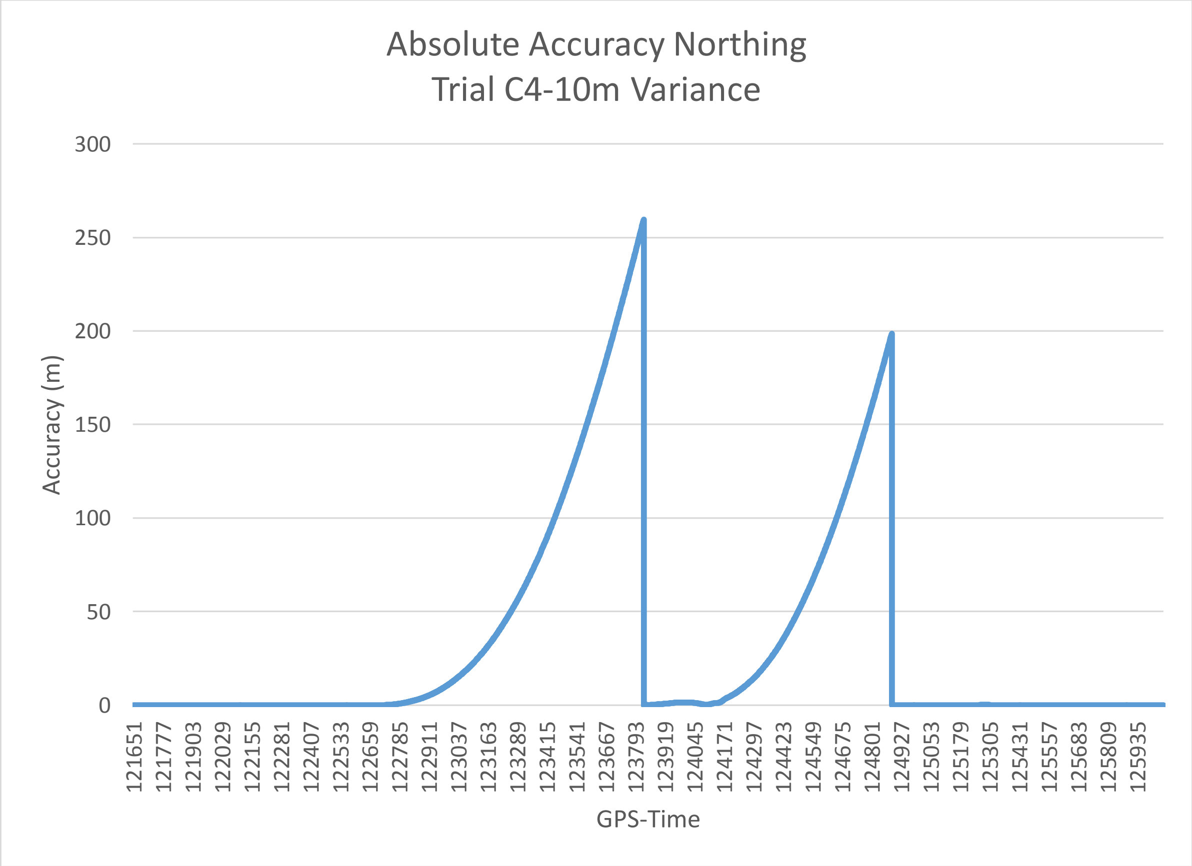
125809
125935
GPS-Time


Absolute Accuracy Northing
Trial C4-10m Variance
Accuracy (m)
0
50
100
150
200
250
300
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
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124171
124297
124423
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125809
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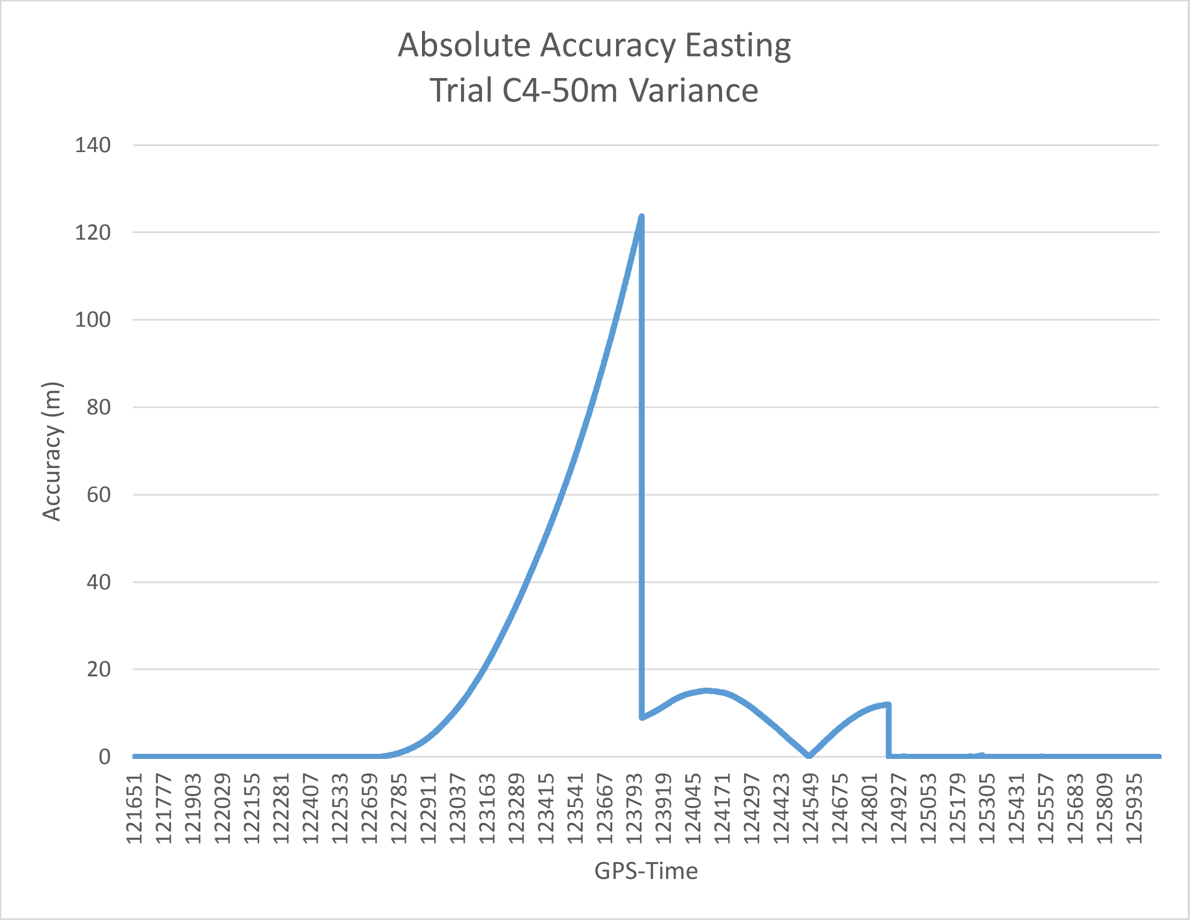
GPS-Time


Absolute Accuracy Easting
Trial C4-50m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
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124171
124297
124423
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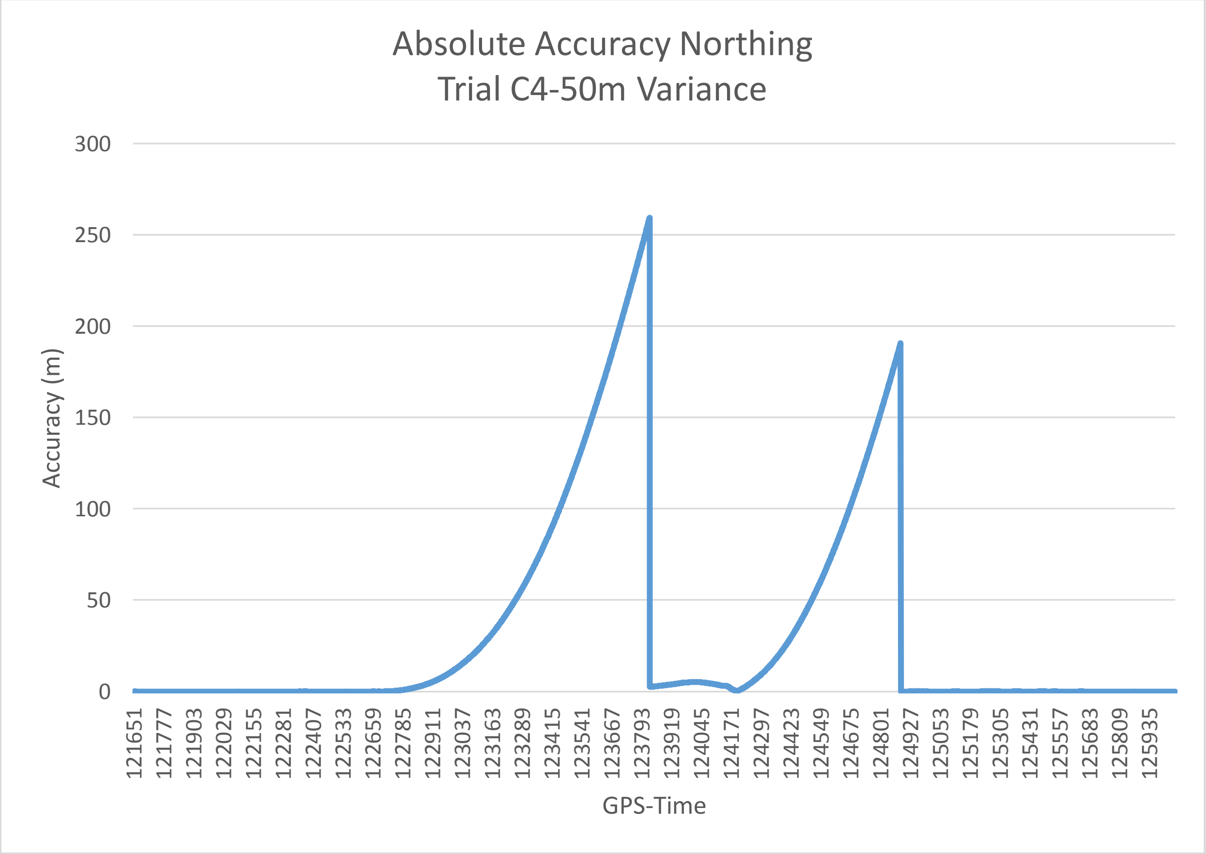
125935
GPS-Time


Absolute Accuracy Northing
Trial C4-50m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
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124171
124297
124423
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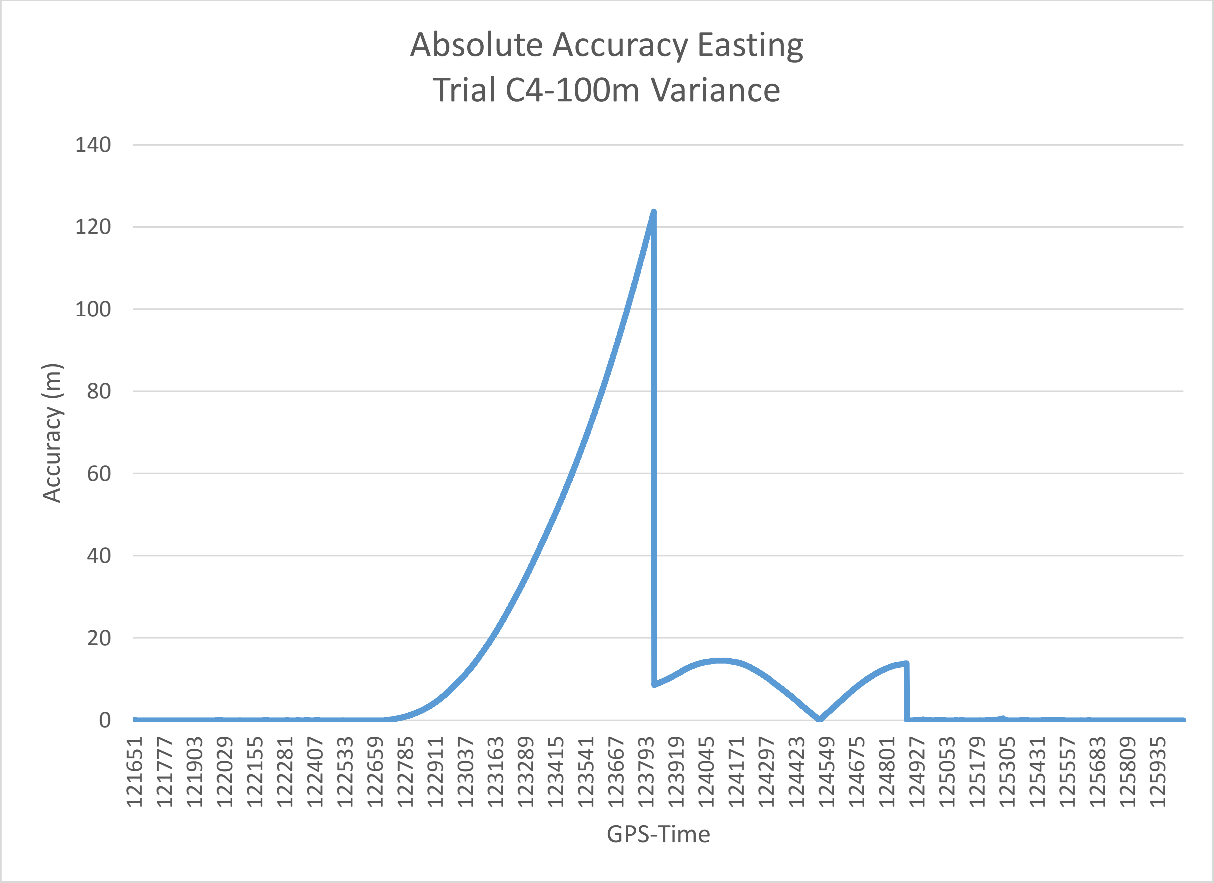
125935
GPS-Time


Absolute Accuracy Easting
Trial C4-100m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
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123163
123289
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124171
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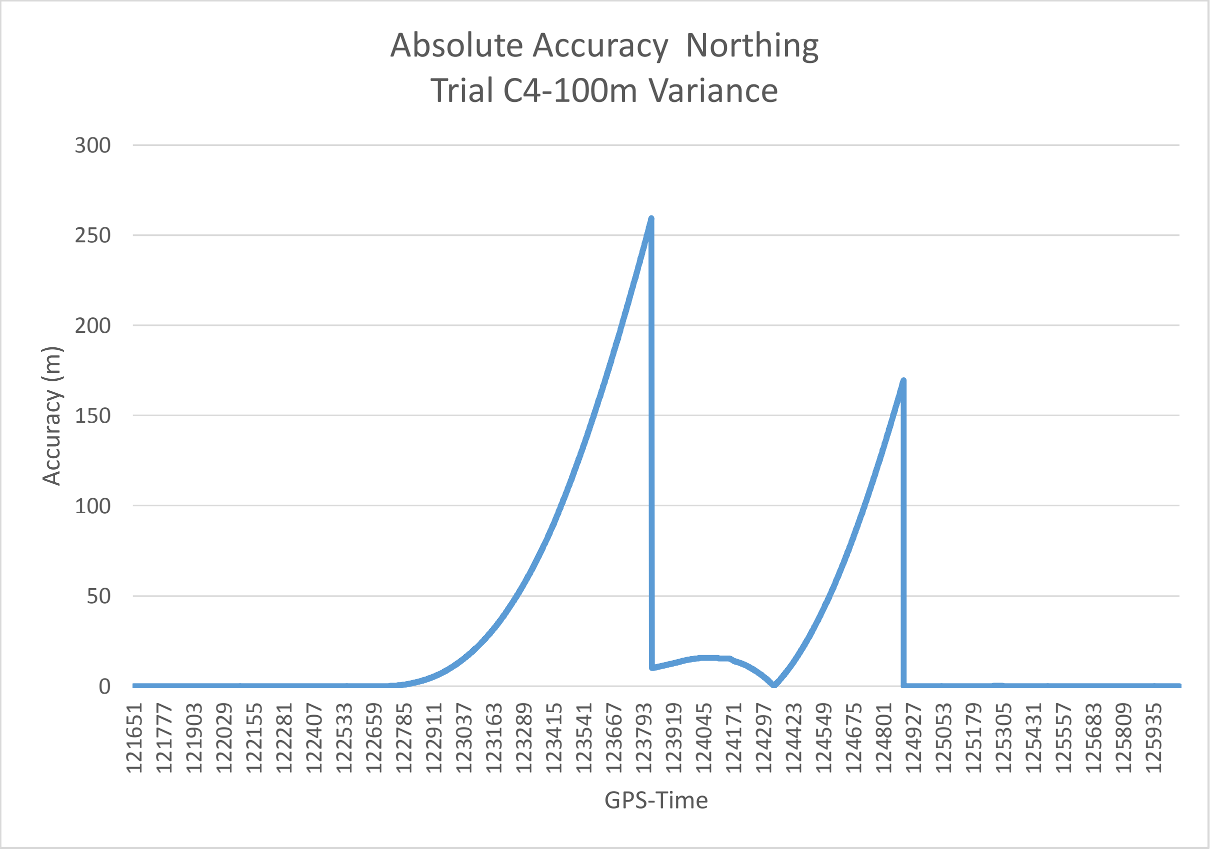
125809
125935
GPS-Time


Absolute Accuracy Northing
Trial C4-100m Variance
Accuracy (m)
300
250
200
150
100
50
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121651
121777
121903
122029
122155
122281
122407
122533
122659
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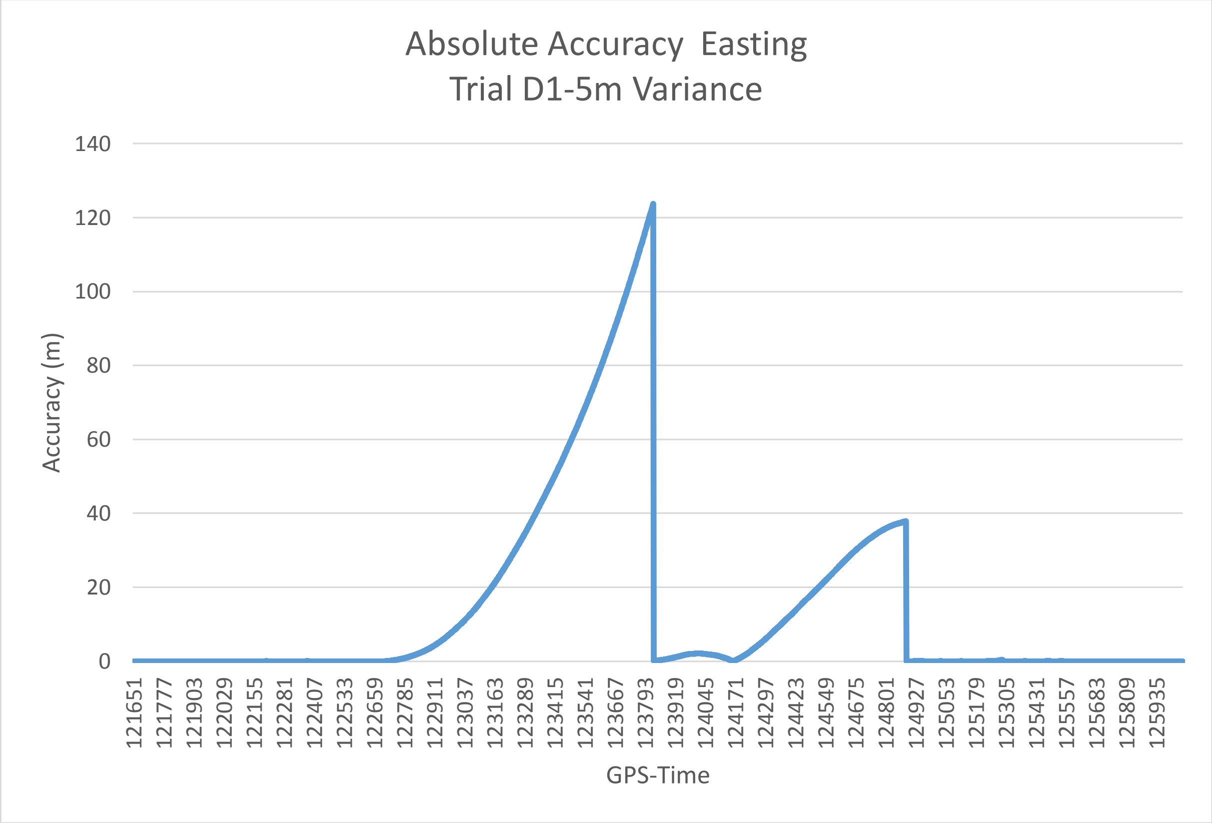
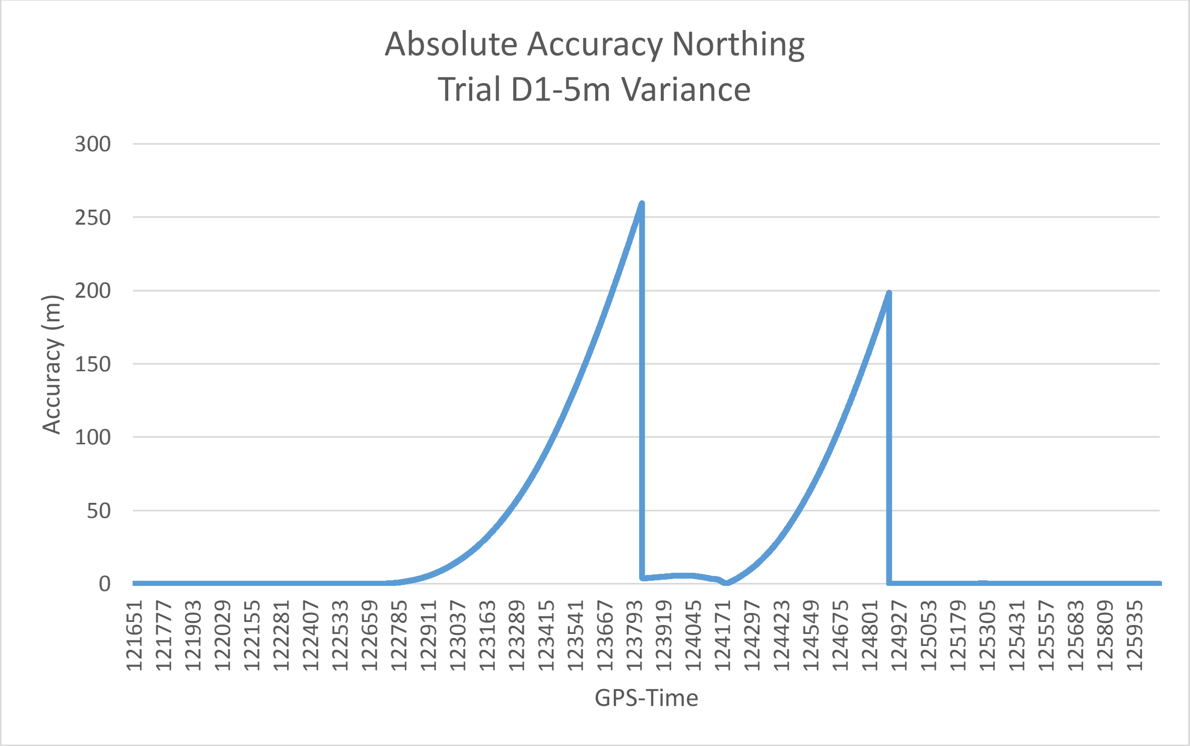
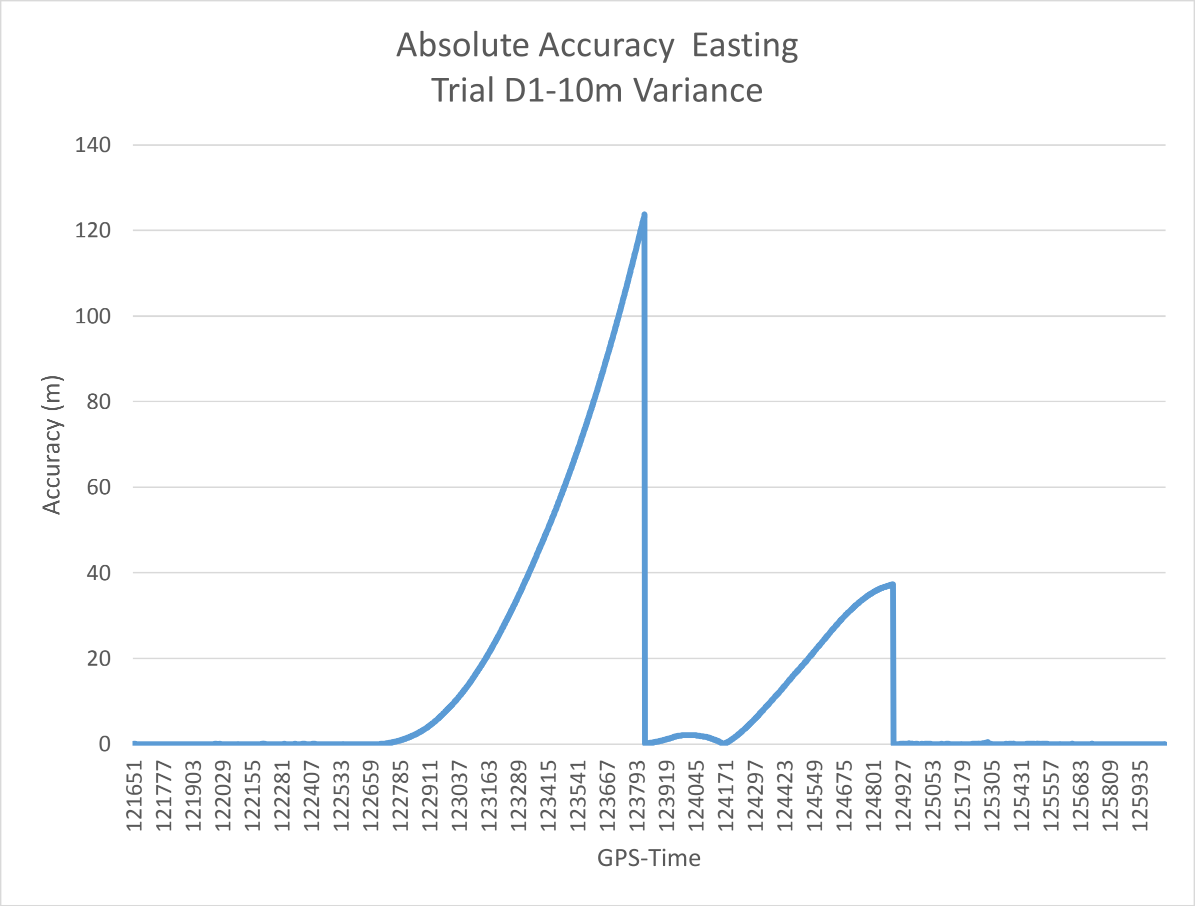
125809
125935
GPS-Time

C14: Trial D1

Manipulated Error: 5m North






Absolute Accuracy Easting
Trial D1-10m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
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124171
124297
124423
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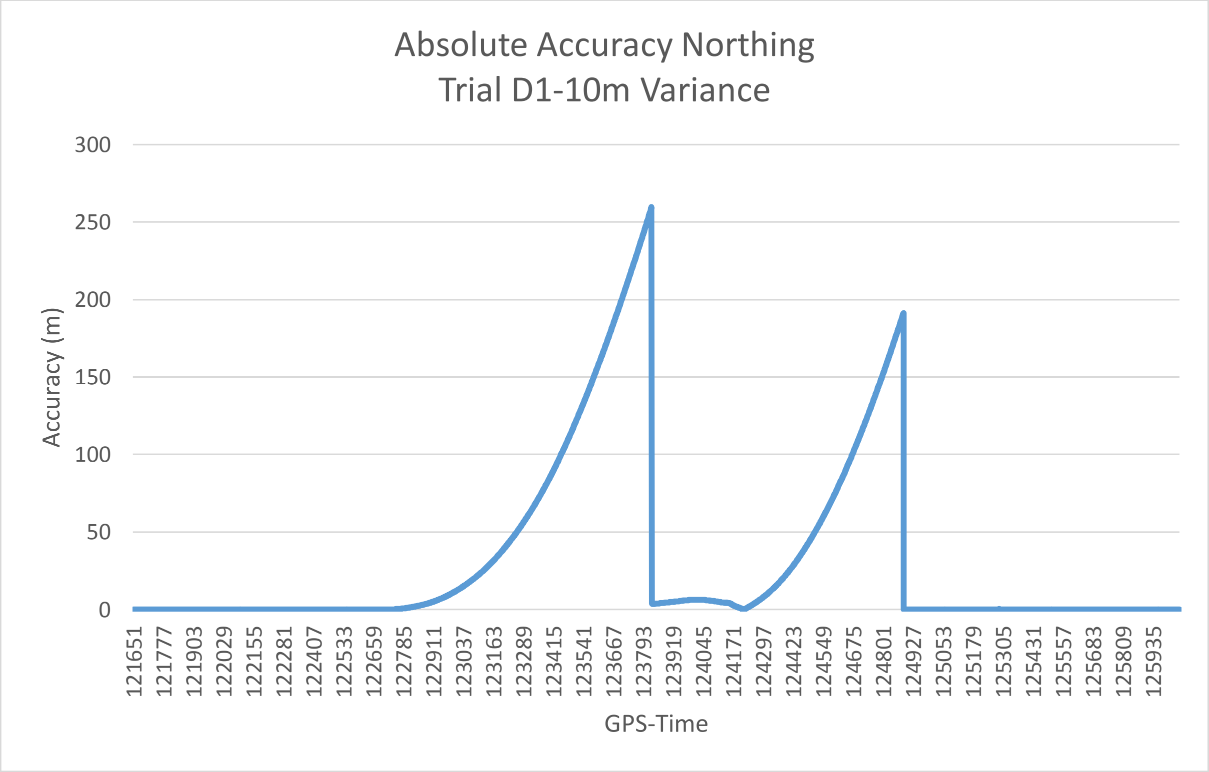
125809
125935
GPS-Time


Absolute Accuracy Northing
Trial D1-10m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
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124171
124297
124423
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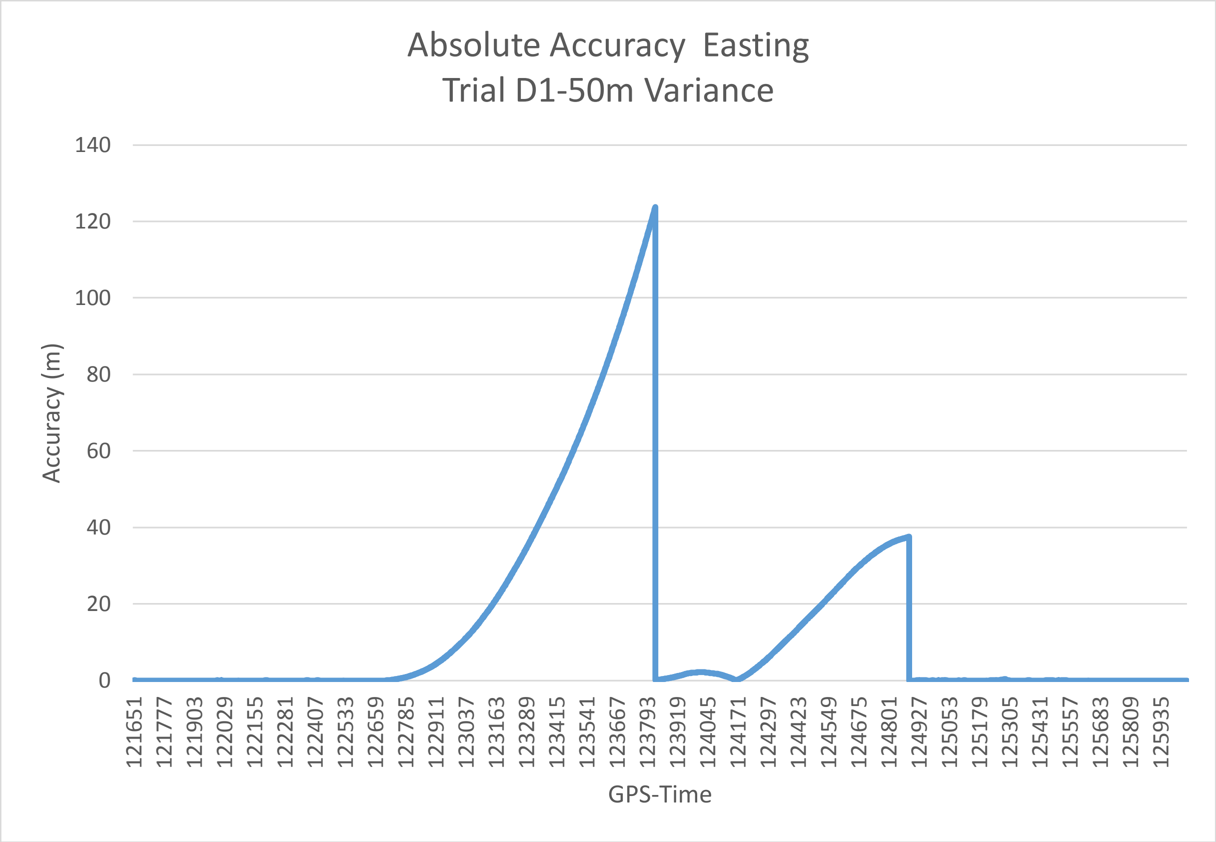
GPS-Time


Absolute Accuracy Easting
Trial D1-50m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
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124171
124297
124423
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124927
125053
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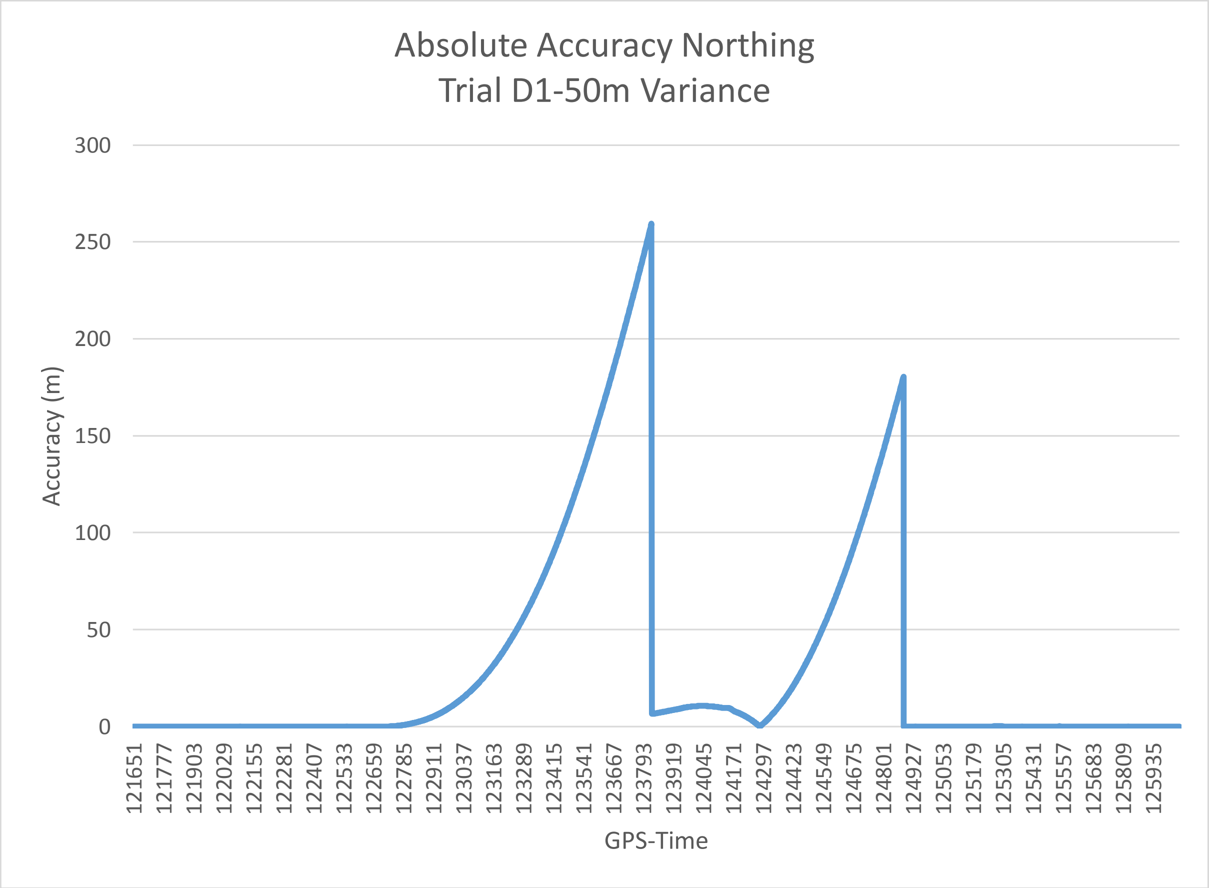
125809
125935
GPS-Time


Absolute Accuracy Northing
Trial D1-50m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
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123541
123667
123793
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124297
124423
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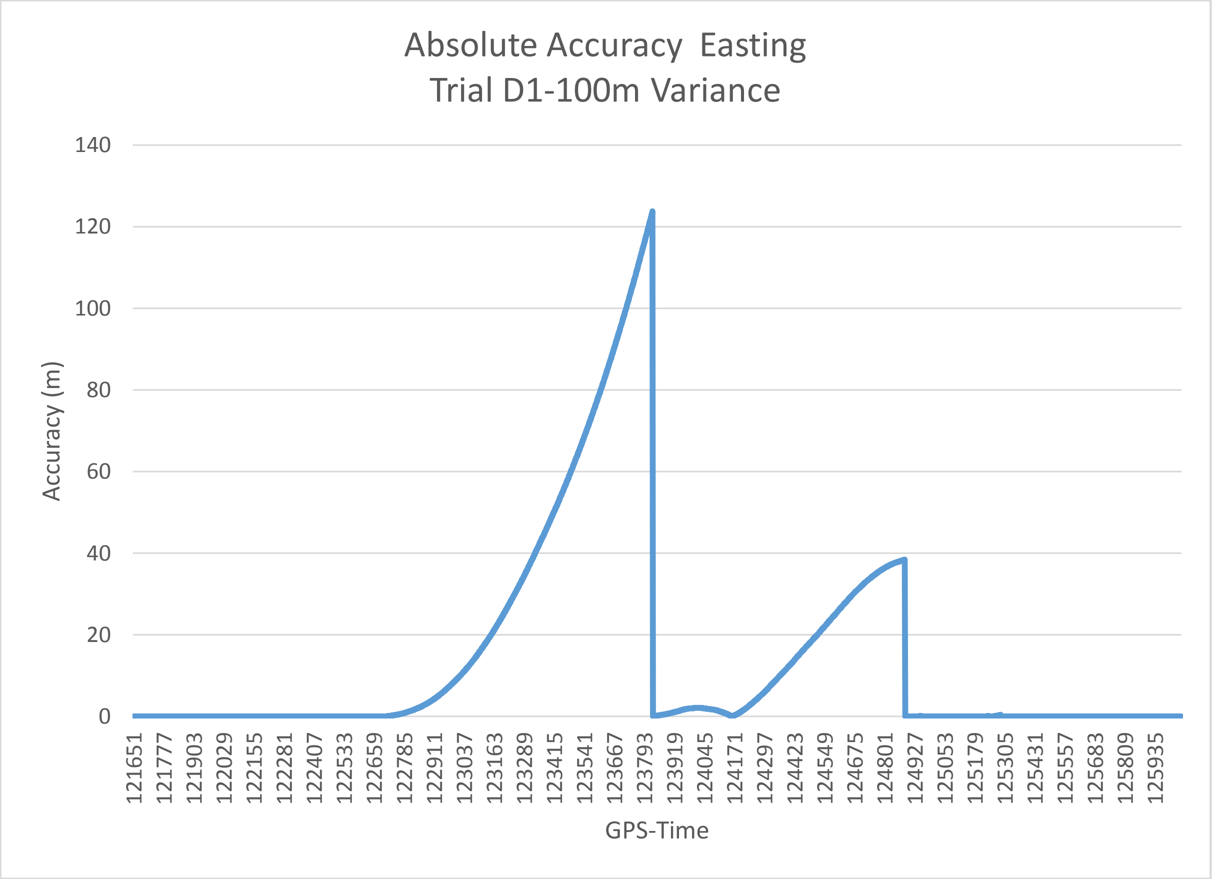
125809
125935
GPS-Time


Absolute Accuracy Easting
Trial D1-100m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
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124045
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124423
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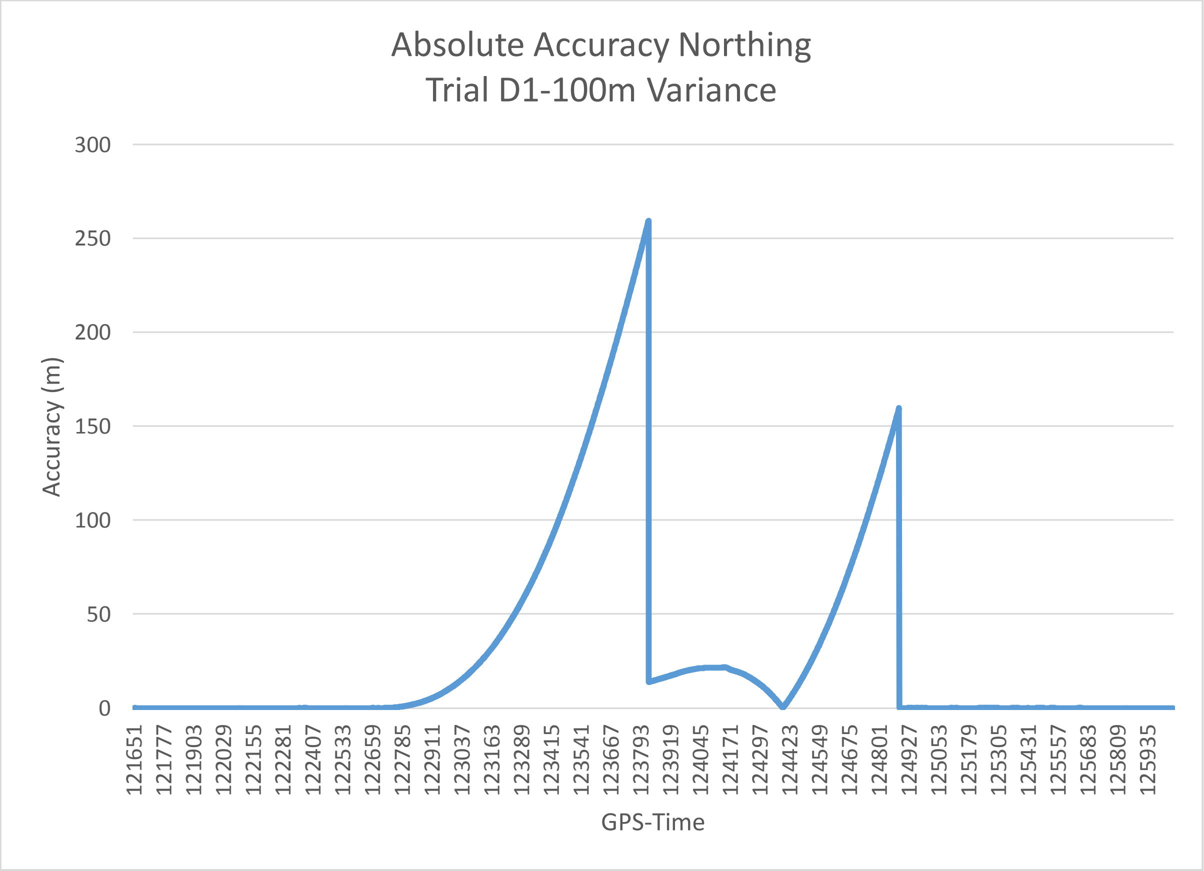
125935
GPS-Time


Absolute Accuracy Northing
Trial D1-100m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
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123037
123163
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124549
124675
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124927
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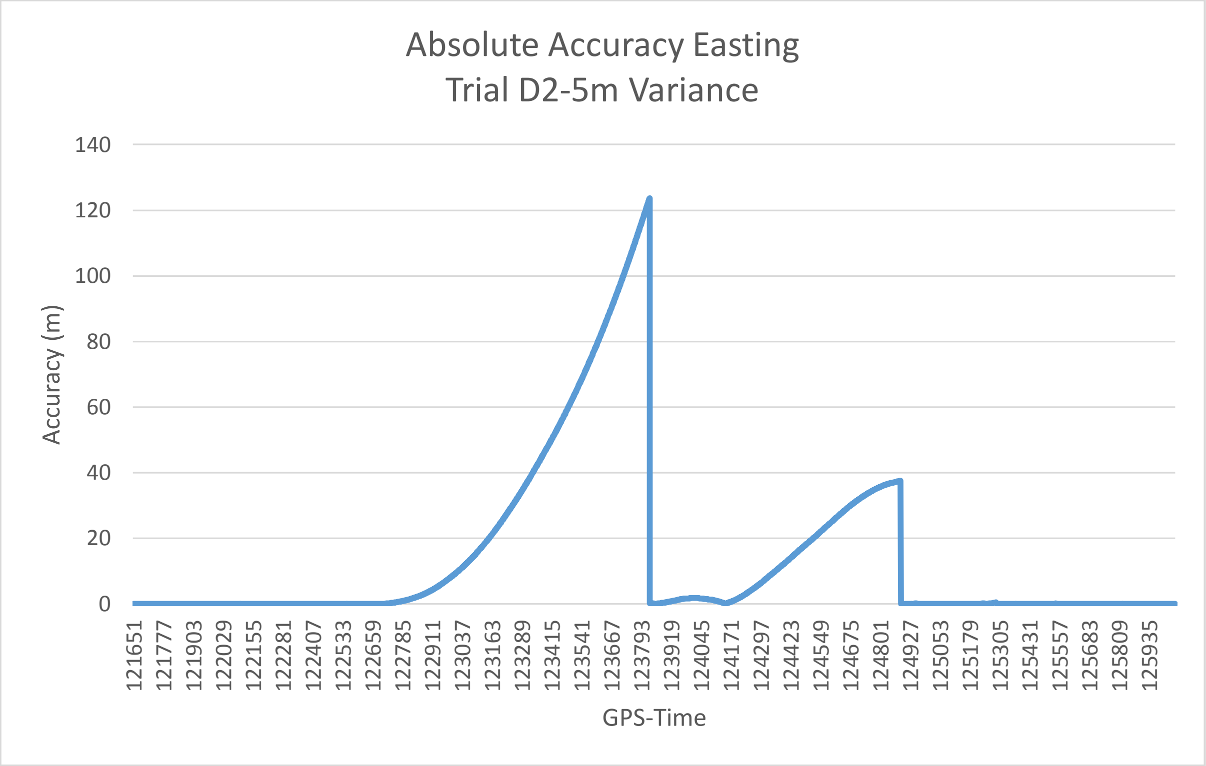
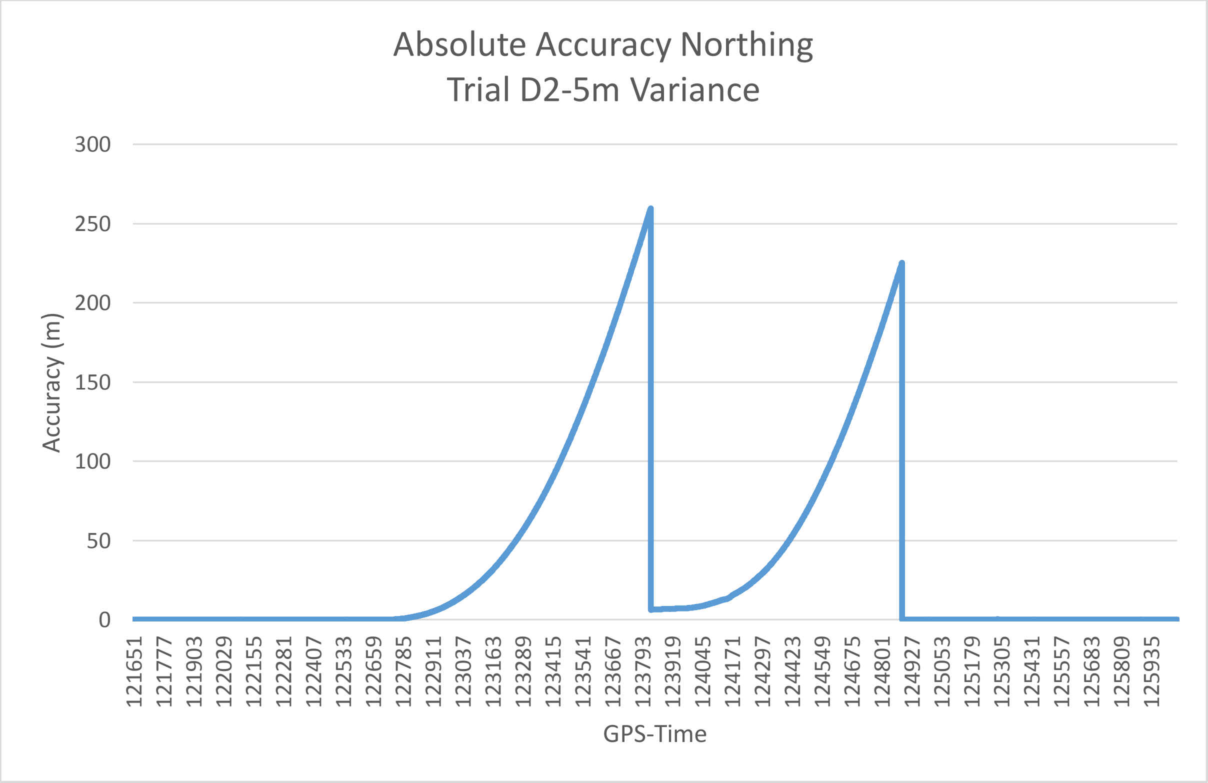
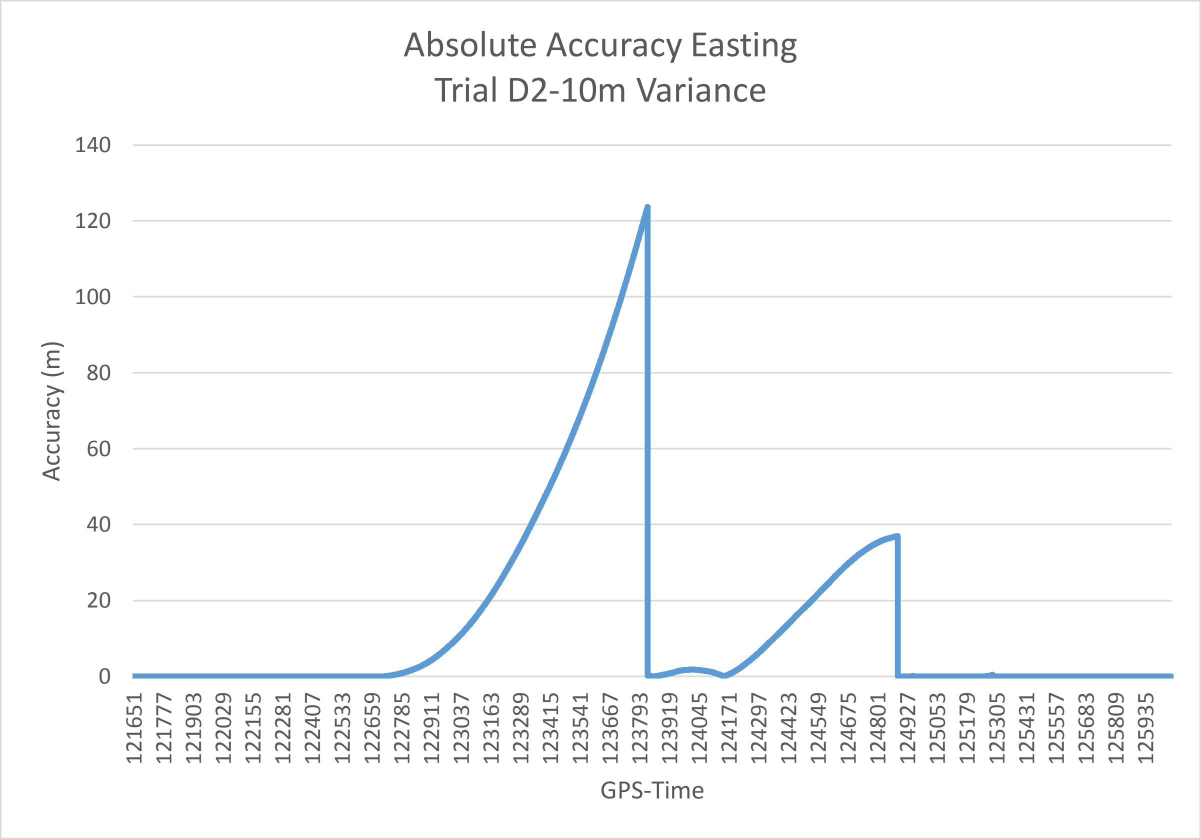
125683
125809
125935
GPS-Time

C15: Trial D2

Manipulated Error: 5m South







Absolute Accuracy Easting
Trial D2-10m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
123541
123667
123793
123919
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124297
124423
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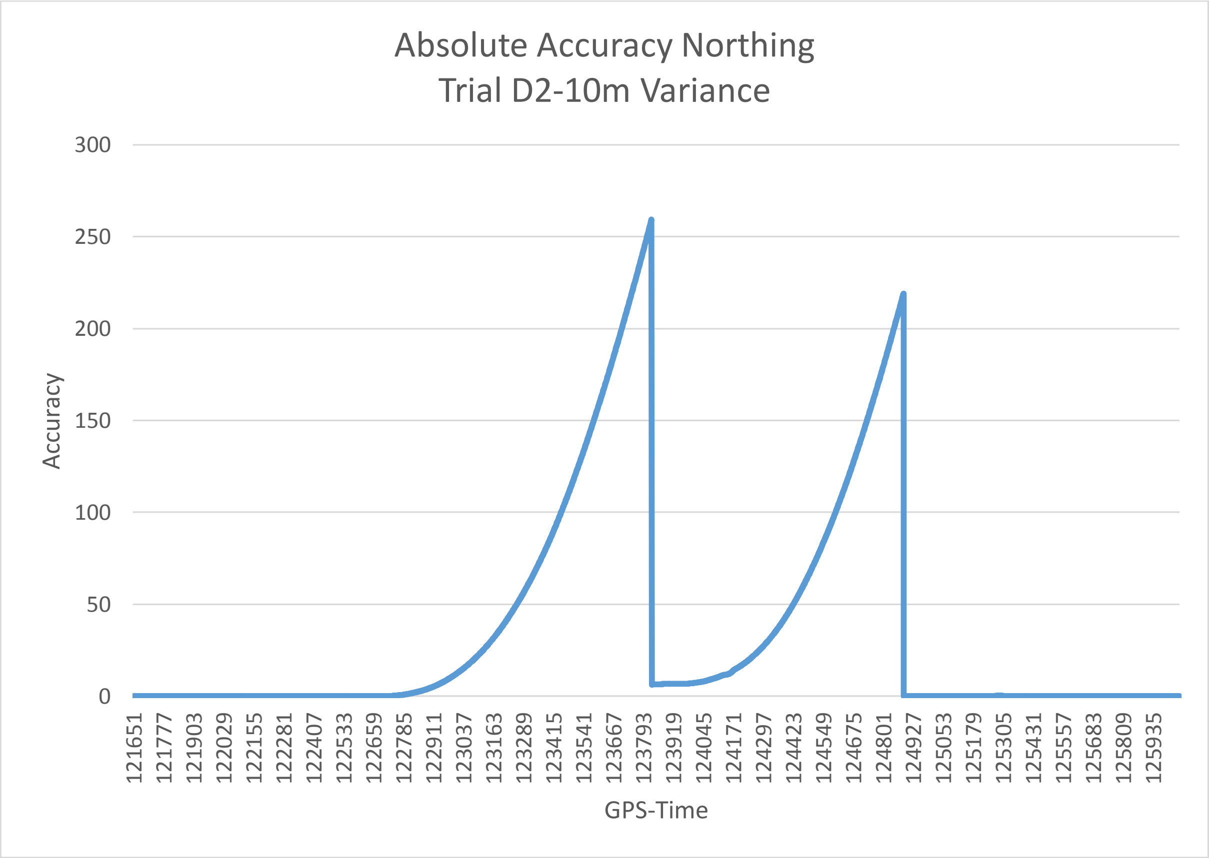
125935
GPS-Time



Absolute Accuracy Northing
Trial D2-10m Variance
Accuracy
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
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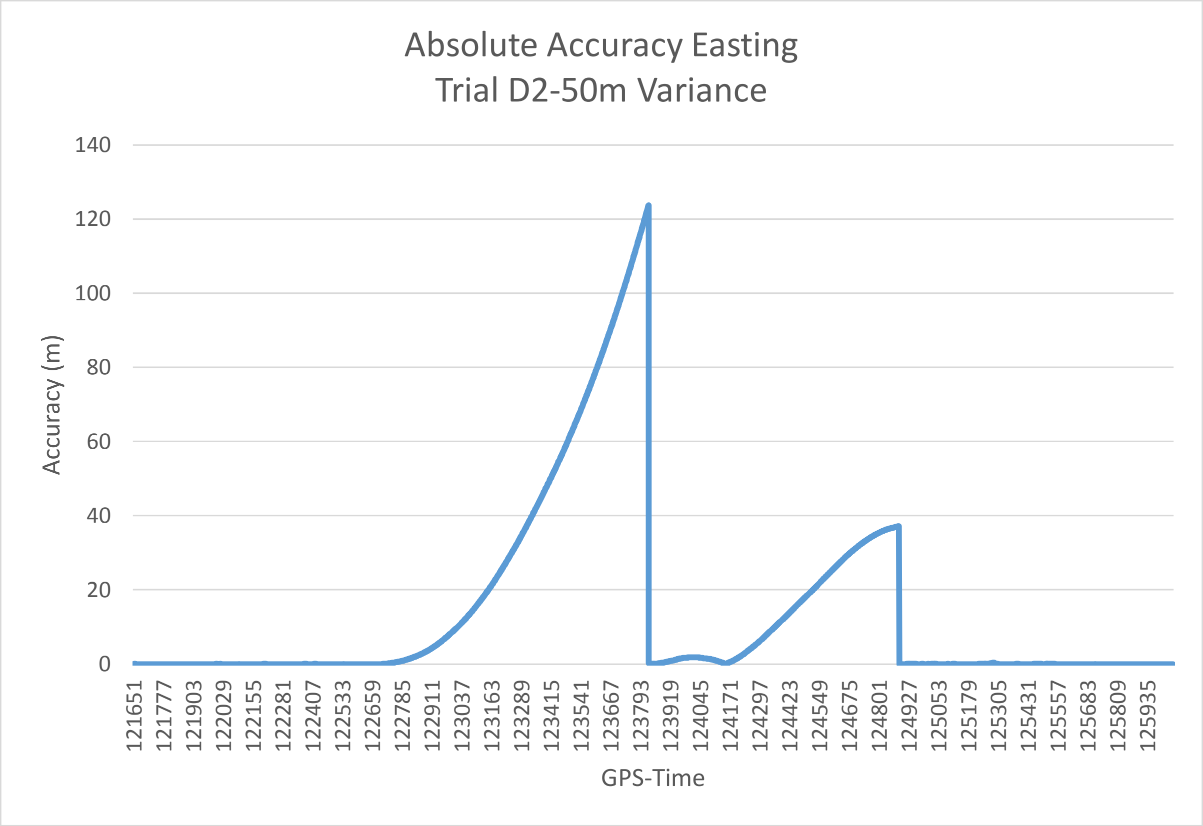
125935
GPS-Time


Absolute Accuracy Easting
Trial D2-50m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
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122911
123037
123163
123289
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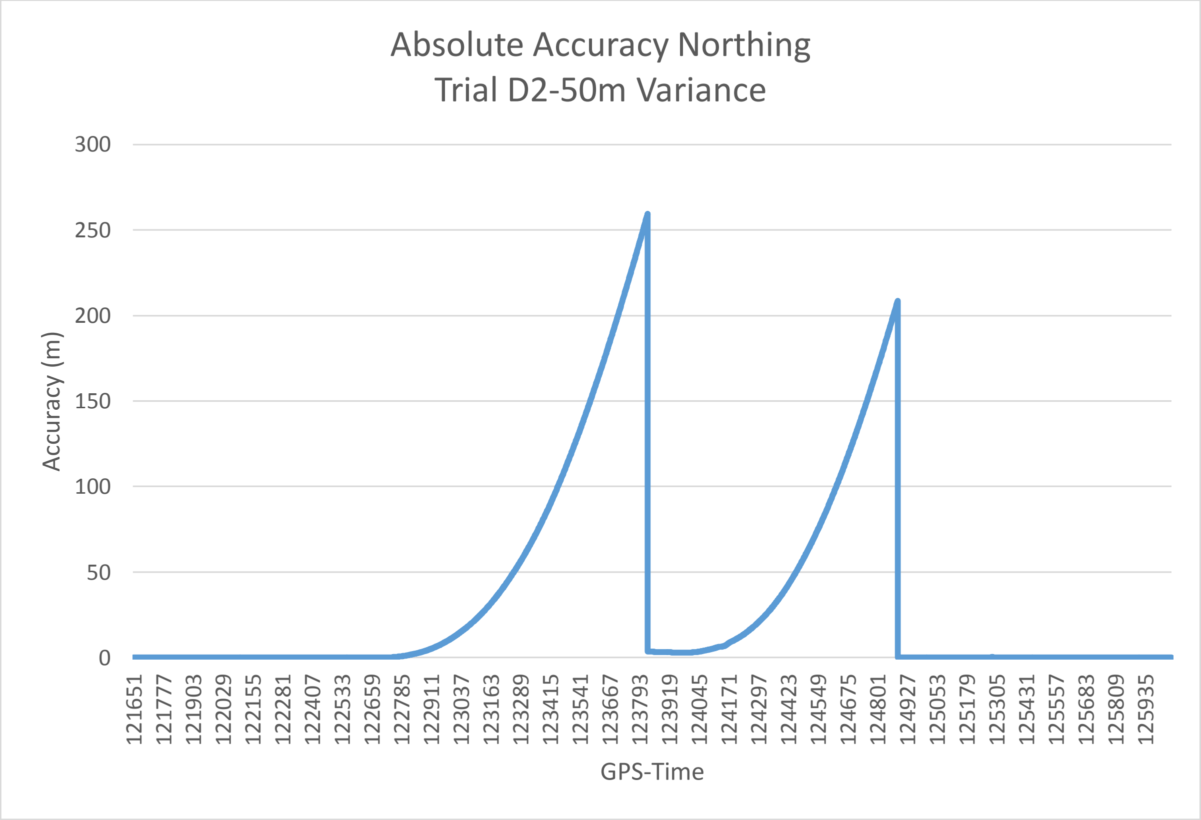
125935
GPS-Time


Absolute Accuracy Northing
Trial D2-50m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
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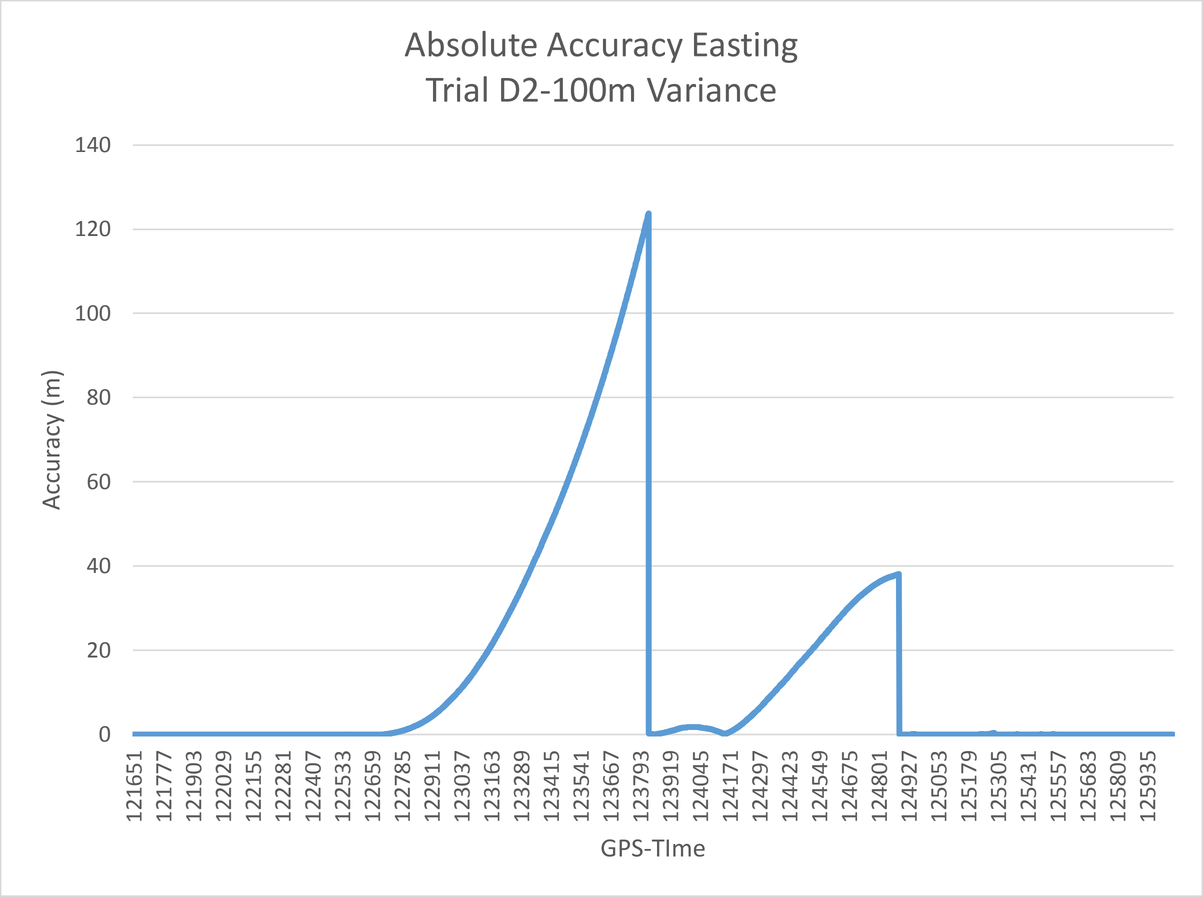
125935
GPS-Time


Absolute Accuracy Easting
Trial D2-100m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
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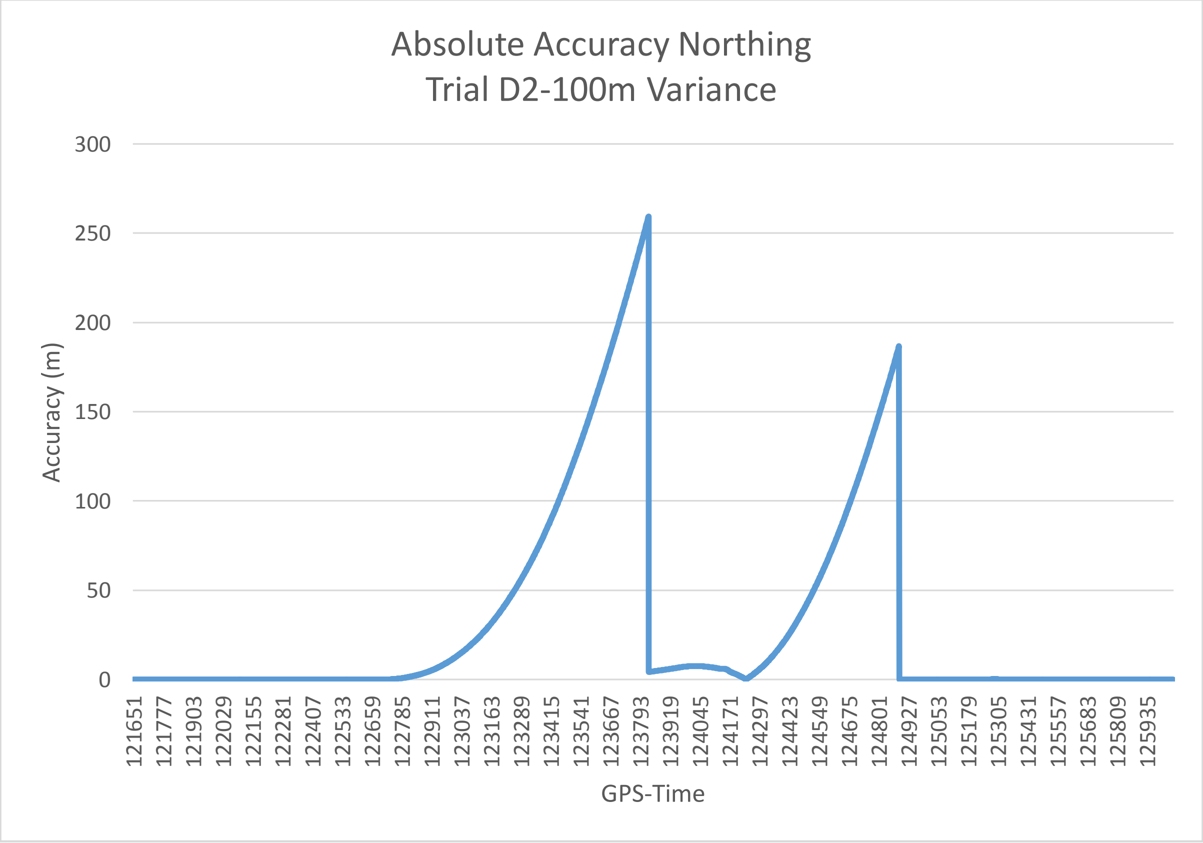
GPS-TIme


Absolute Accuracy Northing
Trial D2-100m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
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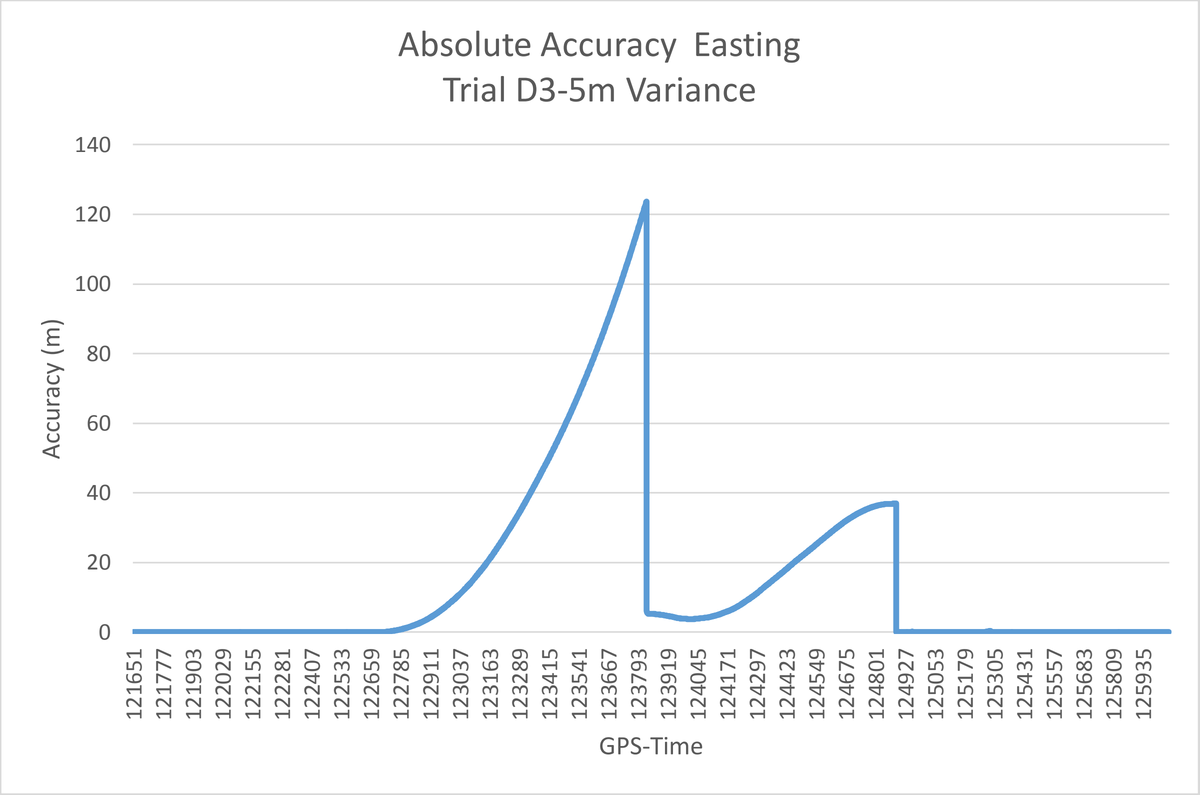
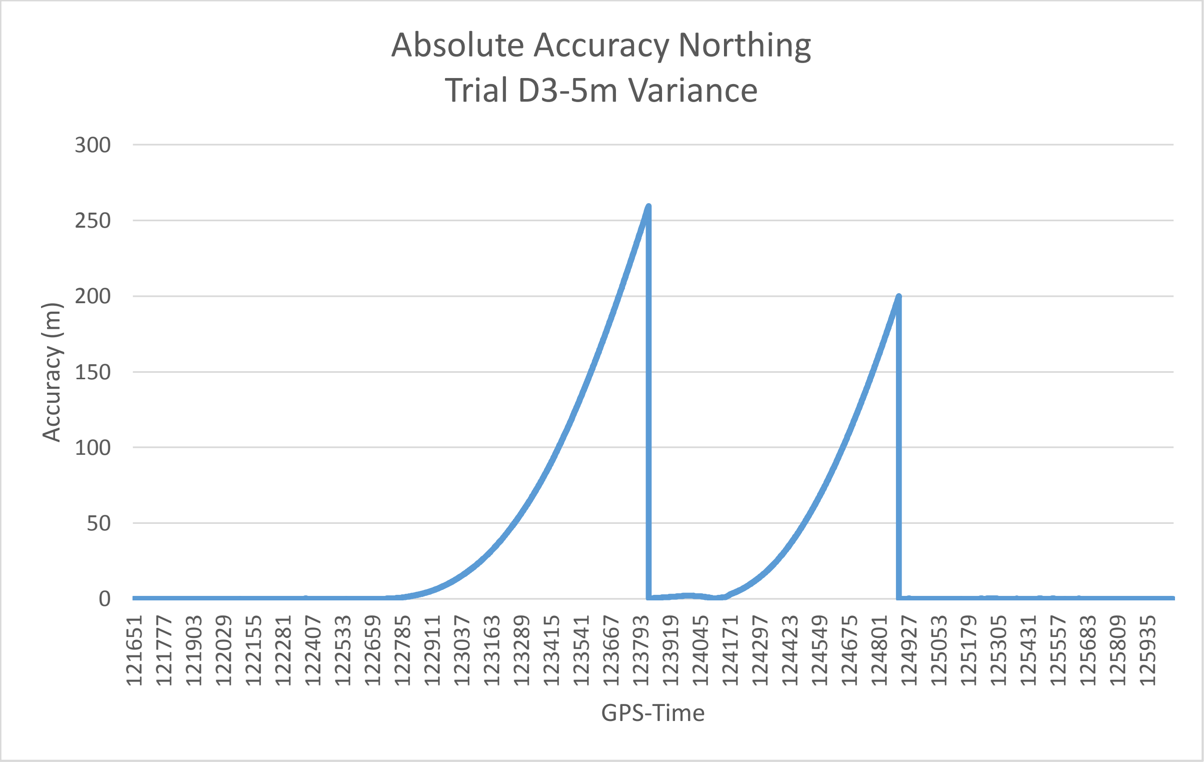
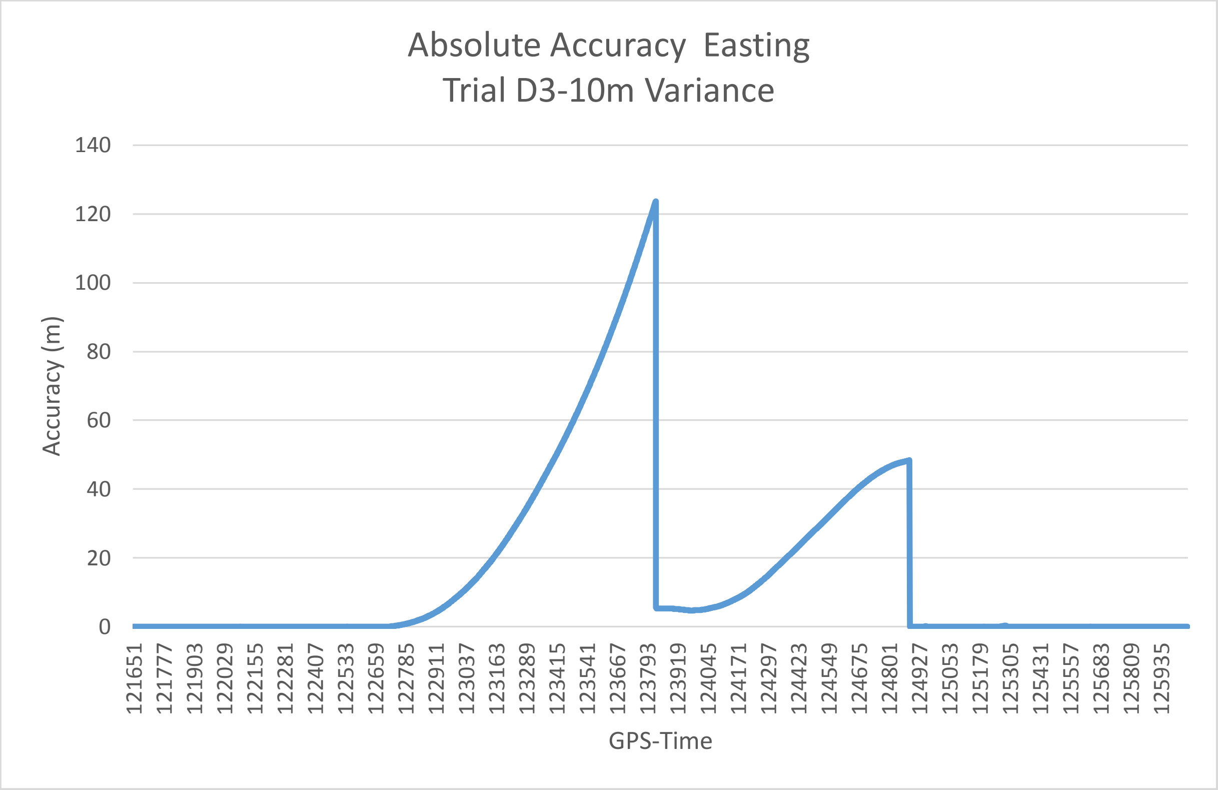
125935
GPS-Time

C16: Trial D3

Manipulated Error: 5m East






Absolute Accuracy Easting
Trial D3-10m Variance
140
120
100
80
60
40
20
0
Accuracy (m)
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
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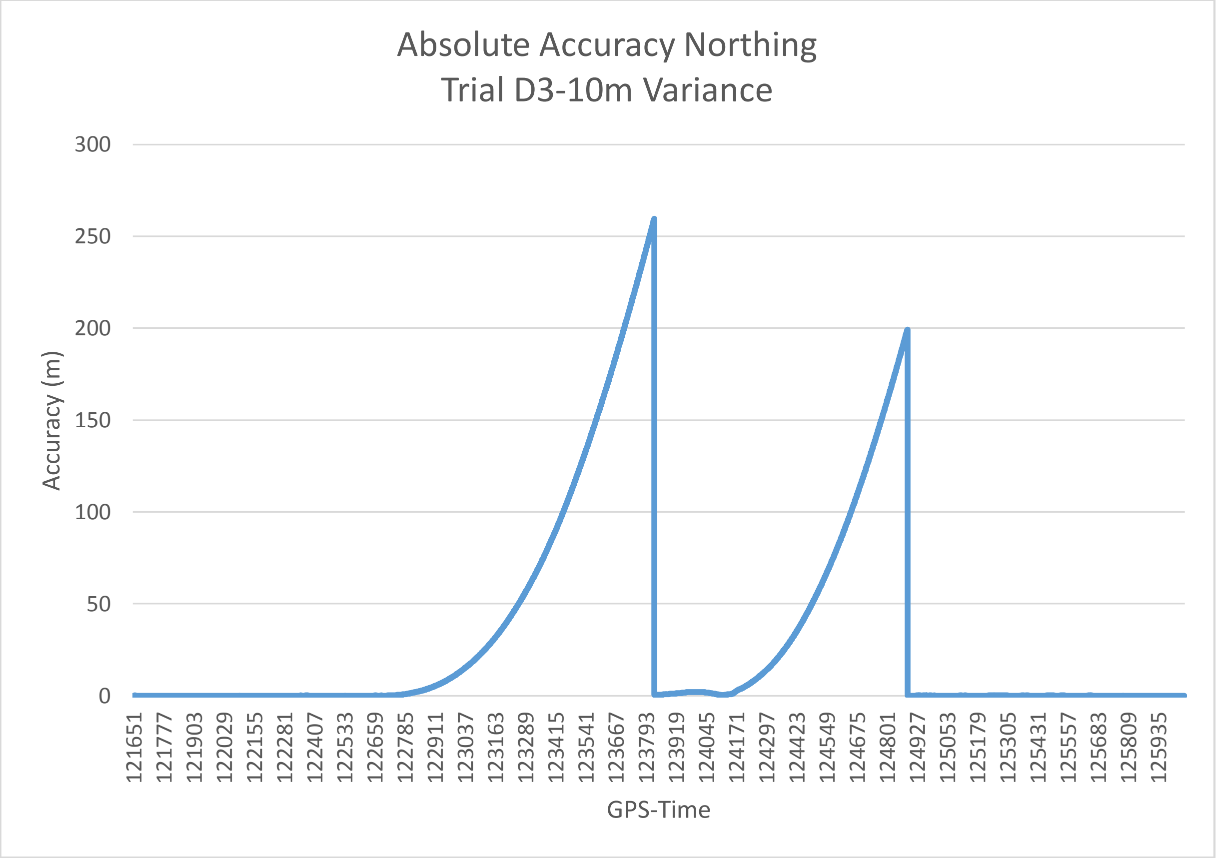
GPS-Time


Absolute Accuracy Northing
Trial D3-10m Variance
300
250
200
150
100
50
0
Accuracy (m)
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
123415
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123667
123793
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124297
124423
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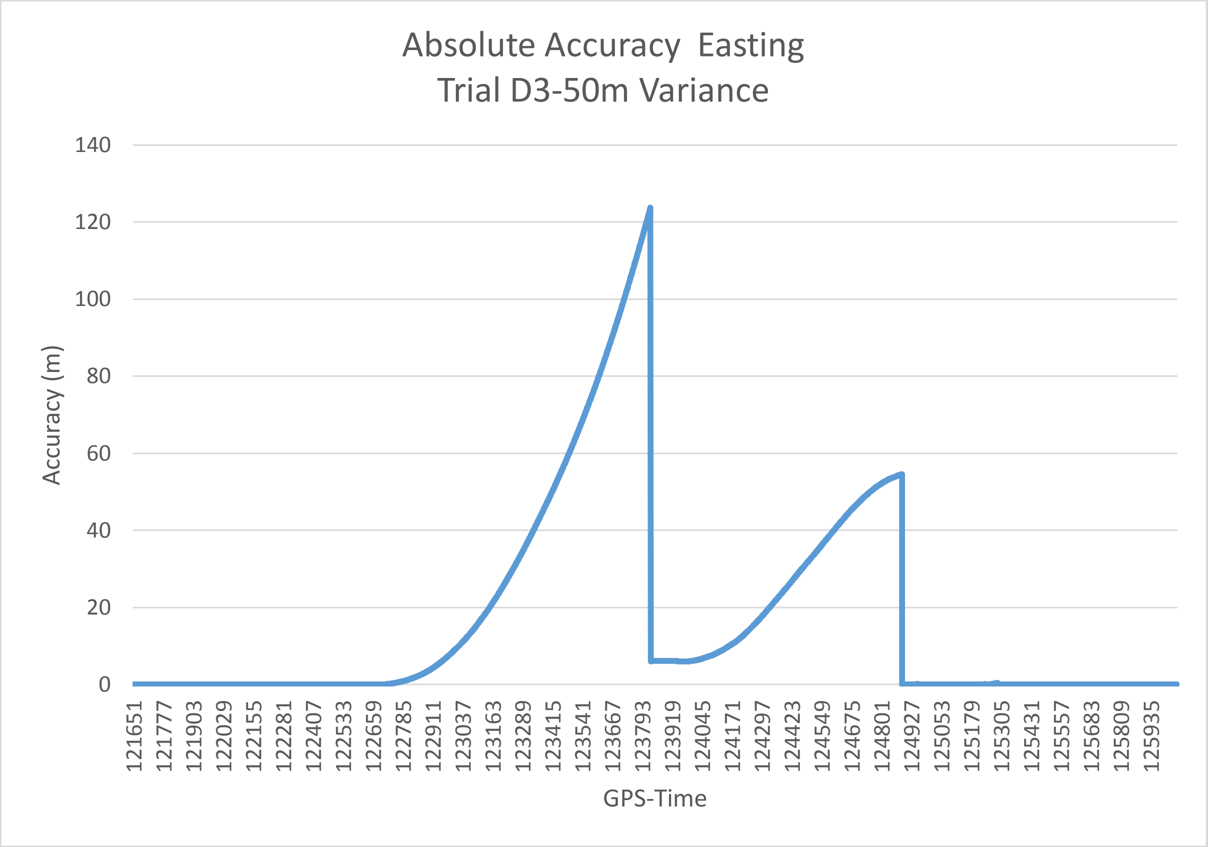
GPS-Time


Absolute Accuracy Easting
Trial D3-50m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
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123289
123415
123541
123667
123793
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124171
124297
124423
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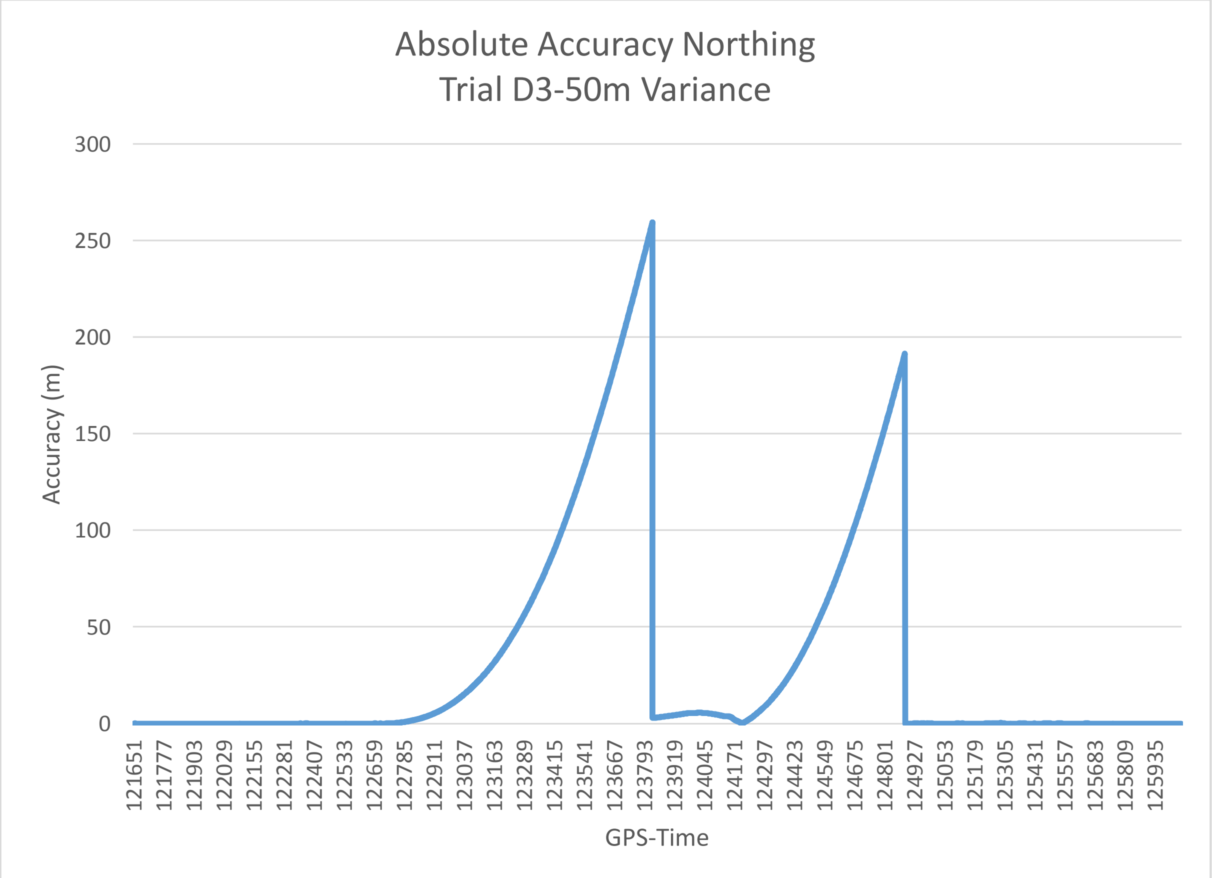
GPS-Time


Absolute Accuracy Northing
Trial D3-50m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
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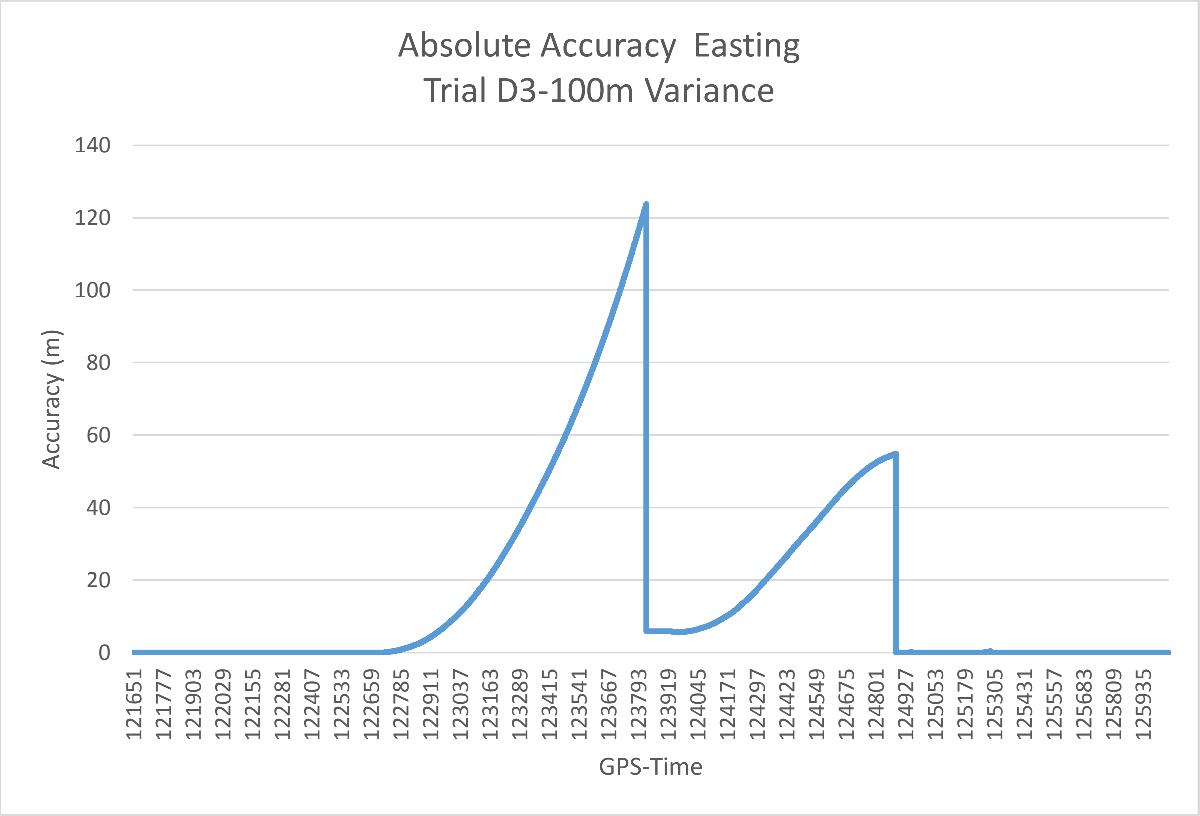
GPS-Time


Absolute Accuracy Easting
Trial D3-100m Variance
140
120
100
80
60
40
20
0
Accuracy (m)
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
123289
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123541
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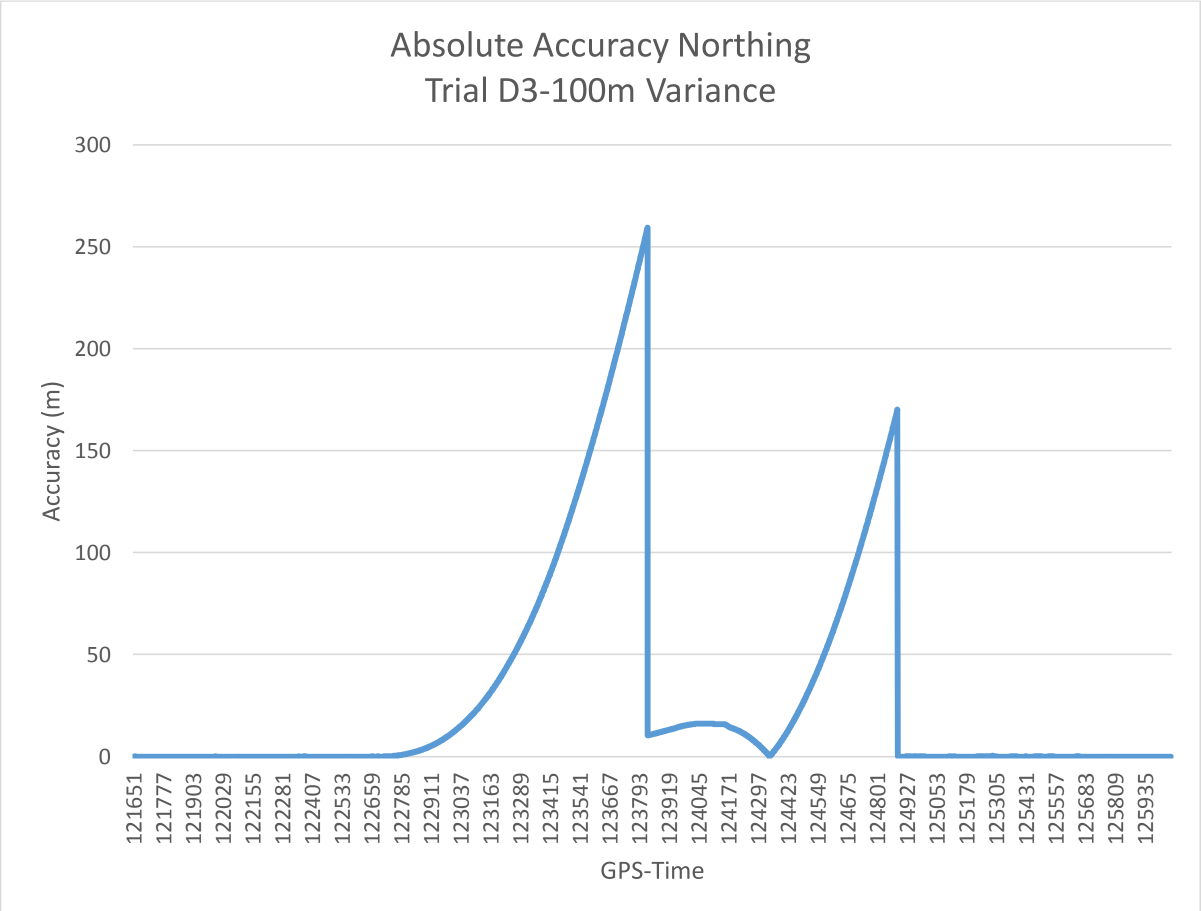
125935
GPS-Time


Absolute Accuracy Northing
Trial D3-100m Variance
300
250
200
150
100
50
0
Accuracy (m)
121651
121777
121903
122029
122155
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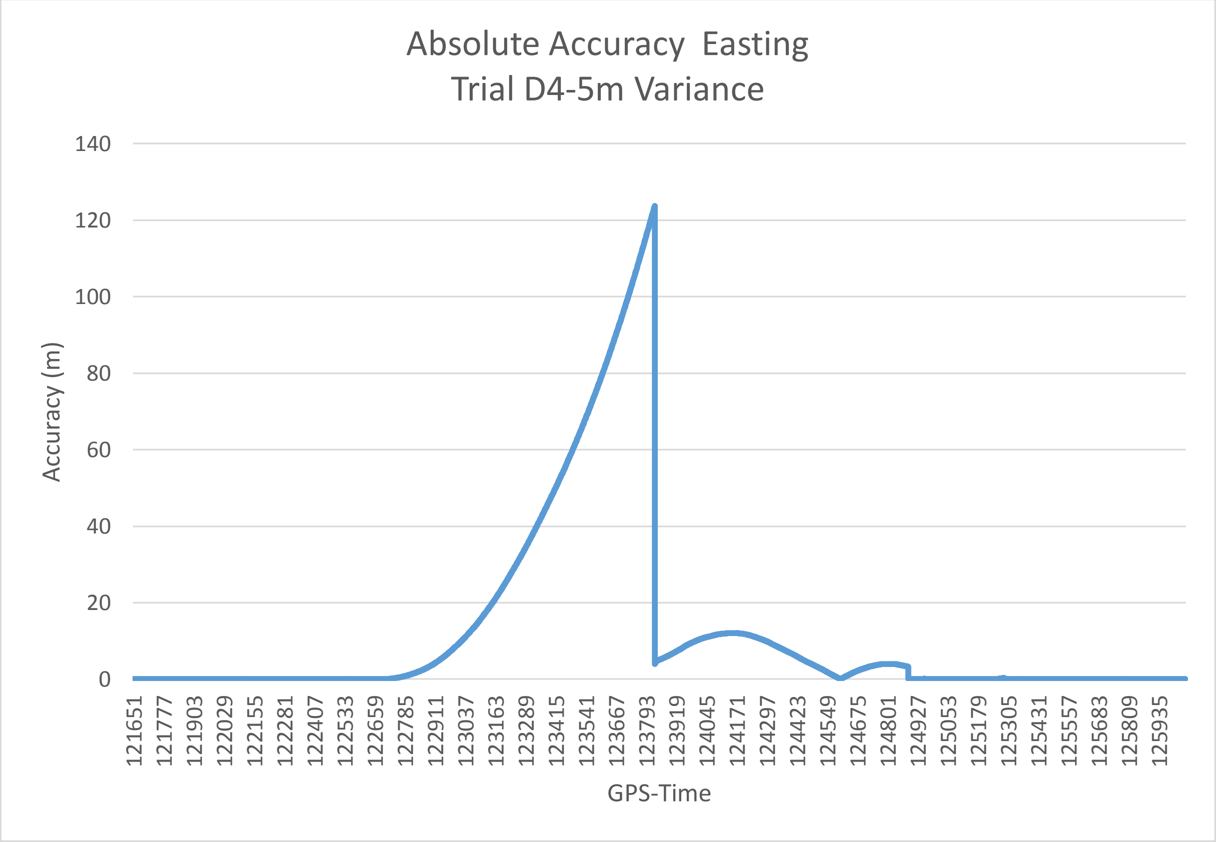
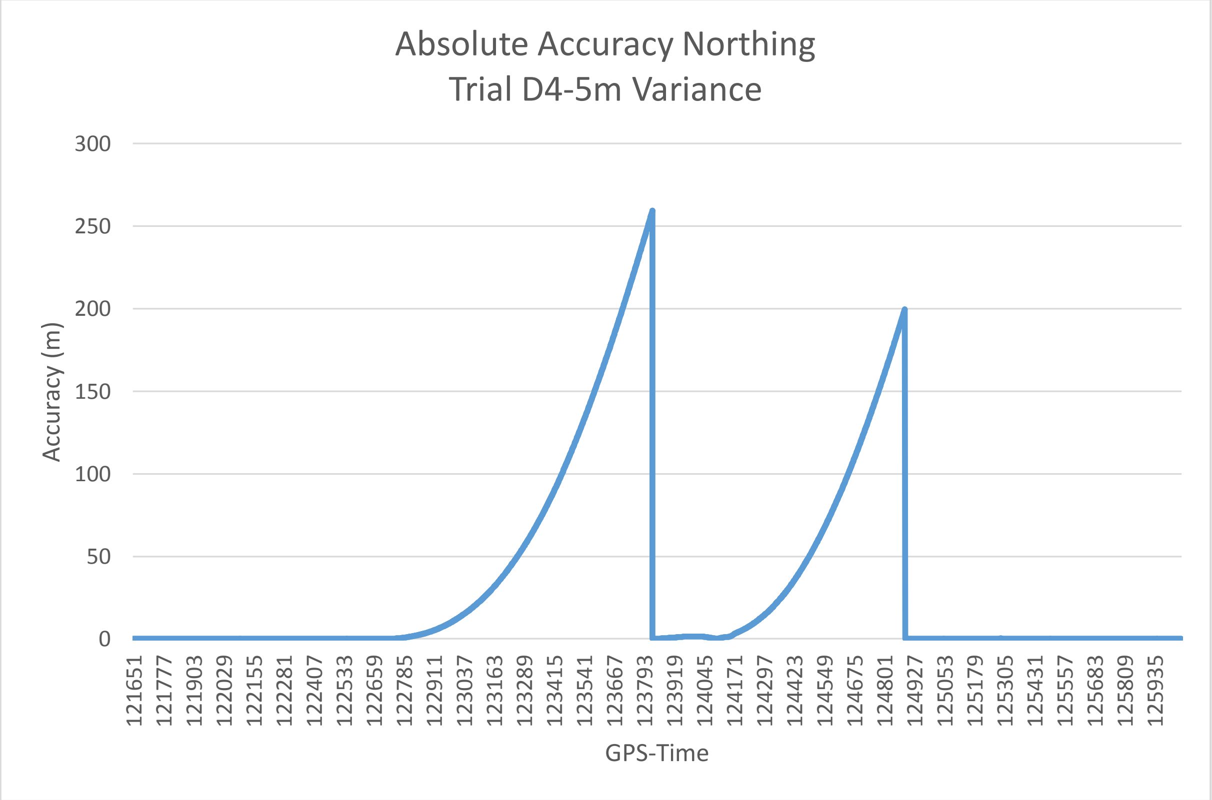
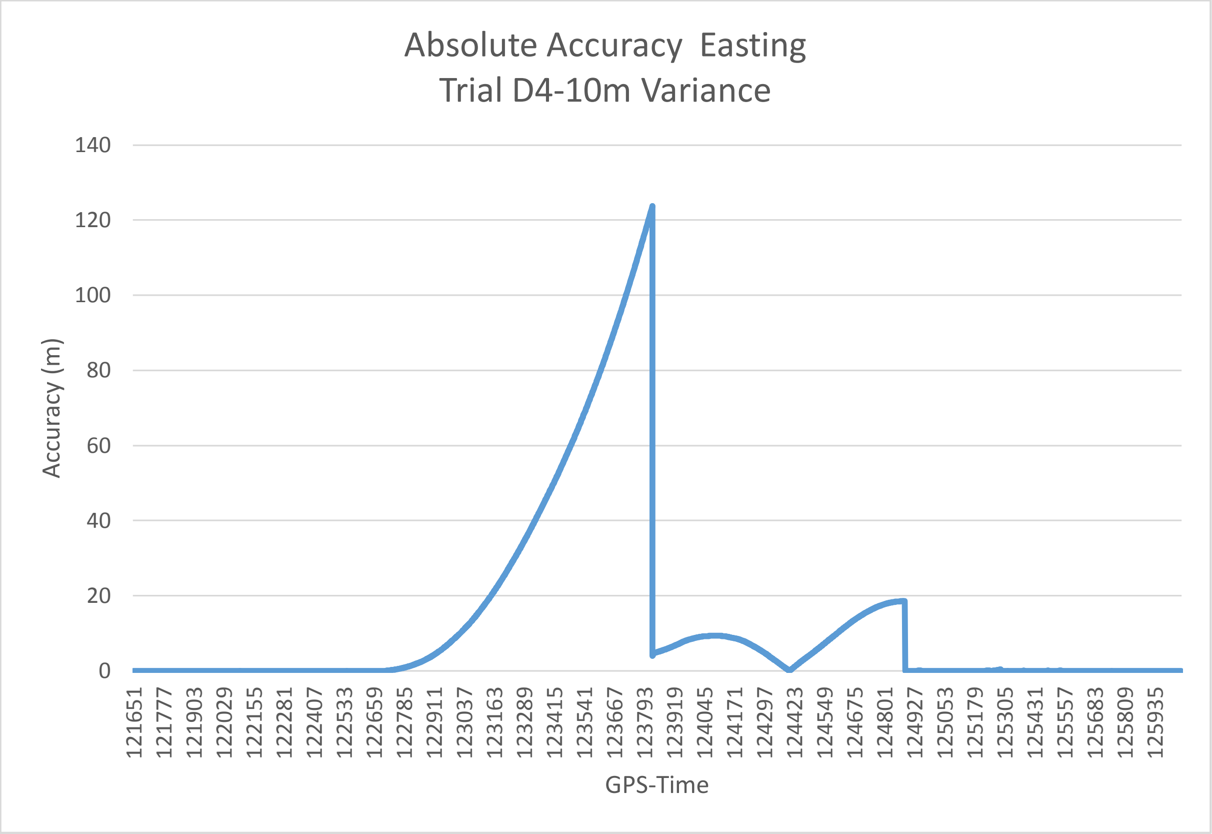
GPS-Time

C17: Trial D4

Manipulated Error: 5m West






Absolute Accuracy Easting
Trial D4-10m Variance
140
120
100
80
60
40
20
0
Accuracy (m)
121651
121777
121903
122029
122155
122281
122407
122533
122659
122785
122911
123037
123163
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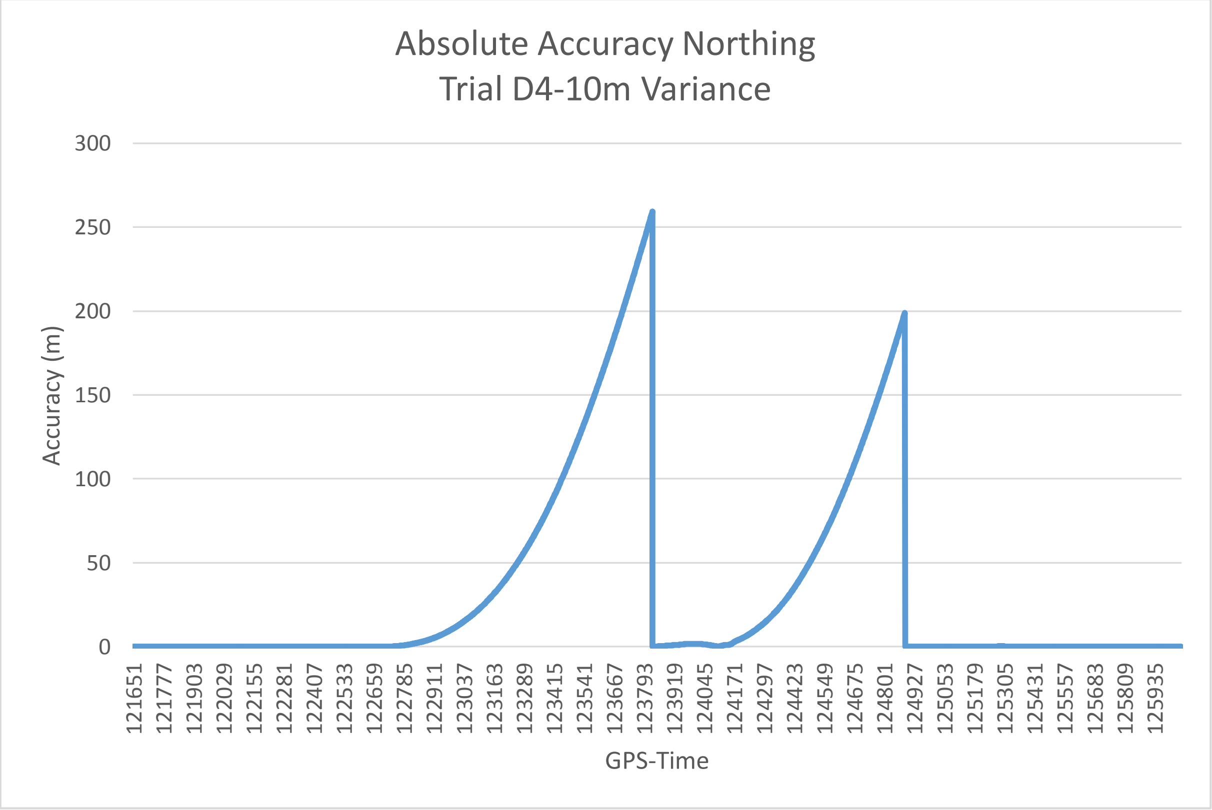
GPS-Time


Absolute Accuracy Northing
Trial D4-10m Variance
300
250
200
150
100
50
0
Accuracy (m)
121651
121777
121903
122029
122155
122281
122407
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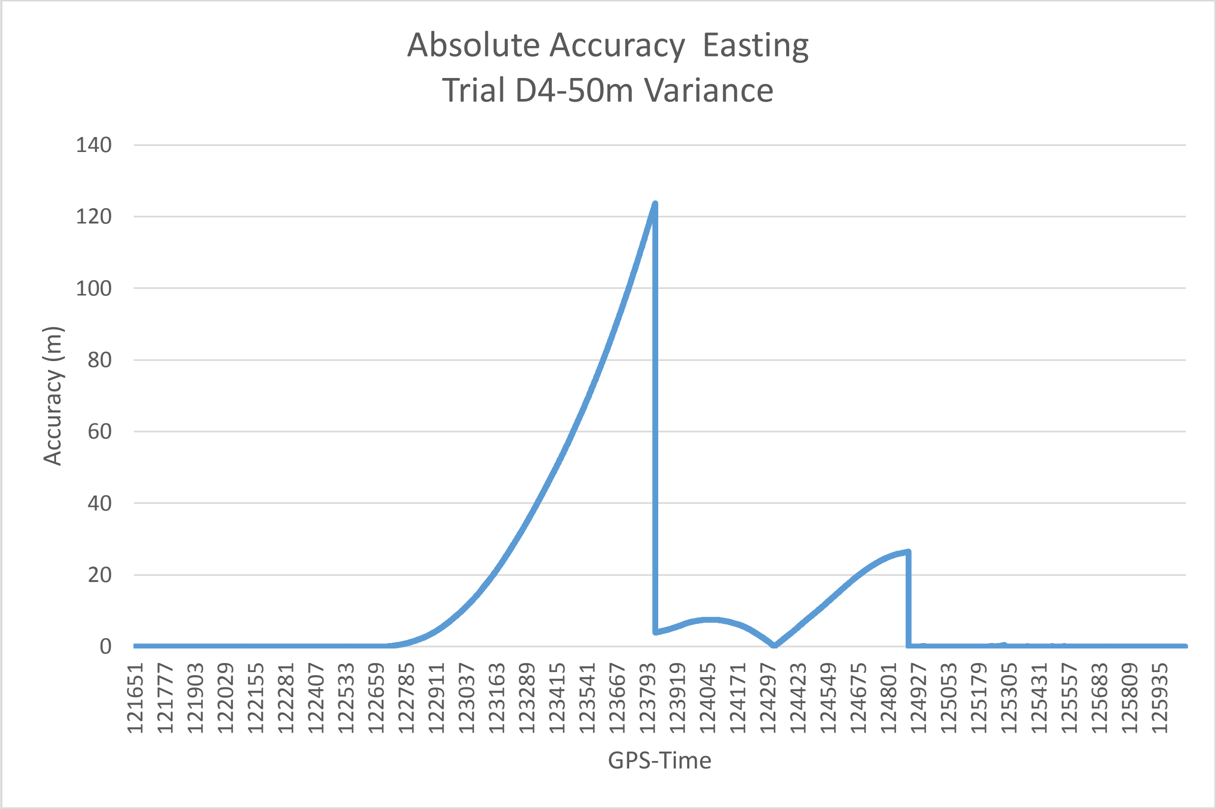
GPS-Time


Absolute Accuracy Easting
Trial D4-50m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
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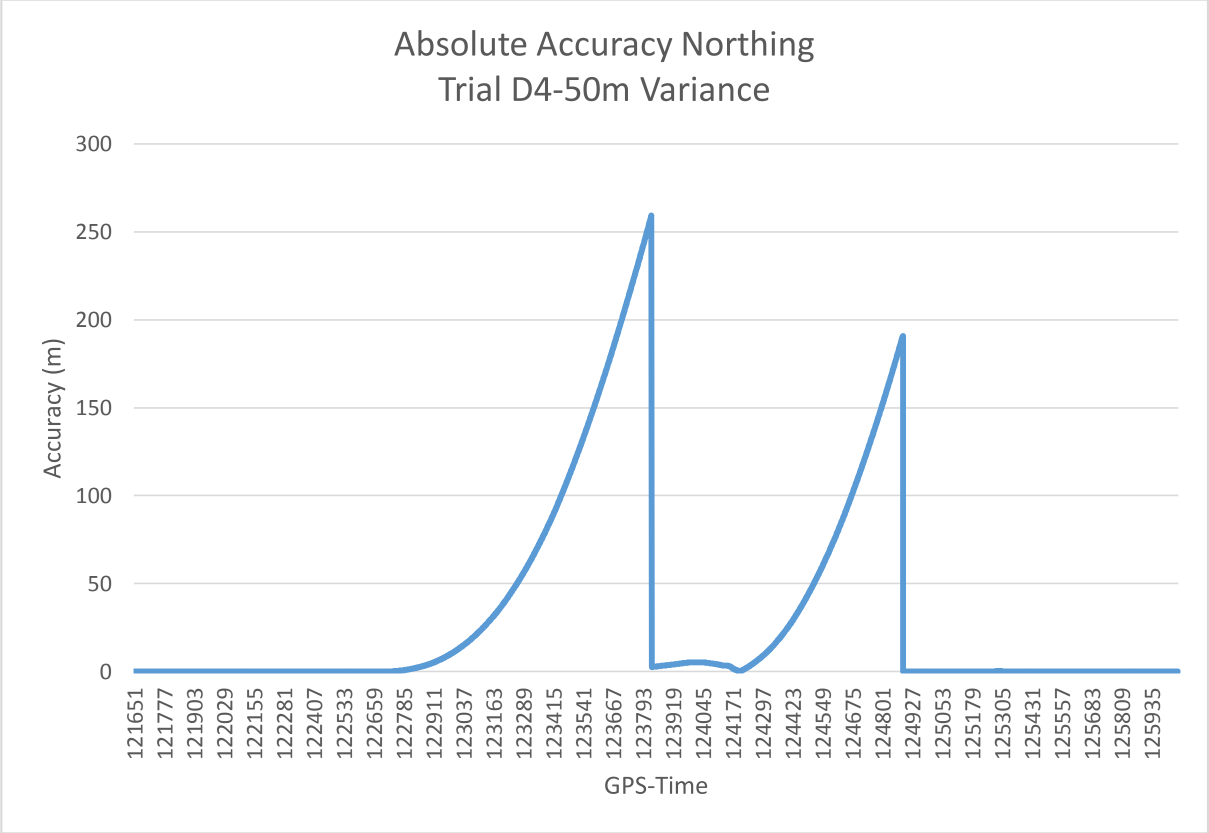
GPS-Time


Absolute Accuracy Northing
Trial D4-50m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
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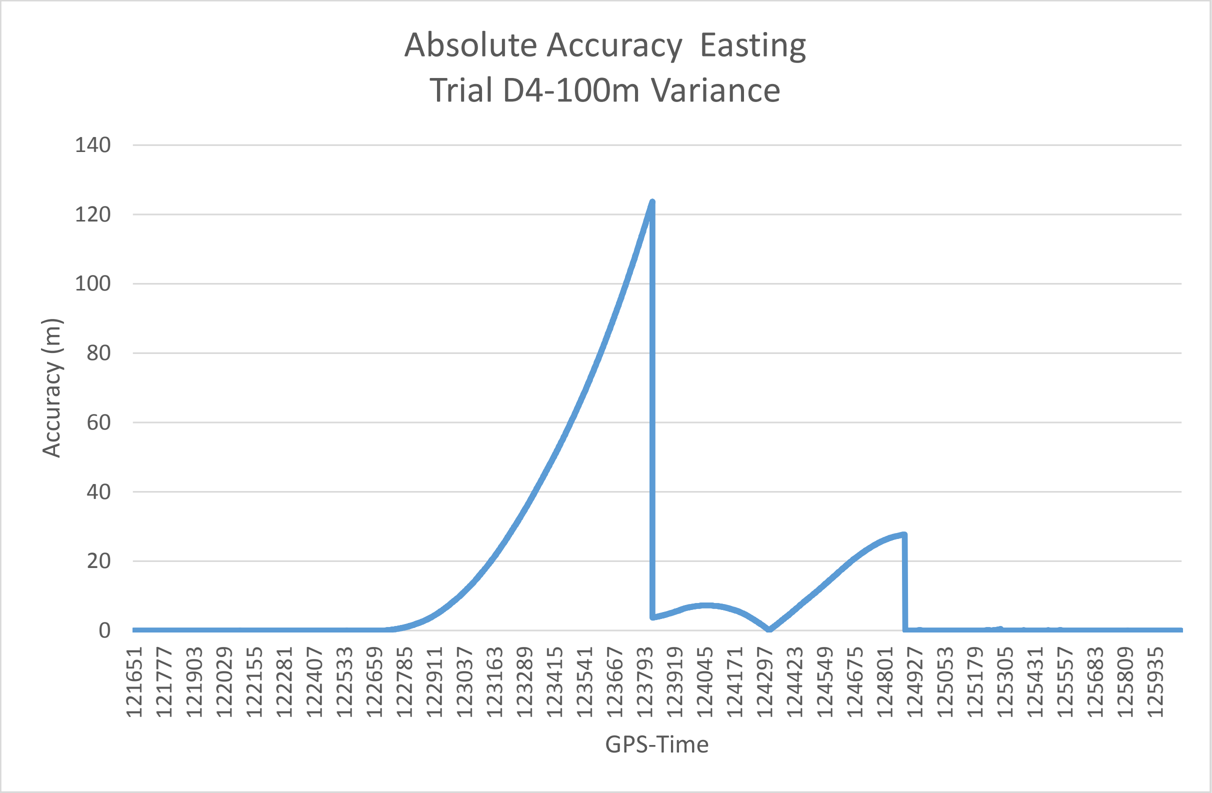
GPS-Time


Absolute Accuracy Easting
Trial D4-100m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
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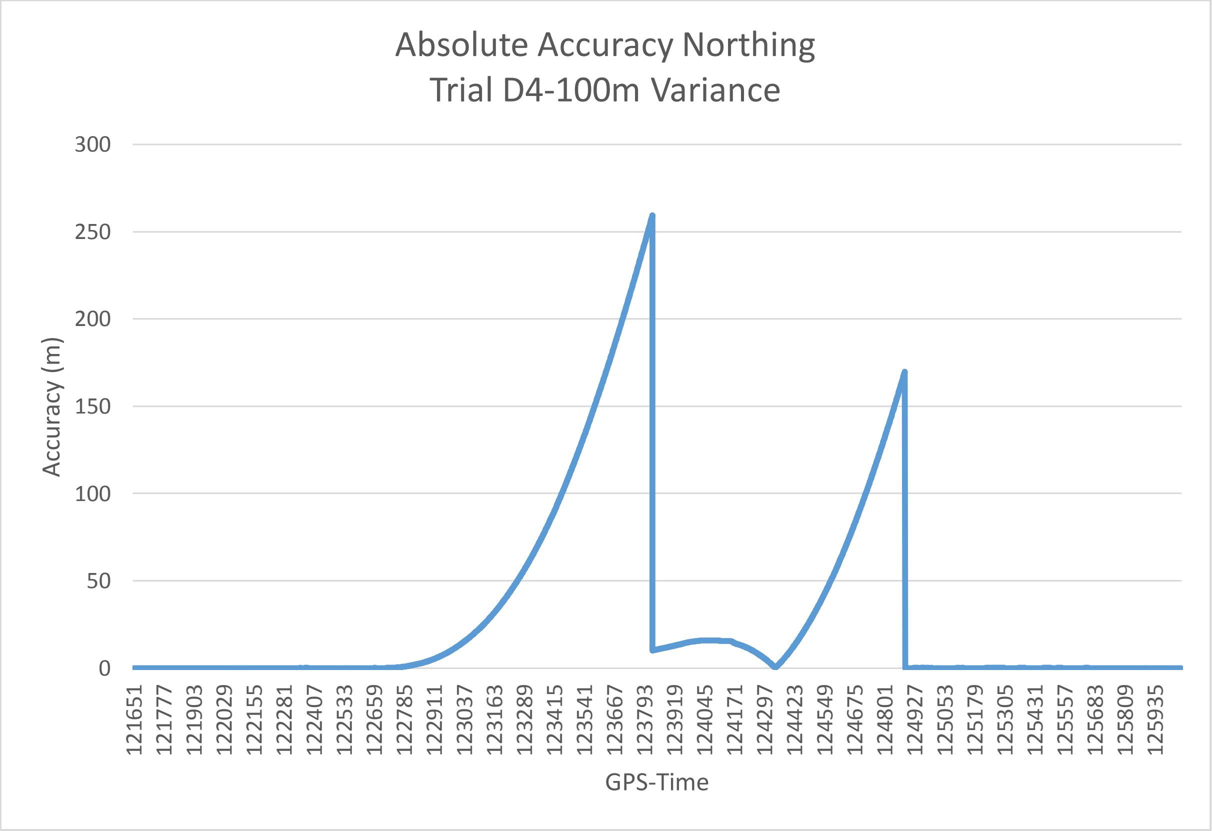
GPS-Time


Absolute Accuracy Northing
Trial D4-100m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
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121903
122029
122155
122281
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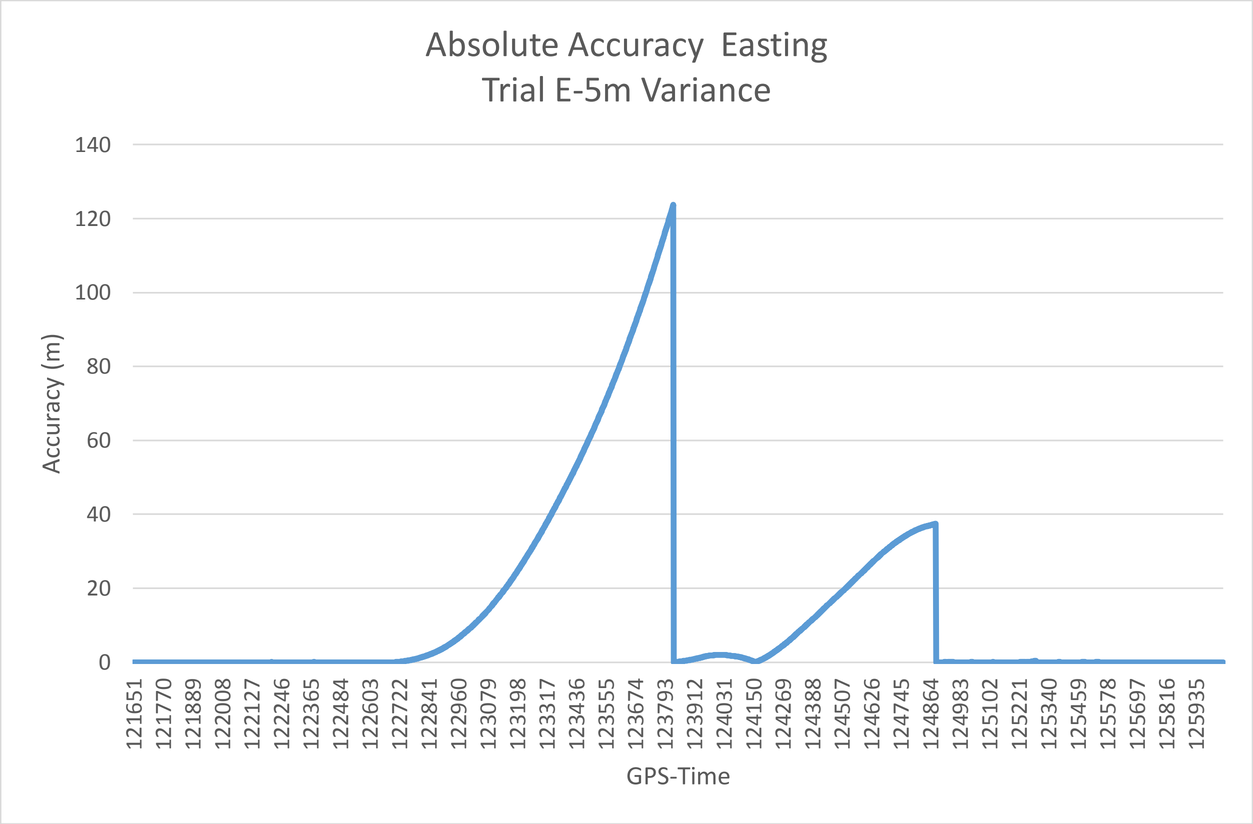
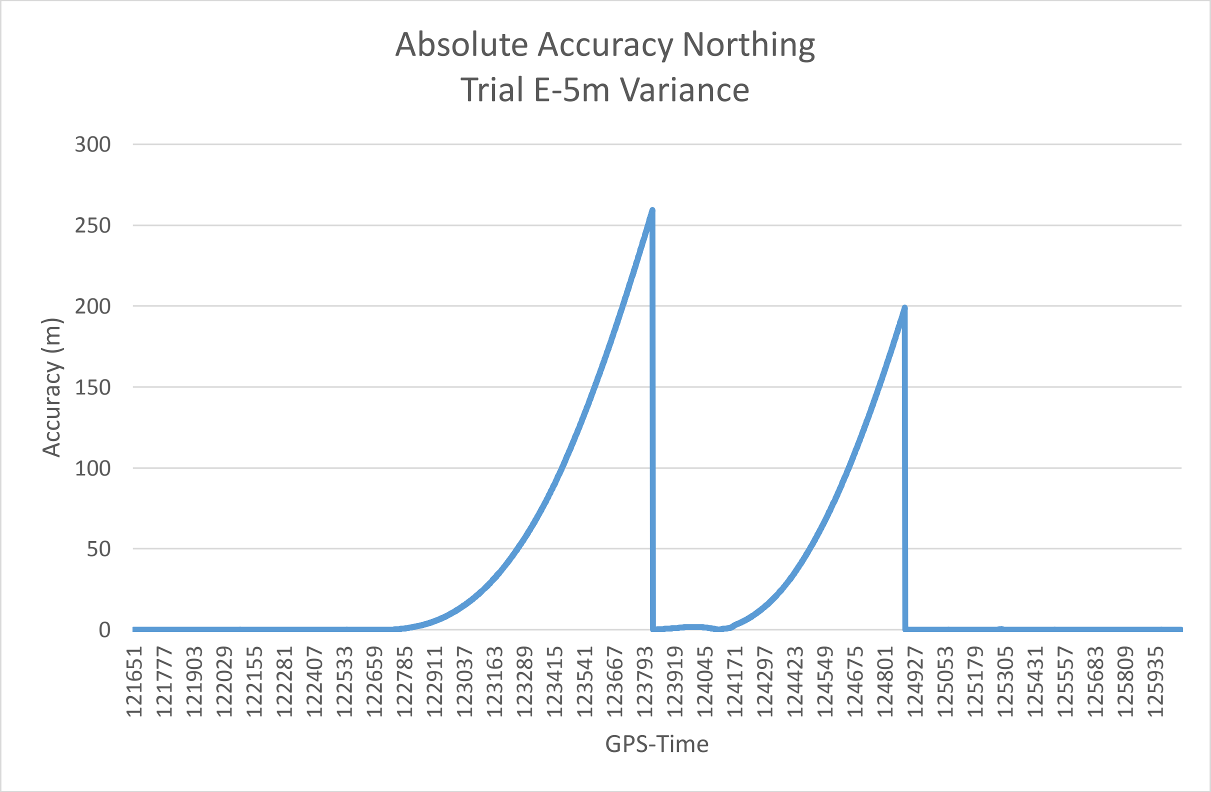
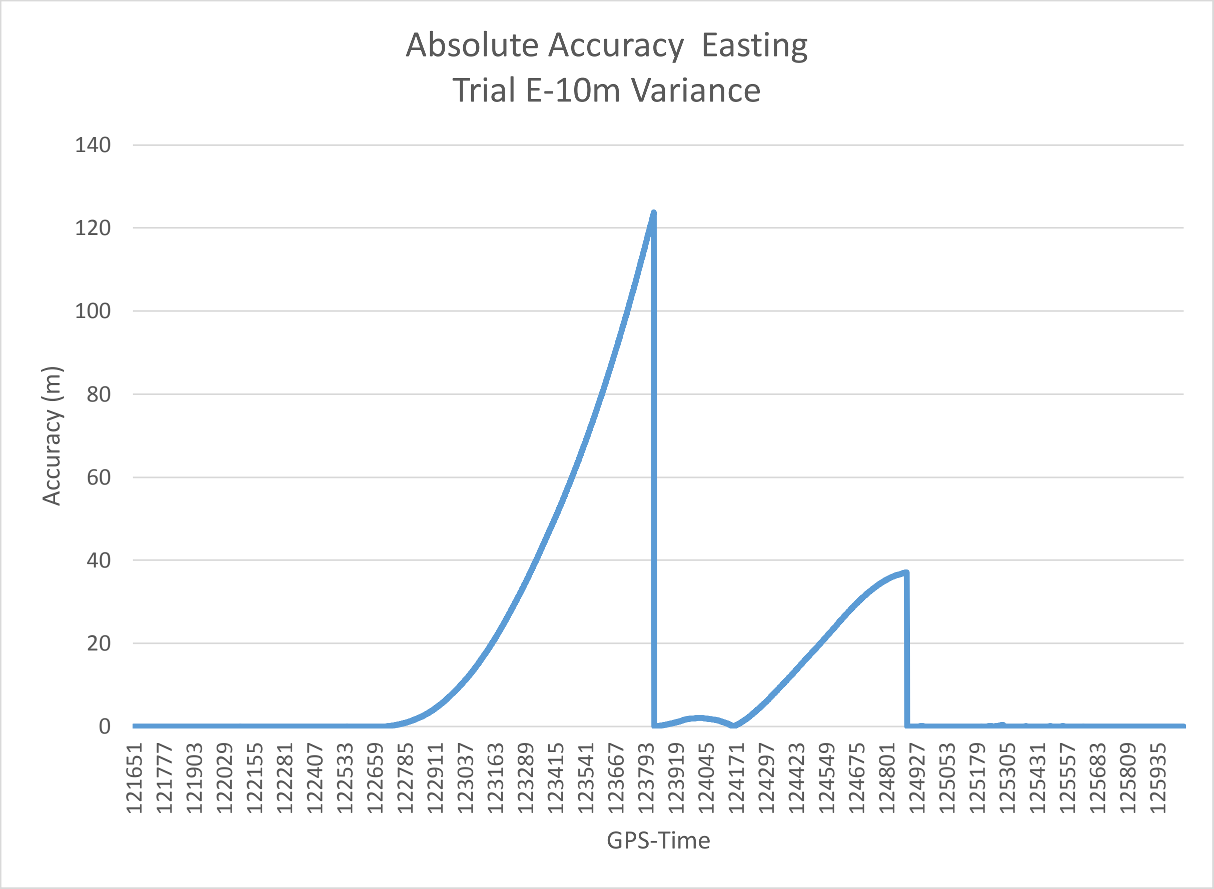
125935
GPS-Time

C18: Trial E

Manipulated Error: 0m






Absolute Accuracy Easting
Trial E-10m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
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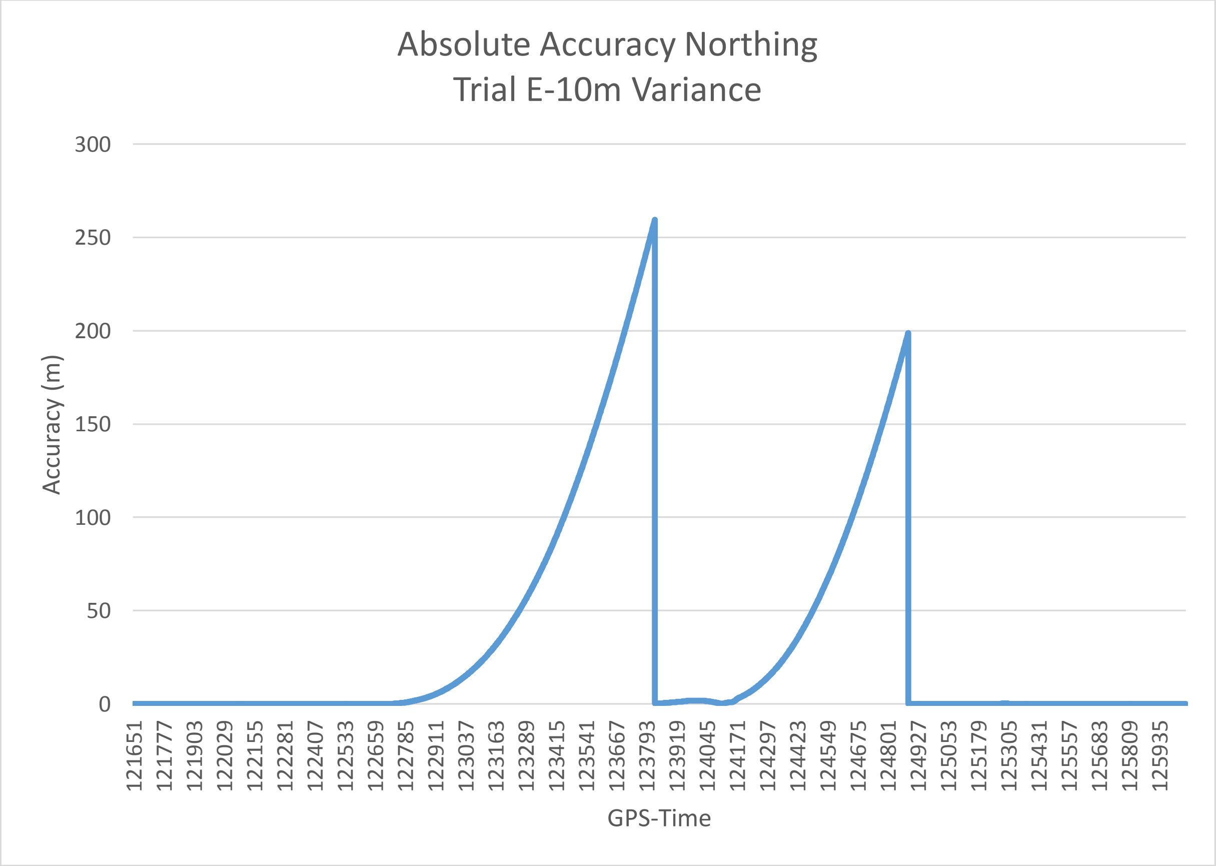
GPS-Time


Absolute Accuracy Northing
Trial E-10m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
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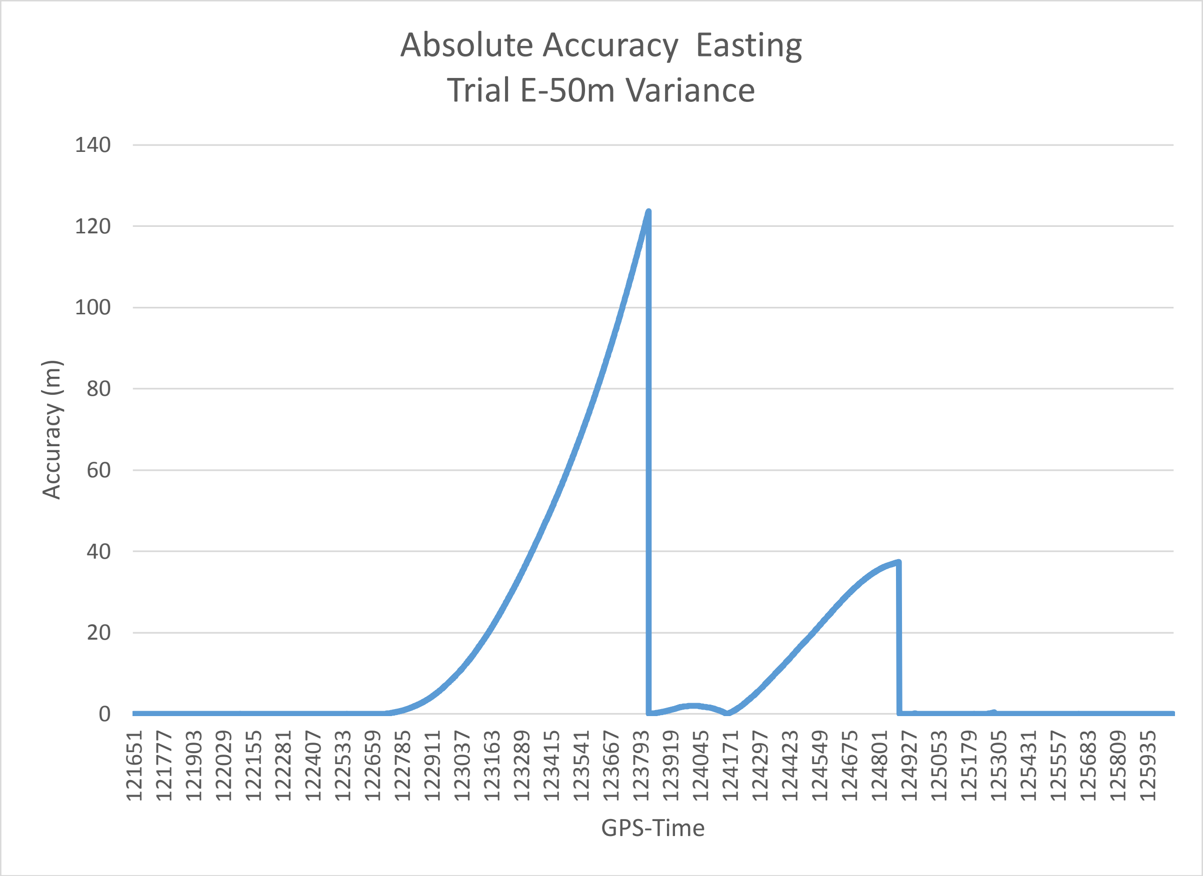
GPS-Time


Absolute Accuracy Easting
Trial E-50m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
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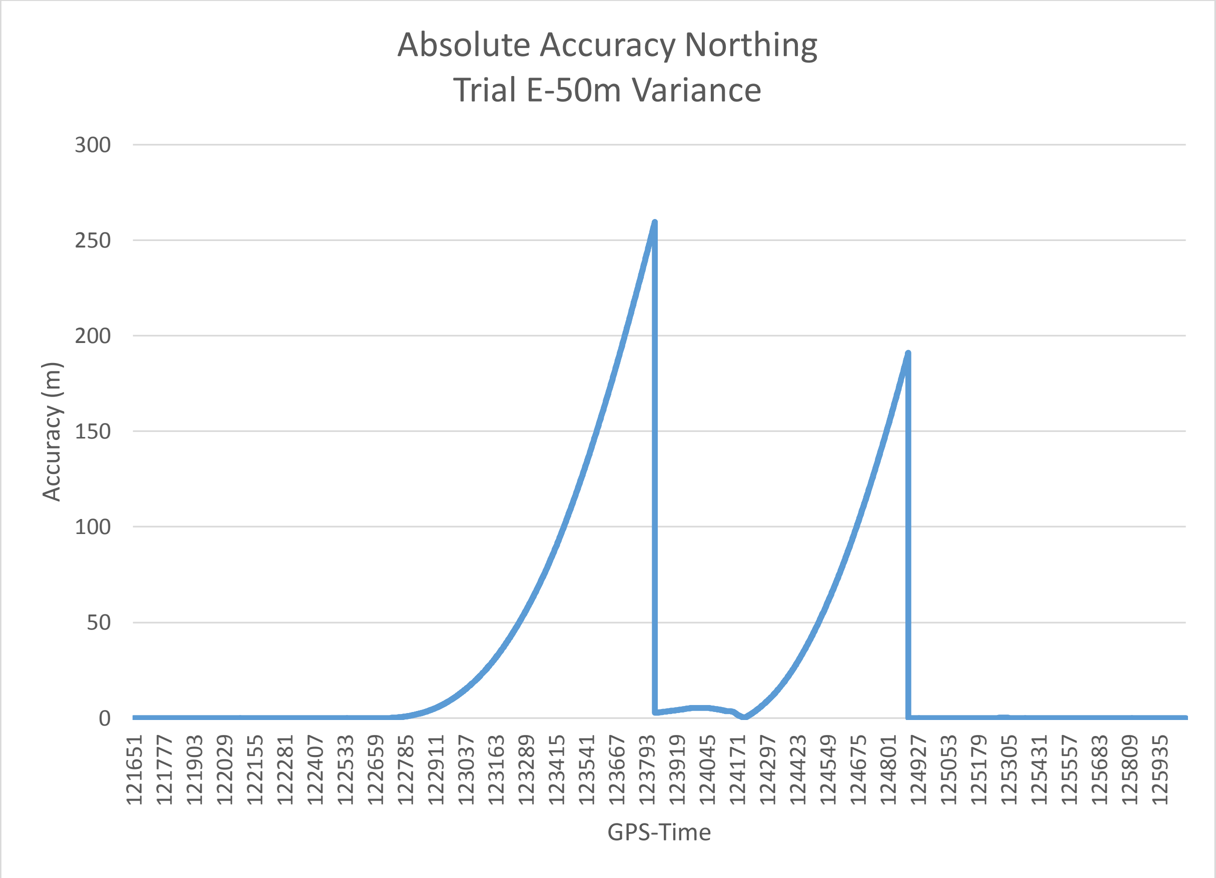
GPS-Time


Absolute Accuracy Northing
Trial E-50m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
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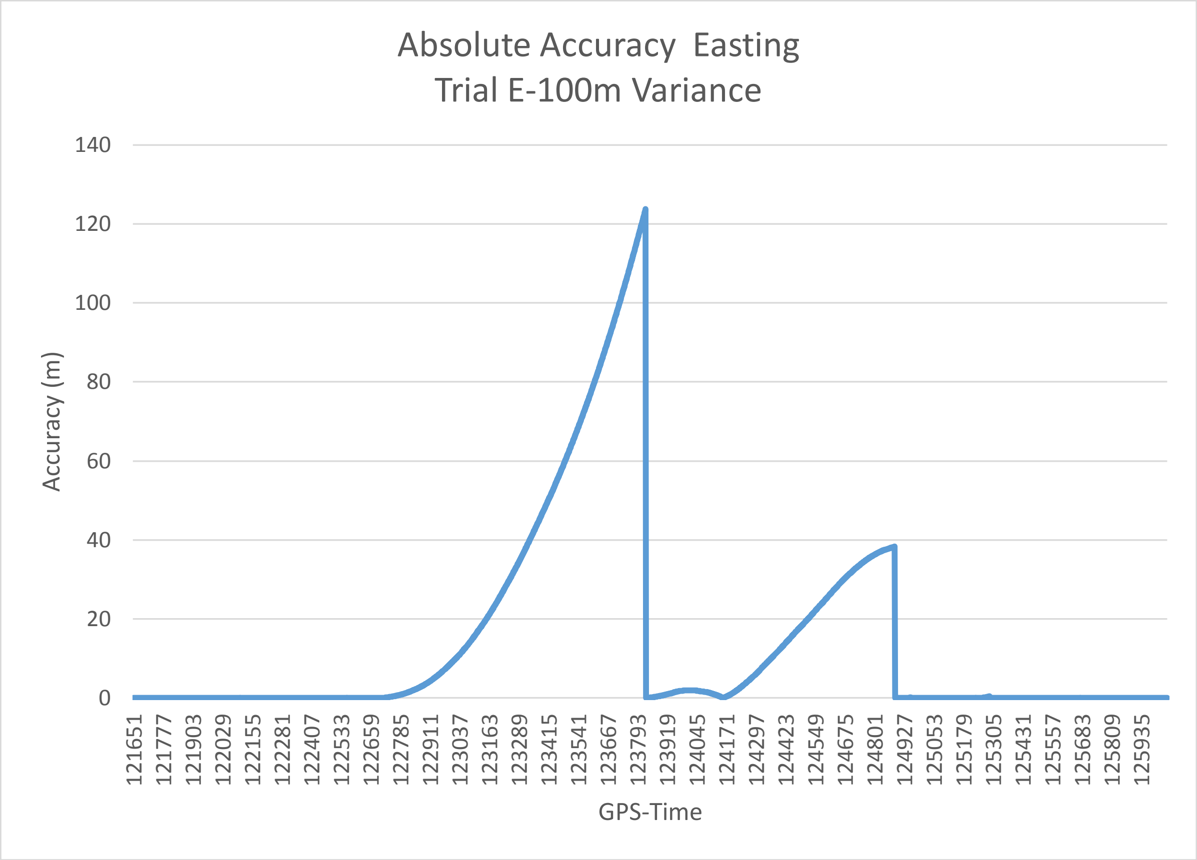
GPS-Time


Absolute Accuracy Easting
Trial E-100m Variance
Accuracy (m)
140
120
100
80
60
40
20
0
121651
121777
121903
122029
122155
122281
122407
122533
122659
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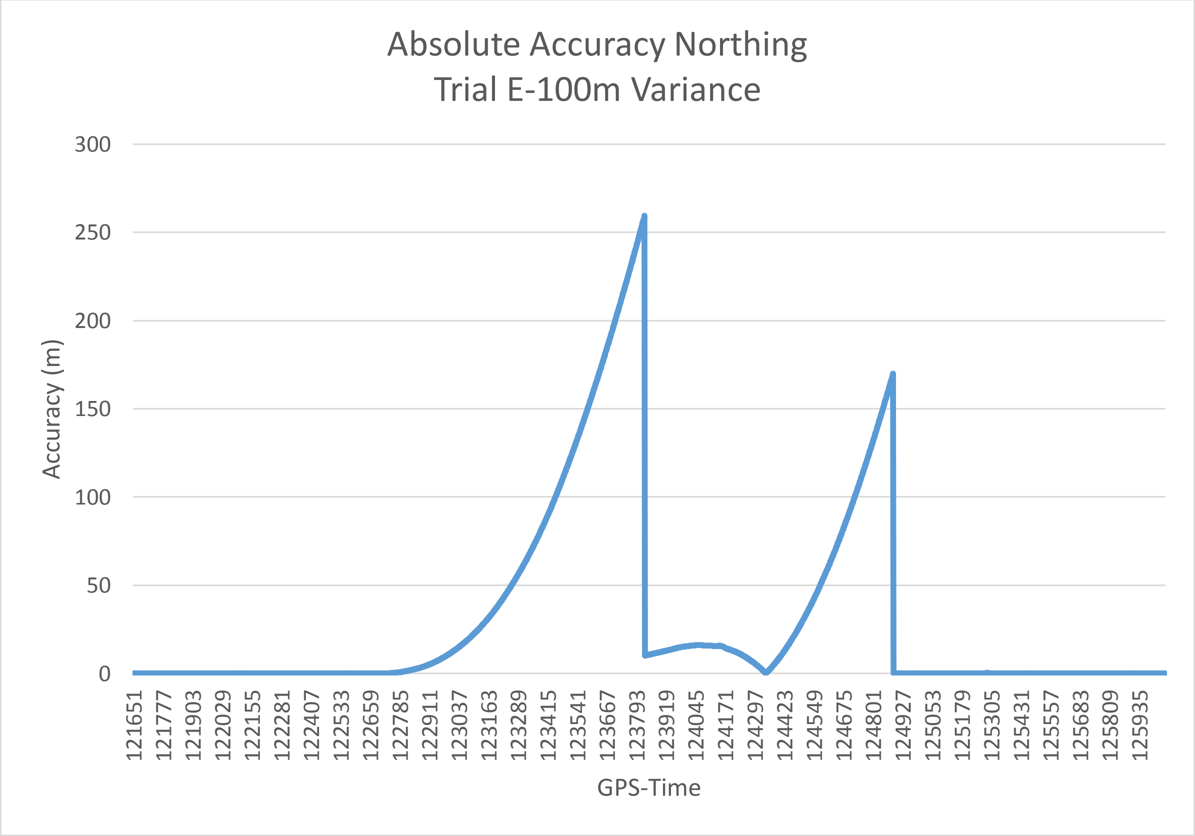
GPS-Time


Absolute Accuracy Northing
Trial E-100m Variance
Accuracy (m)
300
250
200
150
100
50
0
121651
121777
121903
122029
122155
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122407
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